The Human Understanding of Conditionals

Gernot D. Kleiter

Department of Psychology
University of Salzburg, Austria
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Motivation: Conditionals are special – in logic

- Notoriously more complex than AND or OR,
- Russel & Whitehead: Material implication $p \supset q$ is short hand for $\neg p \vee q$
- Axiom scheme of classical PC is written in terms of $\rightarrow$
- Fundamental relationship to inference and the consequence relation: Deduction Theorem

$$\Gamma, A \models B \quad \text{iff} \quad \Gamma \models A \rightarrow B$$

- Adams, Edgington, Bennet, van Benthem, Eells, Gauker, Lewis, Rescher, Skyrms, Stalnaker, ...
- Indicative, deontic, causal, counterfactual, hypothetical, suppositional, nonmonotonic reasoning

Motivation: Conditionals are special – in psychology

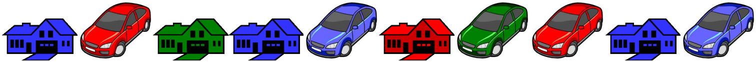
- Confirmation bias—Wason Task
- Matching—argument forms like MP or MT; Evans
- Mental models—categorical syllogims, Johnson-Laird
- Dual process theory—Evans
- Probabilistic approach—Oaksford, Chater, Over
- Mental probability logic—SYSTEM P, Pfeifer & Kleiter

Results from 4 Studies ($N > 370$)

- Understanding of IF—THEN
  - Clear majority: Conditional event
  - Small minority: Conjunction
  - Small minority: Unstable
  - But no: Material implication or biconditional
- Connections to
  - Working memory
  - Developmental hypothesis
  - Wason Task
  - A new probabilistic inference tasks

Experimental Paradigm—Task

Here you see ten cards showing houses and cars. They are red, blue, or green.



The cards are put in one pile, thoroughly shuffled, and

one card is drawn randomly.

How sure can you be that the following sentence holds?

If the card shows a car, then the card shows blue

- Humans understand conditionals as conditional events:
- If the antecedent is false, then the conditional is void
- The human interpretation of conditionals is not truth-functional, cannot be expressed by any combination of $\wedge, \vee, \neg$
- One card i, $i \in \{1, 2, \ldots, 10\}$ is drawn randomly—after thourough shuffling, exchangeability, each card has the same chances (Laplace),
- $P(\text{Car}_i \rightarrow \text{Blue}_i)$
- $P(\text{Blue}_i \mid \text{Car}_i, \text{exchangeability}) = \dfrac{|\text{Blue cards} \cap \text{Car cards}|}{|\text{Car cards}|}$
- Model of a data generating process
- Probability judgments are used as a vehicle to infer the interpretation

Dice Task

- Dice with six sides, red and blue, $\bigcirc$ and $\square$
- 71 items, $n = 65$ students, individual testing, 32 female, 33 make, entity/feature
- Convergence to conditional event interpretation during the course of items $1, 2, \ldots 71$
- Still: Few participants giving conjunction responses
- Practically no material implication or biconditionals

Fugard, Pfeifer, Mayerhofer, Kleiter, 2011, JEP

Card task

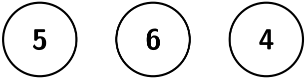
- 52 items, unique classification, thematic objects (cars houses, fishes)
- 80 participants, 40 male, 40 female, 40 object first, 40 feature first
- Computer controlled individual sessions, response times, payed
- Working memory, n-back task

Fugard, Pfeifer, Mayerhofer, Kleiter, unpublished project report

Working Memory: n-back Task

5 6

Working Memory: n-back Task



Working Memory: n-back Task

5 6 4 6

Working Memory: n-back Task

Working Memory: n-back Task

5 6 4 6 4 3

Working Memory: n-back Task

5 6 4 6 4 3 3

Working Memory: n-back Task

5 6 4 6 4 3 3 4

Correct —

Working Memory: n-back Task

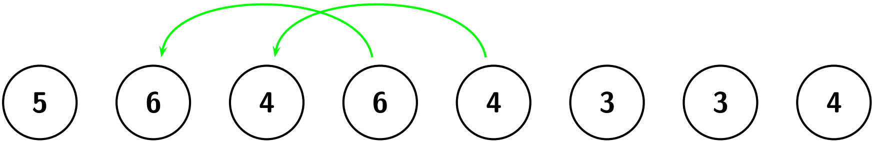
5 6 4 6 4 3 3 4

Correct — **NO**

Working Memory: n-back Task

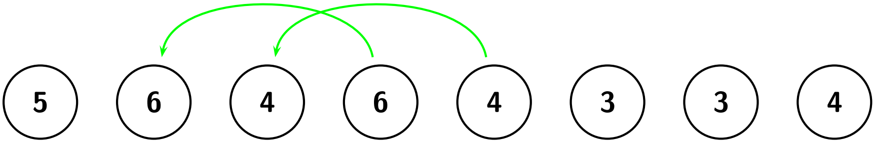
5	6	4	6	4	3	3	4
Correct	—	NO	YES				

Working Memory: n-back Task



Correct — **NO** **YES** **YES**

Working Memory: n-back Task

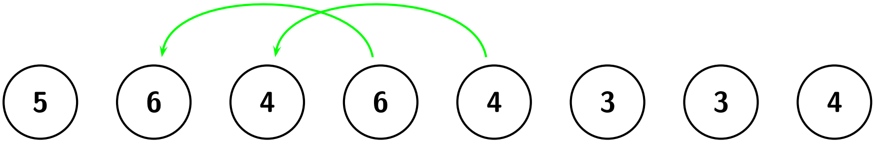


Correct	—	NO	YES	YES	NO

Working Memory: n-back Task

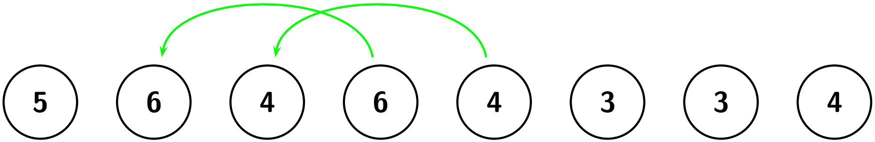
	5	6	4	6	4	3	3	4
Correct	—		NO	YES	YES	NO	NO	

Working Memory: n-back Task



Correct	—	NO	YES	YES	NO	NO	NO

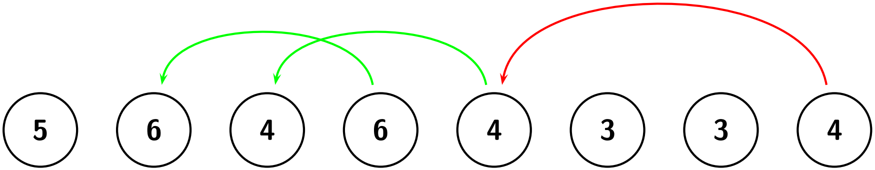
Working Memory: n-back Task



Correct	—	NO	YES	YES	NO	NO	NO

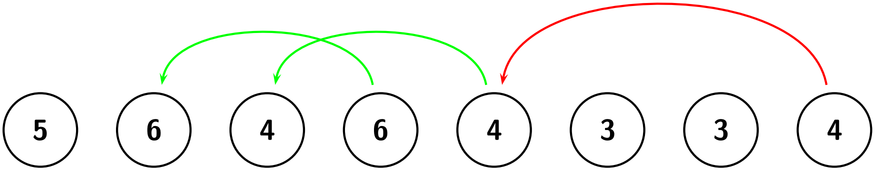
3 back lure

Working Memory: n-back Task



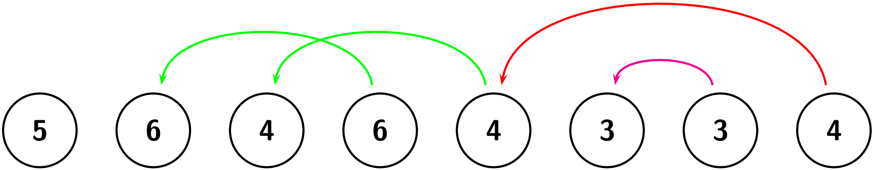
Correct	—	NO	YES	YES	NO	NO	NO
3 back lure							YES

Working Memory: n-back Task



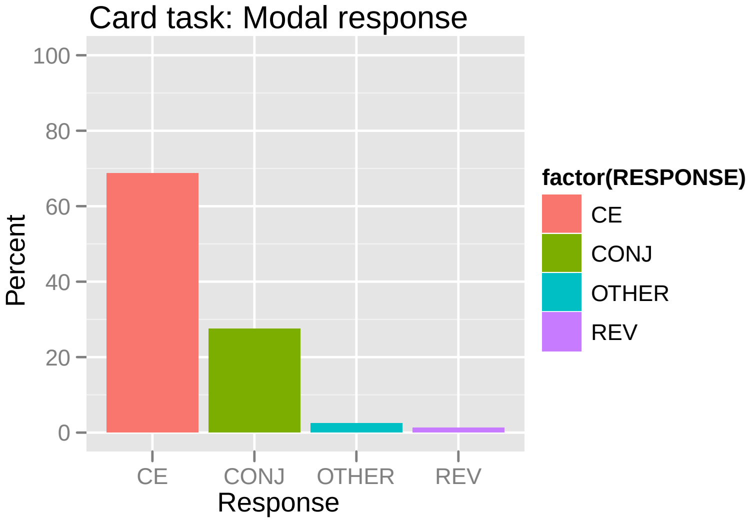
Correct	—	NO	YES	YES	NO	NO	NO
3 back lure							YES
1 back lure							

Working Memory: n-back Task

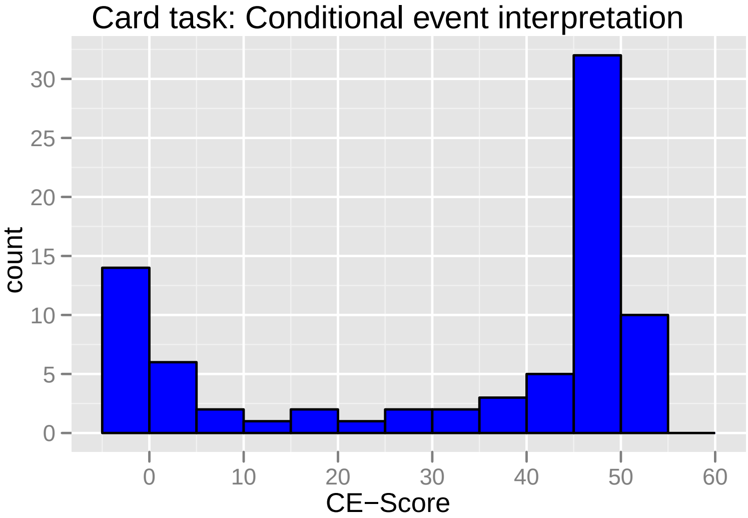


Correct	—	**NO**	**YES**	**YES**	**NO**	**NO**	**NO**
3 back lure							**YES**
1 back lure						**YES**	

Modal response (card task)

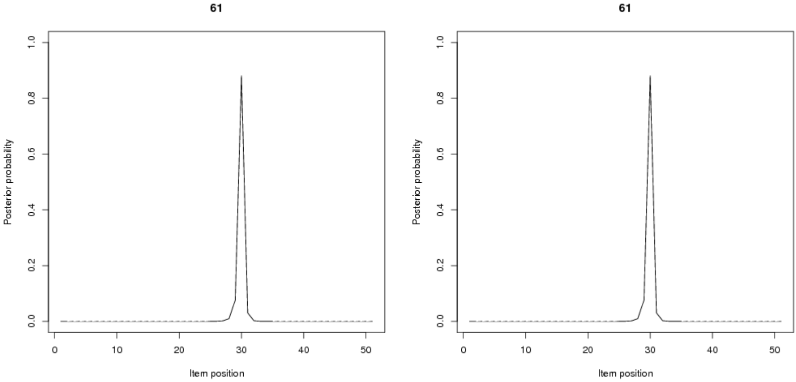


Conditional events, histogram (card task)

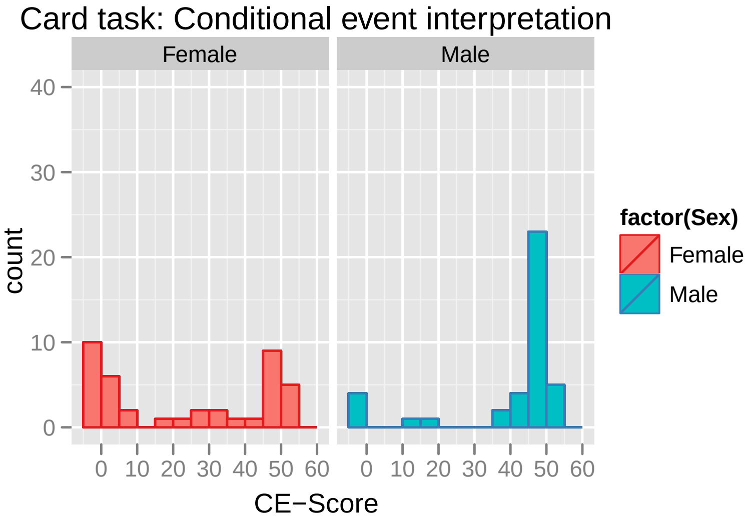


Bayesian change point analysis

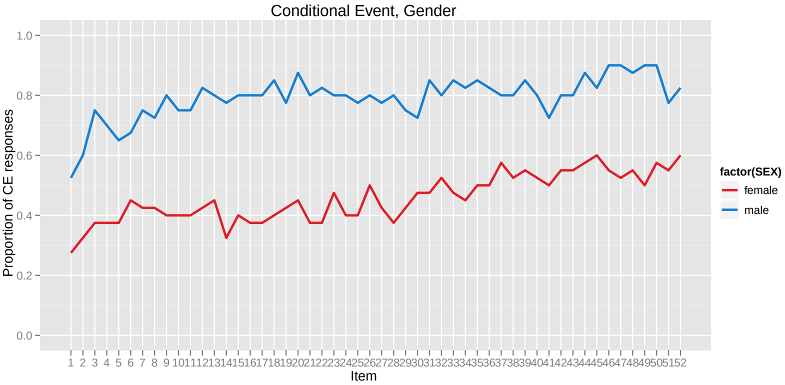
Changing the interpretation from any to conditional event interpretation at one item in the series of 52 items.
Posterior distribution (uniform prior)



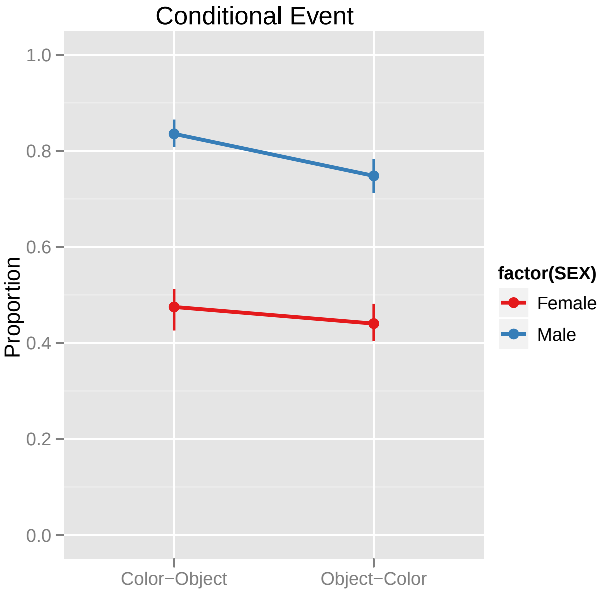
Conditional events, histogram, gender (card task)



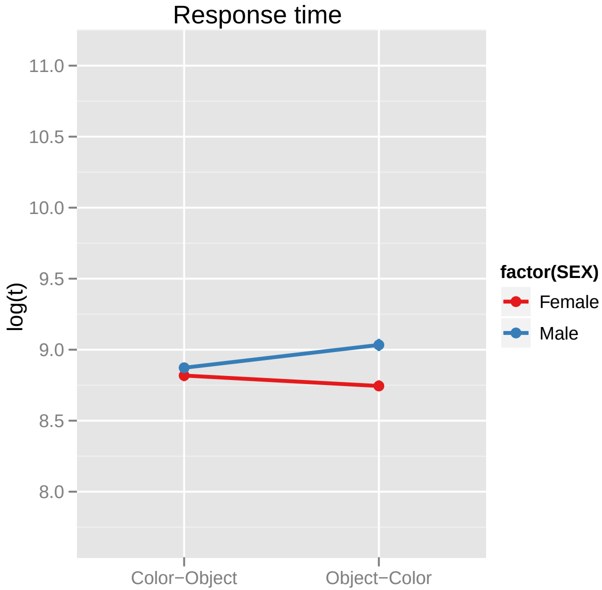
Card task – Conditional Event and Gender



Card task – Conditional Event, Gender and Entity/Feature



Card task


Response time
11.0
10.5
10.0
9.5
9.0
8.5
8.0
log(t)
Color−Object
Object−Color
factor(SEX)
Female
Male

Card task and working memory (n-back)

- No correlation between the interpretation of conditionals and n-back performance—with one exception:
- Lure-3 correlates with conjunction responses, $r = .30$
- Interpretation of natural language conditionals does not require high working memory load.
- Conditionals require a serial Representation, conjunctions are cummutative

Rating scales

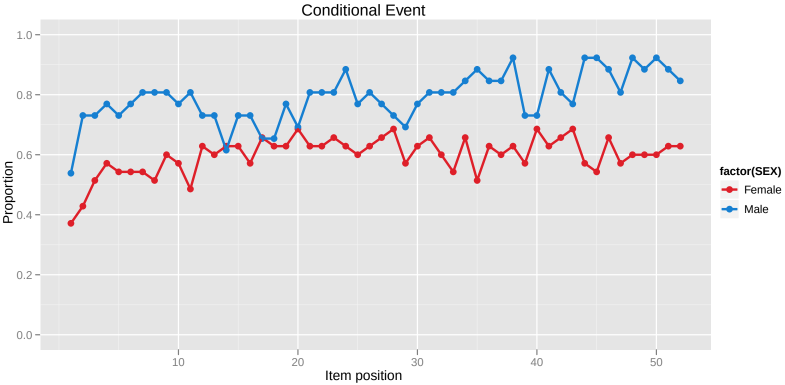
- r(confidence of being correct, number of CE-responses) $= .47$, speaks for the competence model.
- Female participants are slightly less confident, $r = -.27$

Cognitive Developmental

- "Noise" $\longrightarrow$ conjunction $\longrightarrow$ biconditional $\longrightarrow$ material implication (Barrouillet, Gauffroy & Lecas, 2008; Gauffroy & Barrouillet, 2009)
- $8 \longrightarrow 12 \longrightarrow 15 \longrightarrow 22$
- Biconditional with material implication: $A \supset B \quad \wedge \quad B \supset A$
- With conditional event $(B|A) \;\wedge\; (A|B)$ becomes $(A \wedge B) \mid (A \vee B)$
- $$\frac{P(11)}{P(11) + P(10) + P(01)}$$

Conditonal Event and Gender

- Weak trend
- Males give more conditional event responses

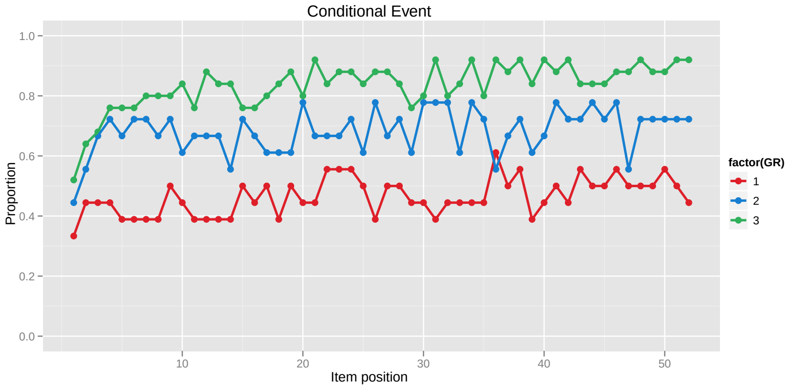


Re-analysis of the Dice Task

- 71 items, 65 Ss, 32 female, 33 male participants
- Male participants give more conditional event responses
- Male participants give more conjunction responses !
- Female participants give more "Other" responses, i.e., neither conditional event and nor conjunction
- "Other" responses take more time
- Gender $\times$ Entity/Feature interaction: Color first facilitates CE responses in female participants
- Males are more systematic (conditional event and conjunction), tend to settle on a fixed strategy. Females do not so easily stick to a rule.
  Males are more confident of being right and tend to use rules.
- In the literature there are no studies using a long series of similar tasks. Thus no data on strategies are available.

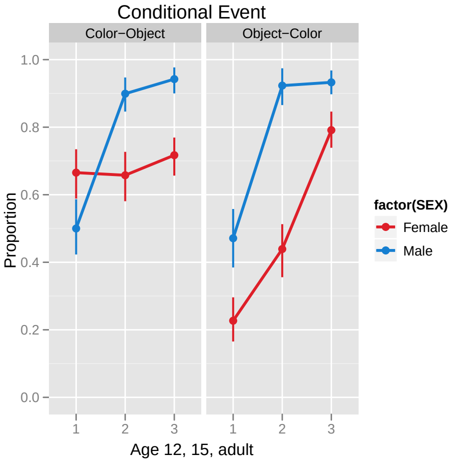
Conditonal Event and Age

- Age groups



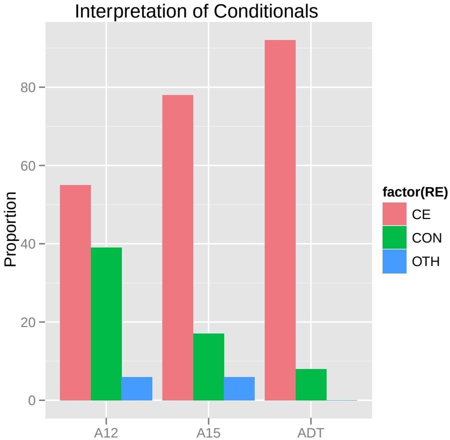
Conditonal Event: Age, gender & object-color

- Convergence to CE with increasing age
- Color-first facilitates CE in girls



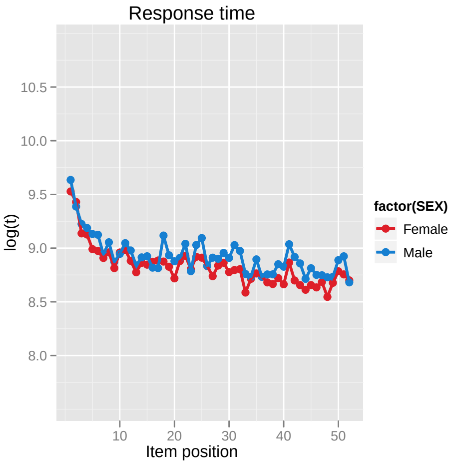
Age: conditonal event, conjunction, and rest

- Increasing conditional events, decreasing conjunctions
- Even 12 years old give many conditional event responses
- Practically no biconditionals, 22 biconditionals out of 3172 responses
- Practically no material implications, 8 out of 3172 responses

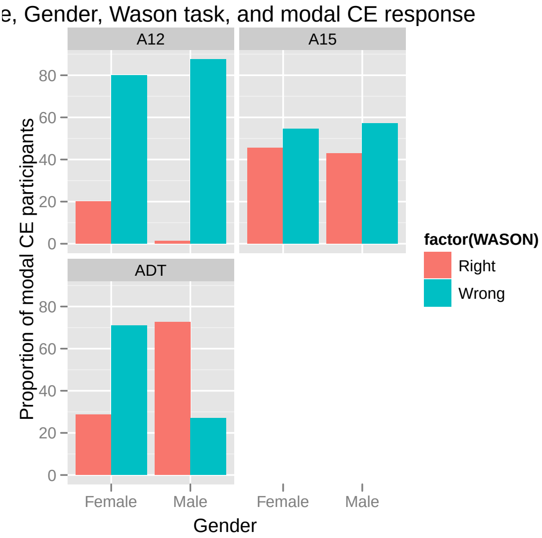


Response time

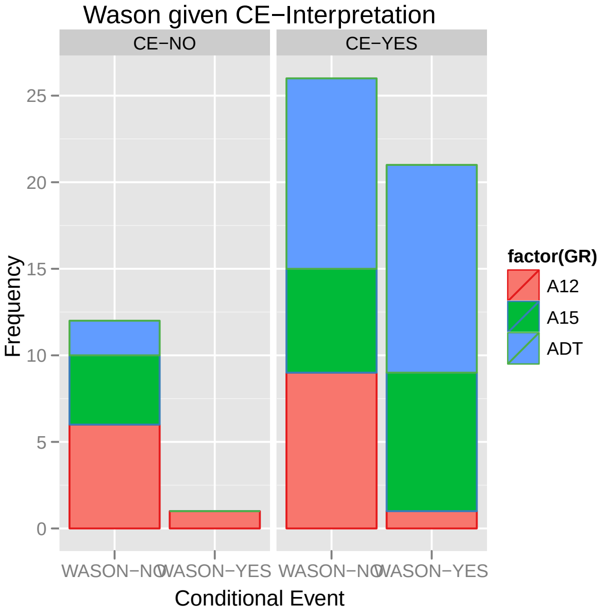
- Women are faster
- Weak trend



Wason task


e, Gender, Wason task, and modal CE response
A12
A15
ADT
Proportion of modal CE participants
0
20
40
60
80
Female
Male
Female
Male
Gender
factor(WASON)
Right
Wrong

Wason task



Wason given CE−Interpretation
CE−NO
CE−YES
Frequency
25
20
15
10
5
0
WASON−NO
WASON−YES
WASON−NO
WASON−YES
Conditional Event
factor(GR)
A12
A15
ADT

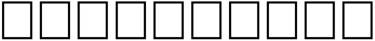
Probabilistic inference task

Here is a deck with 20 cards.

12 cards are red, 8 cards are blue.

You shuffle the cards.

You take the first 10 cards and do the following, card by card:



If the card is red, then you paint a flower on the card.

Now you shuffle all 20 cards again and put them on one deck.

You randomly draw one card.

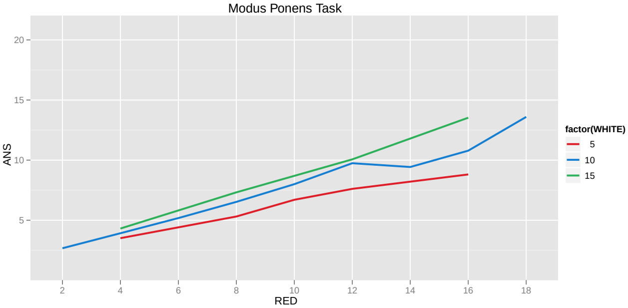
How confident are you, that the card shows a flower?

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

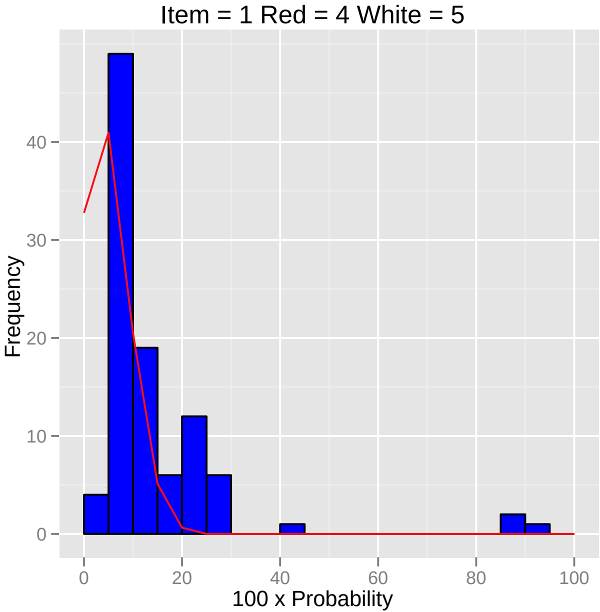
percent

Probabilistic inference task, normative

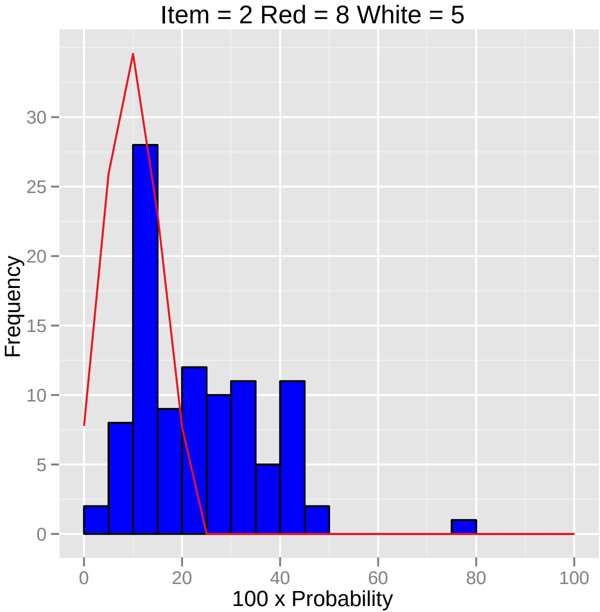
Probabilistic inference task, empirical



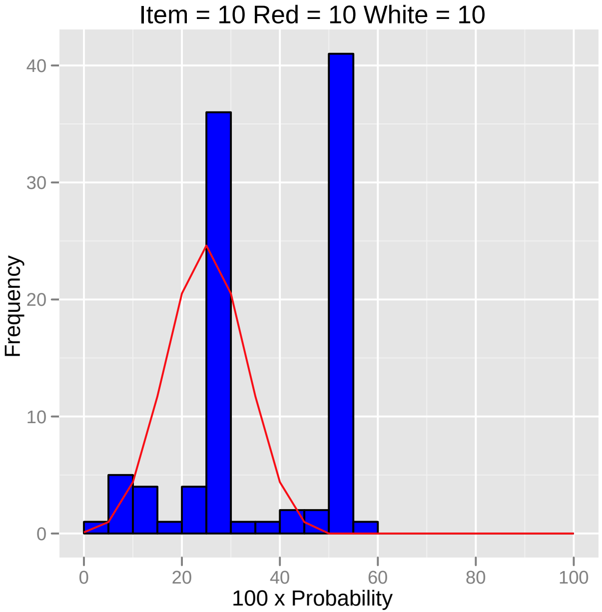
Probabilistic modus monens



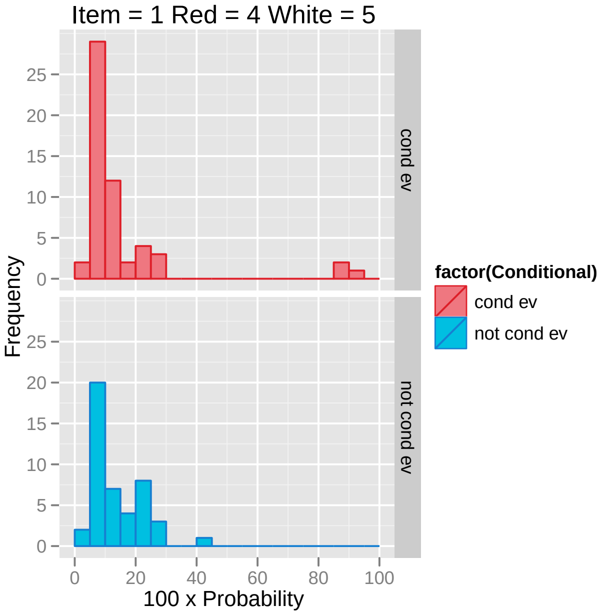
Probabilistics inference task



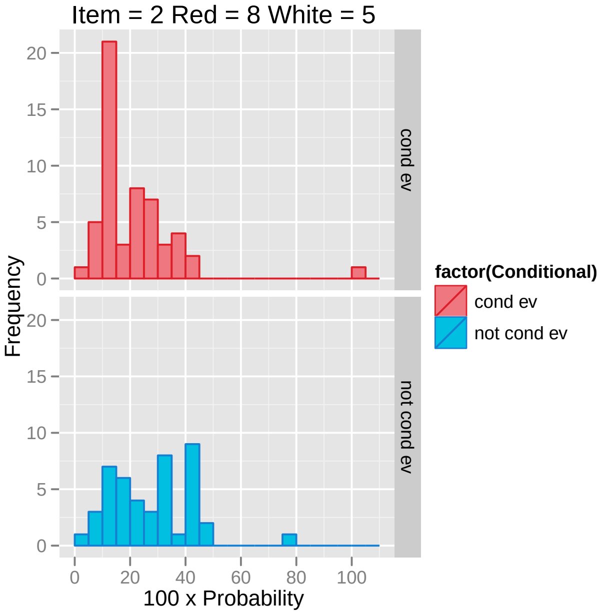
Probabilistic inference task



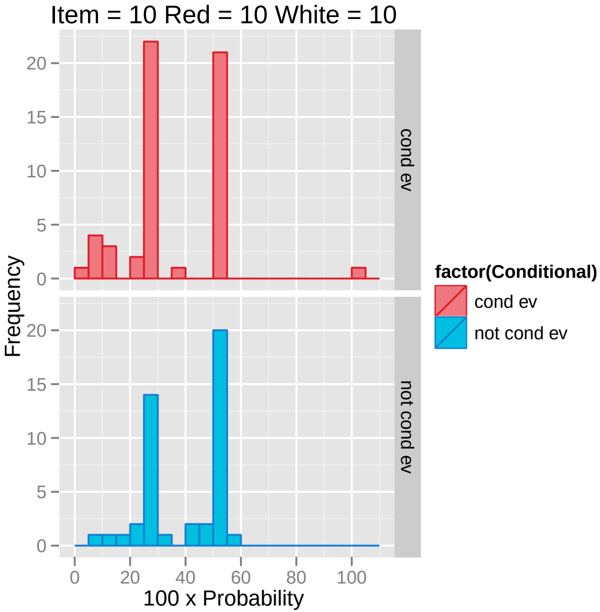
Probabilistic inference task



Probabilistic inference task



Probabilistic inference task



Results, probabilistic inference task

- Shrink and stretch
- Clearly sensitive to the base rates of red cards and the conditionals of flower cards
- About 50 % of the participants are doing very well, about 50 % do not stretch
- There is a (modest) correlation with the conditional event interpretation

Discussion

- Overwhelming empirical support for the conditional event interpretation
- Gender differences: serial/holistic, confidence, rules
- Working memory, serial processing
- No biconditionals
- The conditional event interpretation is necessary but not sufficient to get the Wason right
- Probabilistic reasoning: Shrink and stretch, participants with conditional event interpretation are doing better in the reasoning task

What is next?

- Cognitive model of the representation and processing of conditionals in human reasoning
- Extending to “first-order probability logic” and generalized quantifiers
- Frequency—Proportion—Probability
- Many psychological studies are not on probabilities but on generalized quantifiers
- Bridges to: Decision making, causal reasoning, concepts

Mental Models & Conditional Event

Age	init true	flesh-o indet	incompat rest	Interpretation Probability
1	11		10,01,00	Conjunction $P(11)/[P(11)+P(10)+P(01)+P(00)]$
2	11	00	10,01	Biconditional $P(11)/[P(11)+P(10)+P(01)]$
3	11	01,00	10	Conditional Event $P(11)/[P(11)+P(10)]$

$$\text{P(biconditional)} = \frac{P(\text{true possibilities})}{P(\text{true}) + \sum P(\text{incompatible possibilities})}$$

Do not include the void possibilities

Conditional

Task	Second	Particants	N	Cond event–conj–other	
Dice		Students	66	50–8–8	
Dice		Students	65	45–11–9	
Card	n-back	Students	80		Gender, 3-lure STM
Card	Wason	12,15,adult	61		no biconditionals,Wason
Card	Easy MP	Students	100	82–16–2	Probabilistic reasoning
			372		

Fugard, Pfeifer, Mayerhofer & Kleiter, 2011, JEP;
Fugard, Pfeifer, Mayerhofer & Kleiter;
Gansbiller & Kleiter;

Diepold & Kleiter

Results (Bayesian generalized linear model)

	Conditional	Event
Constant	0.991	0.695
ITEM	-0.023	0.040
AGE	-0.361	0.017
ITEM × AGE	0.021	0.000
SEX Male	-2.391	0.000
ITEM × SEX Male	0.023	0.178
GR × SEXMale	1.674	0.000
ITEM × AGE × SEX Male	-0.012	0.200
EF Object-Color	-2.776	0.000
ITEM × EF Object-Color	-0.009	0.565
AGE × EF Object-Color	1.268	0.000
ITEM × GR × EF Object-Color	-0.007	0.329
SEX Male × EF Object-Color	1.164	0.093
ITEM × SEX Male × EF Object-Color	0.063	0.007
AGE × SEX Male × EF Object-Color	-0.767	0.031
ITEM × AGE × SEX Male × EF Object-Color	-0.010	0.444
1 — ID	-0.000	1.000

Conditional event

Development, Bayesian generalized linear model

	Conditional Event		Response Time	
Constant	-1.781	0.477	9.360	0.000
Item position	0.019	0.000	-0.010	0.000
Age groups	0.961	0.000	-0.138	0.000
Male Gender	1.067	0.000	0.074	0.000
EFObject-Color	-0.322	0.051	-0.126	0.000
ITEM $\times$ EFObject-Color	-0.009	0.098	0.001	0.145
1 — ID	-0.000	1.000	-0.000	1.000
Conditional event			0.109	0.000
AIC	3415.570		3612.957	
BIC	3458.005		3667.516	
N	3172		3172	

Conditional Event:
bayesglm(c $\sim$ ITEM + GR + SEX + EF + ITEM:EF (1|ID), family = binomial)
Response Time:

bayesglm(log(T1 + T2) $\sim$ ITEM + c + GR + SEX + EF + (1|ID))

- Diepold, R. & Kleiter, G. D. (in preparation) Conditional reasoning in a probabilistic inference task task
- Fugard, A. J. B., Pfeifer, N., Mayerhofer, B. & Kleiter, G. D. (2011). How people interpret conditionals: Shifts towards the conditional event *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 37, 635–648.
- Fugard, A. J. B., Pfeifer, N., Mayerhofer, B. & Kleiter, G. D. (Project report). The conditional event interpretation of conditionals
- Gansbiller, J. & Kleiter, G. D. (in preparation). Understanding conditionals: A developmental perspective
- Gauffroy, C. & Barrouillet, P. (2009). Heuristic and analytic processes in mental models for conditionals: An integrative developmental theory. *Developmental Review*, **29**, 249–282.