LogICCC Final Conference
Berlin, 15-18 October, 2011

Logical Models of Reasoning with Vague Information

Highlights from LoMoReVI

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Plan of the Talk

Three parts:

- What has LoMoReVI been about?
  What have LoMoReVIans achieved?
- A glimpse at a particular set of results:
  Giles's dialogue game — extensions and applications
- Final remarks — mainly on interdisciplinarity

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- Reasoning:
  Mathematical and computational tools for and models of reasoning with logically complex sentences.
- Imperfect information:
  Aspects beyond vagueness and impreciseness: uncertainty and truthlikeness; triggering extensions and combinations of logics.

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Developing Mathematical Fuzzy Logic:

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Developing Mathematical Fuzzy Logic:

- P. Cintula, C. Noguera: Implicational (Semilinear) Logics I: A New Hierarchy. Archive for Mathematical Logic 49(4):417-446, 2010.
- P. Hájek: Comments on Interpretability, Decidability and Other Topics on Fuzzy Logic. J. of Logic and Computation, to appear.
- F. Montagna, C. Noguera: Arithmetical Complexity of First-Order Predicate Fuzzy Logics over Distinguished Semantics. Journal of Logic and Computation 20(2): 399-424, 2010.
- M. Baaz, A. Ciabatoni, C. Fermüller: Theorem Proving for Prenex Gödel Logic with Δ: Checking Validity and Unsatisfiability. Submitted [extends two earlier conference papers].
- Handbook of Mathematical Fuzzy Logic
  P. Cintula, P. Hájek, C. Noguera (Eds.)
  Two-volume set summarizes the state of the art of the area;
  11 chapters – 5 of them (co)authored by LoMoReVIans. To appear.

Selected results (ctd.)

Combining and Extending (Fuzzy) Logics :

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Combining and Extending (Fuzzy) Logics :

- M. Cerami, F. Esteva, F. Bou: Decidability of a Description Logic over infinite-valued Product Logic.
  KR 2010, AAAI Press, pp. 203-213, 2010.
- P. Hájek: On the Fuzzy Modal Logic S5(C).
  Fuzzy Sets and Systems 161(18):2389-2396, 2010.
- F. Esteva, L. Godo, R. Rodriguez, T. Vetterlein:
  Logics for approximate and strong entailment. Fuzzy Sets and Systems, to appear.
- F. Bou, M. Cerami, F. Esteva: Finite-Valued Lukasiewicz Modal Logic Is PSPACE-Complete. Proc. of IJCAI 2011, 774-779, 2011.
- C. Fermüller: Revisiting Giles's Game – Reconciling Fuzzy Logic and Supervaluation. In "Games: Unifying Logic, Language, and Philosophy," O. Majer, T. Tulenheimo, A. Pietarinen (eds.), pp. 209 - 227, Springer, 2009.

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Applications to Reasoning with Imperfect Information:

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Applications to Reasoning with Imperfect Information:

- T. Flaminio; L. Godo; E. Marchioni: Belief Functions on MV-algebras of Fuzzy Events Based on Fuzzy Evidence. Proceedings of ECSQARU 2011, Belfast, UK, Weiru Liu (eds.), LNAI 6717, pp. 628-639.
- T. Flaminio, L. Godo, E. Marchioni: On the Logical Formalization of Possibilistic Counterparts of States over n-Valued Lukasiewicz Events. Journal of Logic and Computation 21(3), 429-446, 2011.
- L. Běhounek, O. Majer: A Semantics for Counterfactuals Based on Formal Fuzzy Logic. In M. Peliš, V. Punčochář (eds.): The Logica Yearbook 2010, London, College Publications, pp. 25-41, 2011.
- M. Bílková, O. Majer, M. Peliš, G. Restall: Relevant Agents. In L. Beklemishev, V. Goranko, V. Shehtman (eds.): Advances in Modal Logic, London, 2010, pp. 22–38.
- C. Fermüller, C. Roschger. Bridges Between Contextual Linguistic Models of Vagueness and T-norm Based Fuzzy Logic. Proc. of the 8th Workshop on Uncertainty Processing. T. Kroupa, J. Vejnarova (eds.), 2009, pp. 69-78.

The "LoMoReVI Volume"

Starting point:

LoMoReVI Conference

Čejkovice, Czech Republic

14-17 September 2009

(First official LogICCC Inter-CRP Activity!)

Invited Speakers:

Didier Dubois, Peter Gärdenfors, Nicholas J.J. Smith,
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"Understanding Vagueness —
Logical, Philosophical, and Linguistic Perspectives"

P. Cintula, C. Fermüller, L. Godo, P. Hájek (eds):

- 15 papers on vagueness across all relevant areas:
  philosophy, linguistics, logics, computer science, mathematics
- special feature:
  comments and replies to papers – also cross-disciplinary
- to be published soon by College Publications

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Let **X**/**Y** stand for me/you or for you/me

X asserts	'attack' by **Y**	answer by **X**
$A \supset B$	A	B
$A \vee B$	'?'	A or B (**X** chooses)
$A \wedge B$	'l?' or 'r?' (**Y** chooses)	A or B (accordingly)
$A \,\&\, B$	'?'	A and B

Note: $\neg A$ abbreviates $A \supset \bot$

Answer $\bot$ ('quit') is allowed!

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Dialogue states: $[A_1, \ldots, A_m \| B_1, \ldots, B_n]$

To obtain a logic we additionally need

- winning conditions for atomic states
- regulations defining admissible runs of a game

ad: winning conditions

Giles's idea:

Let the players bet on the truth of their (atomic) claims!

(Yes/no-)experiments — that may be dispersive — decide.

- I pay 1€ to you for each of my false atomic assertions, if you agree to do the same for your atomic assertions

A final states $[p_1, \ldots, p_m \| q_1, \ldots, q_n]$ results in a pay-off of

$$\Big(\sum_{i=1}^{m} \langle p_i \rangle - \sum_{j=1}^{n} \langle q_j \rangle \Big) € \quad \text{for me}$$

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ad: regulations

Constraints on runs of a dialogue like the following suffice:

($R_{\supset}$) if you attack my assertion of $A \supset B$ by claiming A, then I have to assert also B at some state

No particular regulation for the order of moves is required!

Giles's Theorem (extended to Ł-consequence):

$F_1, \ldots, F_n \models_{Ł} G$ iff for every risk value assignment I have a strategy for avoiding expected loss in dialogues starting with my assertion of G and your's of $F_1, \ldots, F_n$.

$F_1, \ldots, F_n \models_{Ł} G \ldots v(F_1) *_{Ł} \ldots *_{Ł} v(F_n) \leq v(G)$ for all v

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Beyond Łukasiewicz logic:

- variants of the game to **G**, **P**, and **CHL**
- closer to the original spirit:
  "From Games to Truth Functions:
  A Generalization of Giles's Game"
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  (very recent work by Christoph Roschger and CF investigating general conditions on dialogue rules and payoff functions, sufficient for extracting a truth functional semantics)
- Online tool in preparation . . . (contact *Christoph Roschger*)

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- The various research fields (often) have very different aims and methodology. It were inadequate to try to melt these into one "science of vagueness"!
- To look into other disciplines is fascinating not in-spite-of, but rather because of major differences. Pondering upon the aims and limits of one's own field can be rewarding!

Trans-disciplinarity:

- Examples of trans-disciplinary work on vagueness:
  - linguists using concepts from ToV, like supervaluation
  - philosopher's interested in empirical findings
  - psychologists looking at FL (Hersch/Caramazza[1])
  - logicians studying logics arising from ToV concepts

[1] A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language. Journal of Experimental Psychology 105(3):254-276, 1976

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- "Local trans-disciplinarity of mathematical logic itself":
  take your favorite logic and study
  – (algebraic, standard, categorial, game . . .) semantics
  – proof theory
  – computational aspects
  – embeddings/translations to and from other logics
  – . . .

LoMoReVI has been trans-disciplinary in both senses!

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