Computational Foundations of Social Choice

Felix Brandt

LogICCC Final Meeting
Berlin, September 2011

DSS
Decision Sciences & Systems

EUROPEAN SCIENCE FOUNDATION
SETTING SCIENCE AGENDAS FOR EUROPE

Project Participants

Principal Investigators:

AI TCS — Felix Brandt (University of Munich)

AI LOG — Ulle Endriss (University of Amsterdam)

AI — Jeffrey Rosenschein (The Hebrew University)

TCS — Jörg Rothe (University of Düsseldorf)

ECON — Remzi Sanver (Instanbul Bilgi University)

Associated Partners:

AI ECON — Vincent Conitzer (Duke University)

TCS — Edith Elkind (University of Southampton, now Nanyang TU Singapore)

TCS LOG — Edith Hemaspaandra (Rochester Institute of Technology)

TCS — Lane Hemaspaandra (University of Rochester)

AI LOG — Jerome Lang (University of Toulouse)

ECON — Jean-Francois Laslier (Ecole Polytechnique Paris)

AI — Nicolas Maudet (Universite Paris-Dauphine)

Nederlandse Organisatie voor Wetenschappelijk Onderzoek
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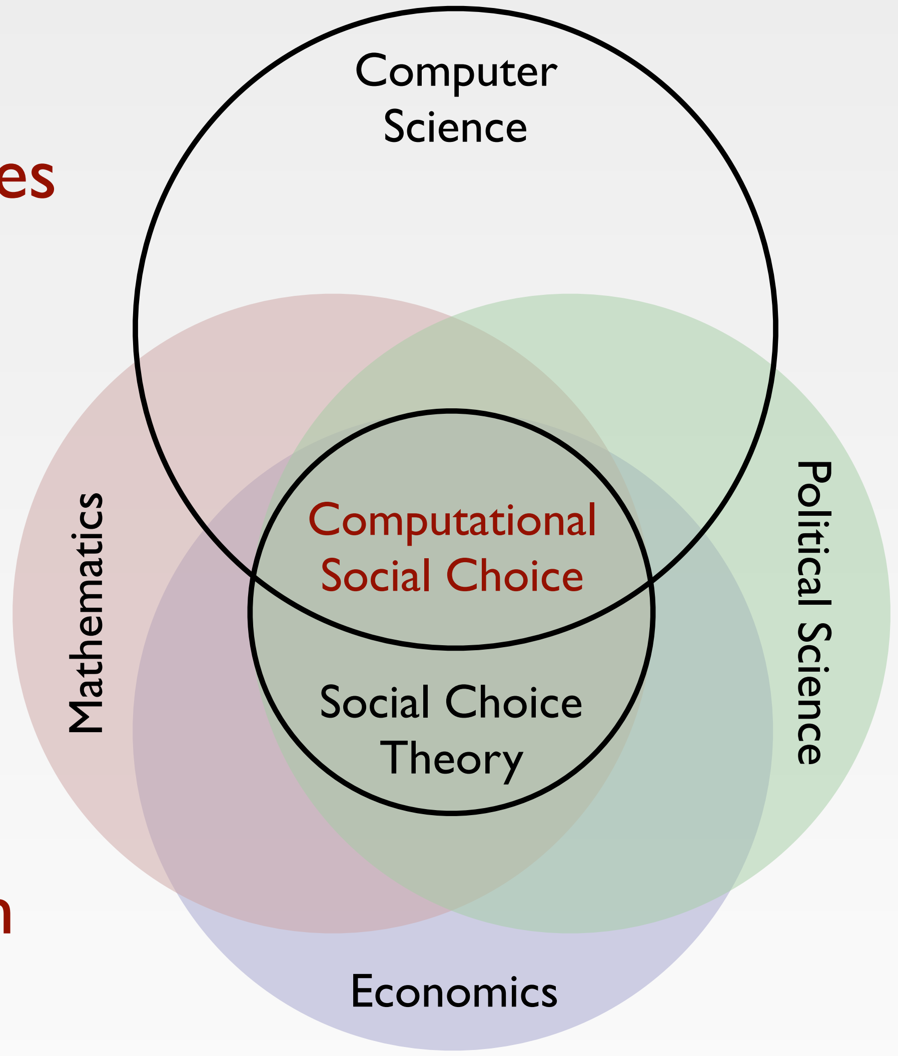
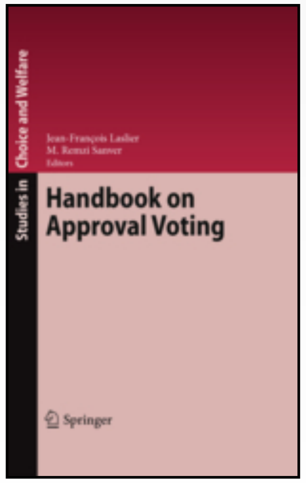
Computational Social Choice

- Core Topics/Research Highlights:
  1. Computational aspects of evaluating voting rules
     - efficient algorithms, approximation, complexity, etc.
  2. Computational hardness of manipulation
     - typical case analysis, heuristics, bribery, control
  3. Computational aspects of fair division
     - cake cutting, indivisible goods, efficient algorithms
  4. Social choice in combinatorial domains
     - multiple referenda, committees, pref. representation
  5. Computational aspects of coalitional formation
     - weighted voting games, power indices, matching
  6. Epistemic issues
     - incomplete information, communication complexity, privacy



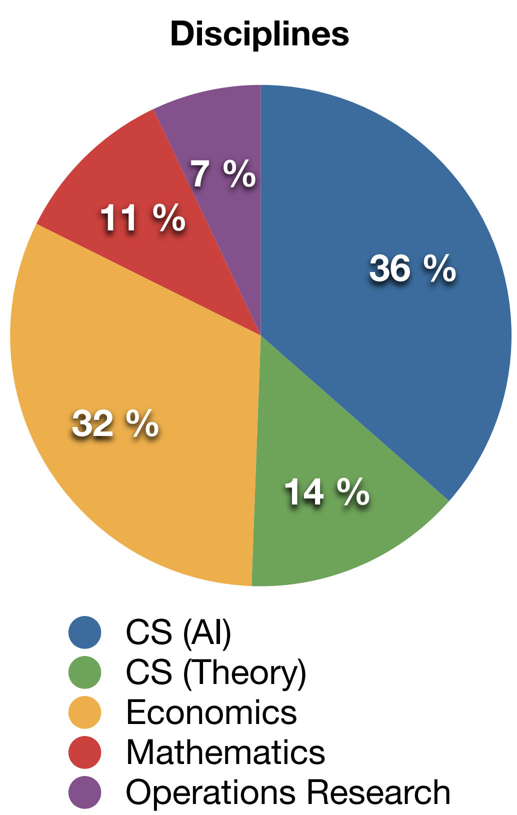
Publication Impact

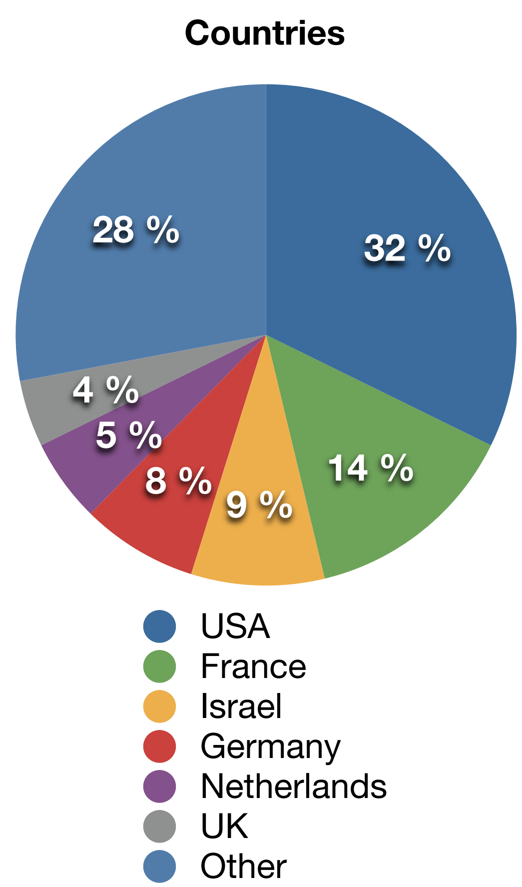
- AI: **19** IJCAI-2011 papers by seven CFSC members!
  - AP Conitzer received IJCAI Computers & Thought Award
  - IJCAI Workshop on Social Choice and Artificial Intelligence
    - organized by CFSC members Elkind, Endriss, & Lang
- CS Theory: Papers in *Information and Computation, Theoretical Computer Science, Information Processing Letters, SODA*, etc.
- Social Sciences: Papers in *Journal of Economic Theory, Social Choice and Welfare, Theory and Decision, Mathematical Social Sciences, Mathematical Logic Quarterly, Synthese*, etc.
- *Handbook on Approval Voting*, edited by AP Laslier and PI Sanver with chapter co-authored by CFSC members E. Hemaspaandra, L. Hemaspaandra and Rothe



Dagstuhl Seminar

- Organized by CFSC members Brandt, Conitzer, Hemaspaandra, Laslier, and mathematician William S. Zwicker in March 2010
- 44 participants (including ten of twelve CFSC members)
- Special issue of Mathematical Social Sciences



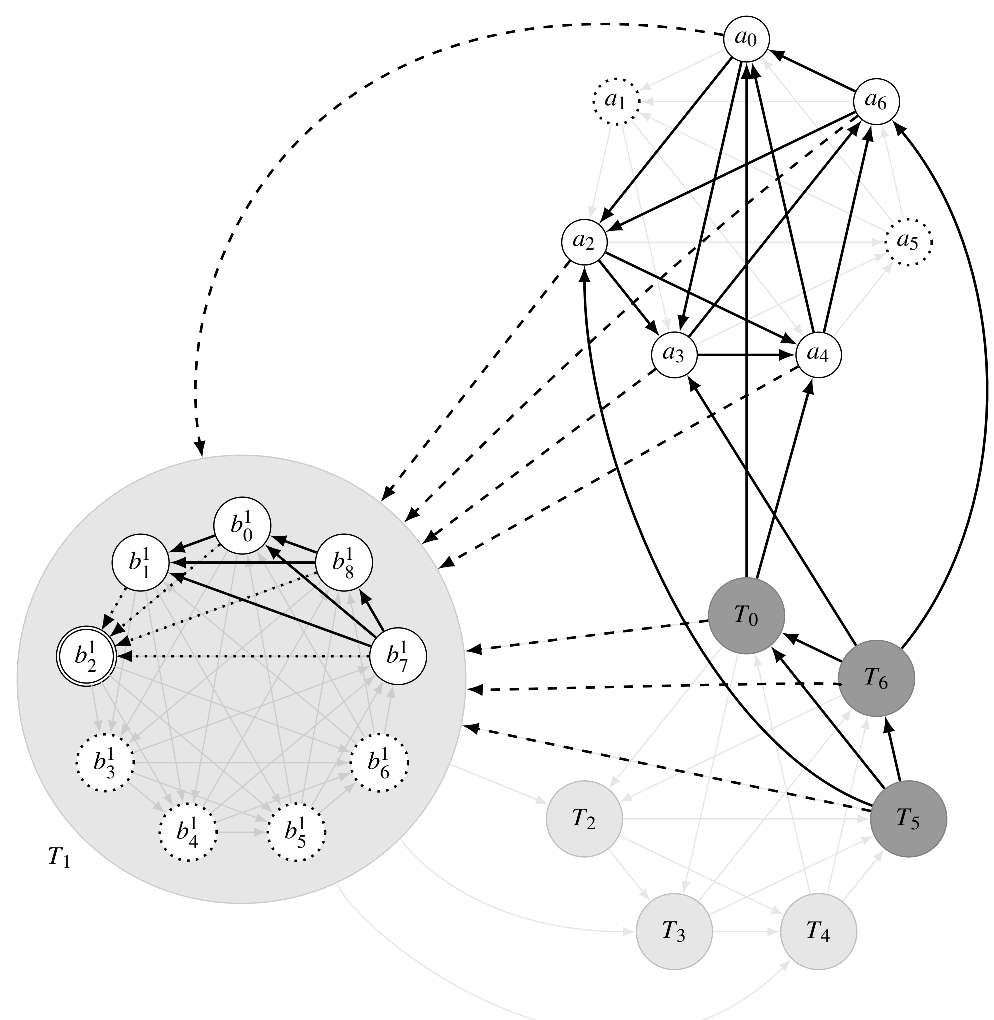


COMSOC 2010

- Organized by CFSC members Conitzer and Rothe in September 2010
- 93 participants (including nine of twelve CFSC members)
- Invited speakers:
    - Gabrielle Demange
    - Matthew O. Jackson
    - Bettina Klaus
    - Herve Moulin, and
    - Hannu Nurmi

- Tutorial by Agnieszka Rusinowska (from LogICCC CRP on Social Software)

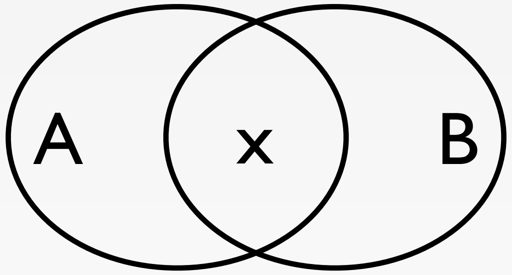
The Tournament Equilibrium Set



Rational Choice Theory


Amartya K. Sen

- Let U be a universe of alternatives.
- Alternatives are chosen from feasible subsets.
  - Throughout this talk, the set of feasible sets $\mathcal{F}(U)$ contains all finite and non-empty subsets of U.
- A choice function is a function $S : \mathcal{F}(U) \to \mathcal{F}(U)$ s.t. $S(A) \subseteq A$.
- Two typical consistency conditions
  Let A, B be feasible sets and $x \in A \cap B$.
  - Contraction (α): if $x \in S(A \cup B)$ then $x \in S(A) \cap S(B)$
  - Expansion (γ): if $x \in S(A) \cap S(B)$ then $x \in S(A \cup B)$



- Sen (1971) proved that the conjunction of both properties is equivalent to the fundamental economic notion of rationalizability.

From Choice to Social Choice

- Let N be a finite set of voters and $\mathcal{R}(U)$ the set of all transitive and complete relations over U.
- A social choice function (SCF) is a function $f : \mathcal{R}(U)^N \times \mathcal{F}(U) \rightarrow \mathcal{F}(U)$ such that $f(R, A) \subseteq A$.
  - For a given preference profile, every SCF induces a choice function and all consistency conditions can be readily applied.
- Useful conditions on SCFs
  - IIA (Independence of Irrelevant Alternatives): Choice only depends on preferences over alternatives in the feasible set.
  - Pareto-optimality: If a is unanimously strictly preferred to b, then b is not chosen.
  - Non-dictatorship: There should be no voter whose most preferred alternative is always uniquely chosen.

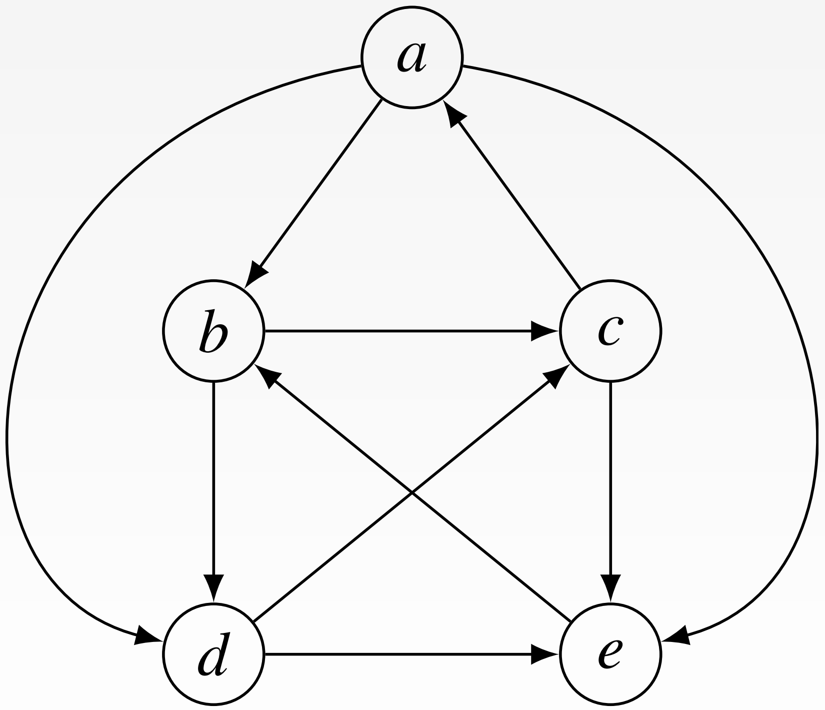
Arrow's Impossibility

Kenneth J. Arrow

- Theorem (Arrow, 1951; Sen, 1971): There exists no SCF that simultaneously satisfies IIA, Pareto-optimality, non-dictatorship, α, and γ whenever there are more than two alternatives.
  - In the context of SCFs, IIA is only a mild framework requirement (Bordes and Tideman, 1991) and dropping it offers little relief (Banks, 1995).
  - Dropping Pareto-optimality offers little relief (Wilson, 1972).
  - Dropping non-dictatorship is unacceptable.
  - Dropping γ offers little relief (Sen, 1977).
- Dropping α allows for reasonable SCFs!

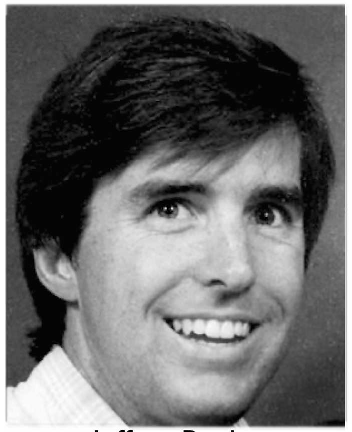
Majoritarian SCFs

- An SCF is majoritarian if its outcome only depends on the pairwise majority relation > within the feasible set.
  - Majoritarianism implies all Arrovian conditions except α and γ.
  - We assume for convenience that individual preferences are strict and there is an odd number of voters.
  - Hence, the pairwise majority relation is asymmetric and complete, i.e., it can be represented by a tournament graph.



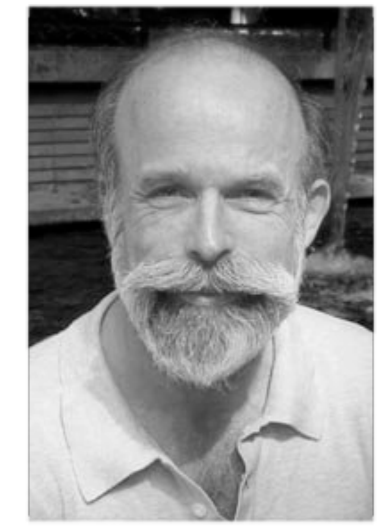

Peter C. Fishburn

Positive Results


Jeffrey Banks

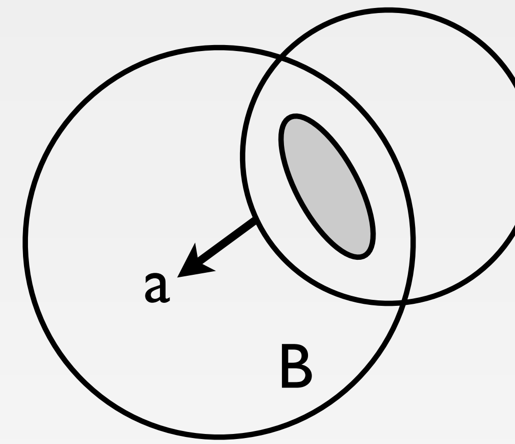
- Theorem (Moulin, 1986): The uncovered set, proposed independently by Fishburn (1977) and Miller (1980), is the smallest majoritarian SCF satisfying γ.
- γ can be weakened to *strong retentiveness.*
- Theorem (B., 2011): The Banks set, proposed by Banks (1985), is the smallest majoritarian SCF satisfying strong retentiveness.
- *Strong retentiveness* can be further weakened to *retentiveness.*
- Conjecture (Schwartz, 1990): The tournament equilibrium set (TEQ) is the smallest majoritarian SCF satisfying retentiveness.

Tournament Equilibrium Set

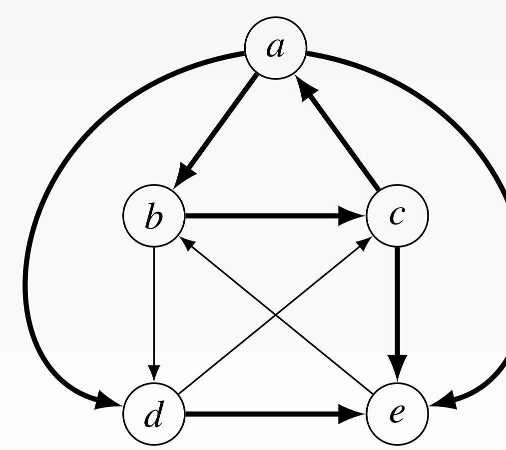


Thomas Schwartz

- Let S be an arbitrary SCF.
  - A non-empty set of alternatives B is **S-retentive**, if $S(\{b \mid b > a\}) \subseteq B$ for all $a \in B$.
  - Idea: No alternative in the set should be "properly" dominated by an outside alternative.



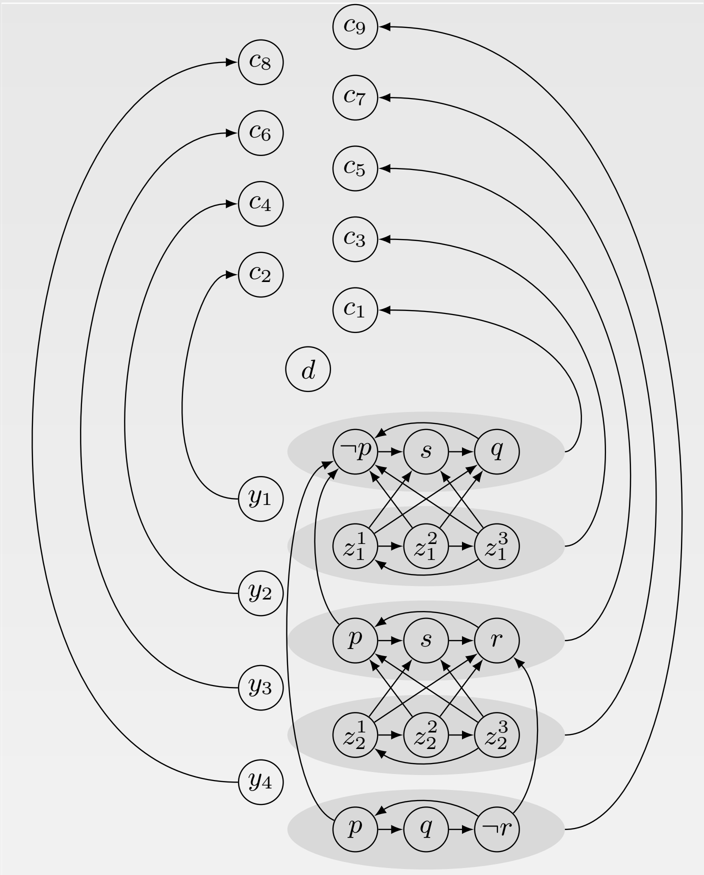
- $\mathring{S}$ is a new SCF that yields the **union of all minimal S-retentive sets.**
  - $\boxed{TEQ = \mathring{TEQ}}$
    - recursive definition
    - unique fixed point of ring-operator
    - Schwartz's conjecture states that every tournament contains a unique minimal TEQ-retentive set.
    - Example: TEQ = {a,b,c}



The Mystery of TEQ

- Theorem (Laffond, Laslier, Le Breton, 1993; Houy, 2009): TEQ satisfies monotonicity (and a host of other desirable properties) iff Schwartz's conjecture holds.
- Theorem (B., Harrenstein; 2011): TEQ satisfies $\widehat{\alpha}$ and $\widehat{\gamma}$ (and thus is set-rationalizable and self-stable) iff Schwartz's conjecture holds.
- Theorem (B., 2011): TEQ is group-strategyproof (according to Kelly's preference extension) iff Schwartz's conjecture holds.
- All or nothing: Either TEQ is a most appealing SCF or it is severely flawed.

Computing TEQ



- Theorem (B., Fischer, Harrenstein, Mair; 2010): Deciding whether an alternative is contained in TEQ is NP-hard.
  - best known upper bound is PSPACE!
  - simple heuristic relying on Schwartz's conjecture (B. et al., 2010)
  - fixed-parameter tractable with respect to decomposition degree (B., Brill, Seedig; 2011)
- We defined an infinite hierarchy of efficiently computable SCFs that "converge" towards TEQ and share most of its conjectured desirable properties (B., Brill, Fischer, Harrenstein; 2010)
  - yields an infinite number of weaker versions of Schwartz's conjecture; we proved the second one
  - anytime algorithm for computing TEQ (based on Schwartz's conjecture)

Schwartz's Conjecture

- There exists no counterexample with less than 13 alternatives; checked all 154 billion tournaments (B. et al., 2010).
  - TEQ satisfies all nice properties when there are less than 13 alternatives.
- We did not find a counterexample by searching billions of random tournaments with up to 50 alternatives.
  - Checking significantly larger tournaments is computationally intractable.
- Over the years, we discarded various incorrect proof attempts of Schwartz's conjecture by ourselves and other researchers.
- Many non-trivial weakenings of Schwartz's conjecture are known to hold (Good, 1971; Dutta, 1988; B. et al, 2010; B., 2011)
  - Recently, I proposed a weakening of Schwartz's conjecture which is more accessible, but still highly non-trivial (B., 2008).

A counterexample to conjectures of Brandt and Schwartz

Maria Chudnovsky[1]
Columbia University
New York, NY, USA

Ilhee Kim
Princeton University
Princeton, NJ, USA

Gaku Liu
Princeton University
Princeton, NJ, USA

Sergey Norin
McGill University
Montreal, QC, Canada

Alex Scott
Oxford University
Oxford, UK

Paul Seymour[2]
Princeton University
Princeton, NJ, USA

Stephan Thomassé
Université Montpelier 2
Montpelier, France

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- Non-constructive proof relying on probabilistic argument by Erdös and Moser (1964)
  - Neither the counter-example nor its size can be deduced from proof.
  - Smallest counter-example of this type requires about 10^{138} alternatives.
  - Verifying whether a tournament of this size constitutes a counter-example is not feasible.
    - The number of atoms in the universe is approximately 10^{80}.

- What does this mean?
  - In principle, TEQ is severely flawed.
  - If there does not exist a substantially smaller counter-example, this has no practical consequences.
  - The 21-year-old conjecture of a political scientist has been refuted using extremal graph theory.

Final Words

- In April 2011, during an extensive debate on whether plurality rule, which is known for various flaws, should be replaced with another somewhat more complicated voting rule (not TEQ!), British Prime Minister David Cameron responded to arguments from academics by saying:

> *Politics shouldn't be some mind-bending exercise. It's about what you feel in your gut.*