Joint evolution of cooperation and participation in public-good games

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A framework
for studying the evolution of cooperation

- Most of game-theoretical studies on the evolution of cooperation have been based on models with discrete strategies, linear payoff function, obligatory participation (fixed group size), and the (multi-player) *Prisoner's Dilemma*

- Recent studies show the importance of
  - continuously varying traits
  - nonlinear payoff function
  - voluntary participation (resulting in varying group size)
  - consideration of a broad range of (multi-player) games

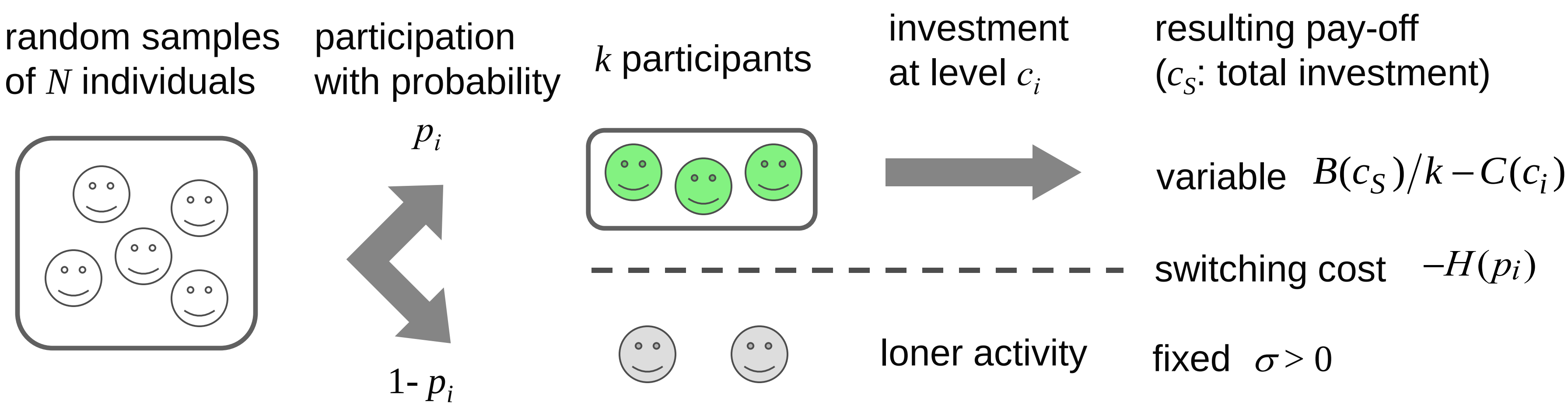
- Here we provide a synthetic framework for studying the evolution of cooperation encompassing all of these factors

Model: the continuous voluntary public-good game

- Standard public-good game with continuously varying investment and participation
- Consider individuals i with two-dimensional continuous strategy (c_i, p_i)

  c_i : cooperative investment

  p_i : participation probability



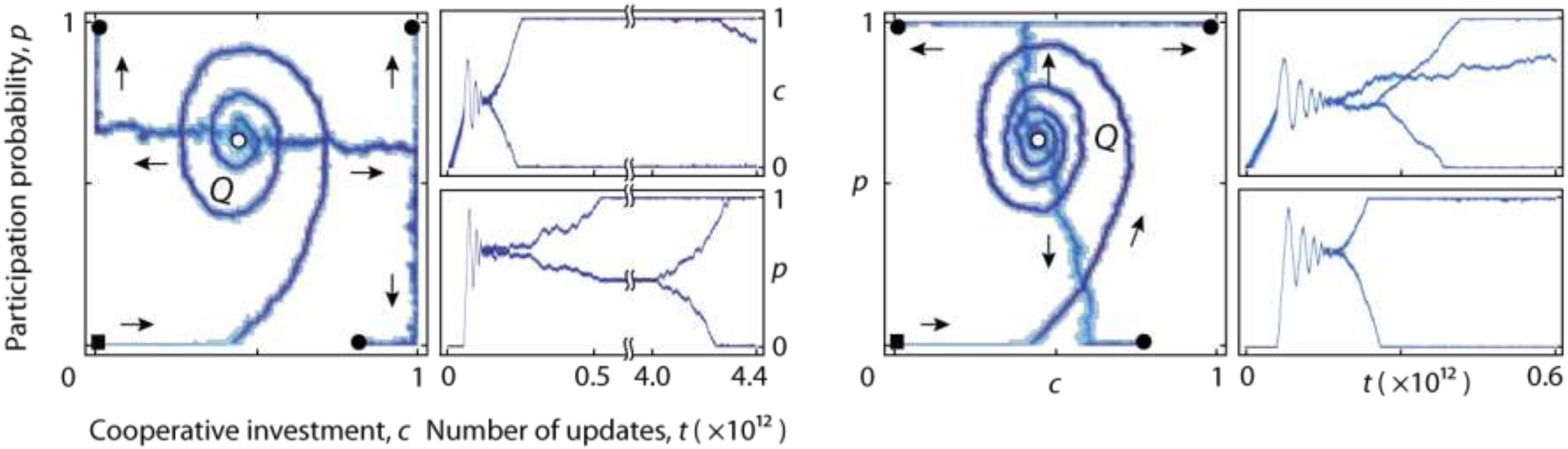
$$B(c_S) = \beta_2 c_S^2 + \beta_1 c_S$$

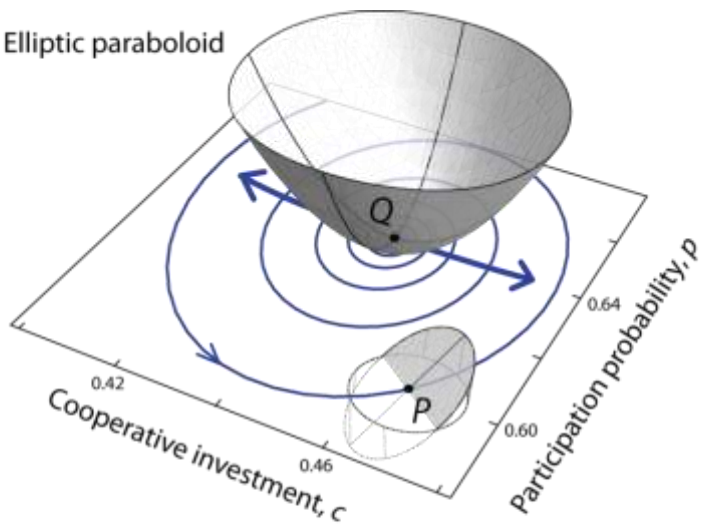
$$C(c_i) = \gamma_2 c_i^2 + \gamma_1 c_i$$

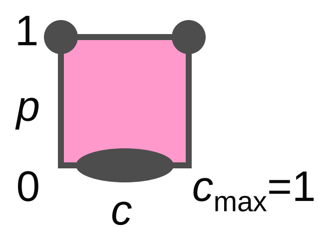
$$H(p_i) = \eta_2 p_i (1 - p_i)$$

$$\frac{d}{dc_x}\left[\frac{B(Nc_x)}{N} - C(c_x)\right]\Bigg|_{c_x=0} = B'(0) - C'(0) > 0$$

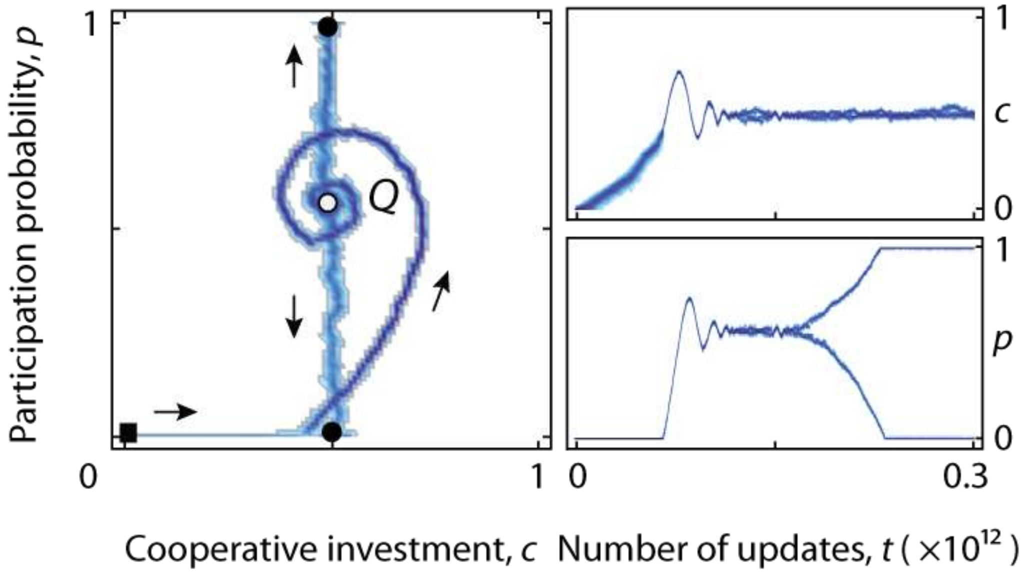
Trimorphism: the evolutionary origin of full cooperator, full defector, and non-participant

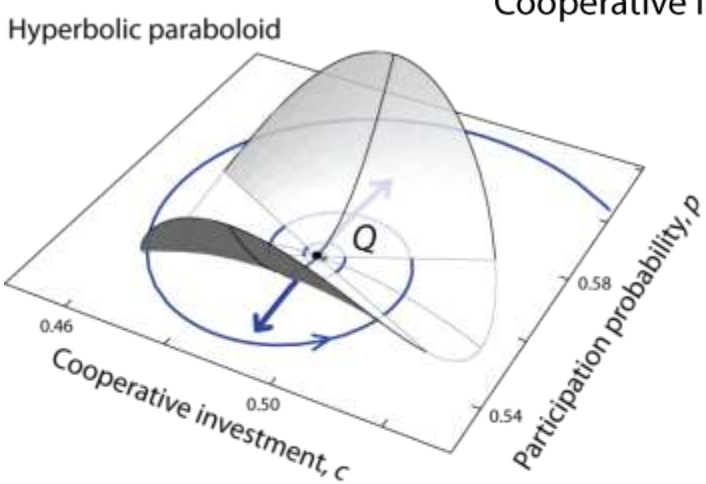


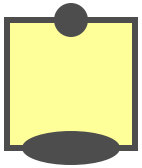




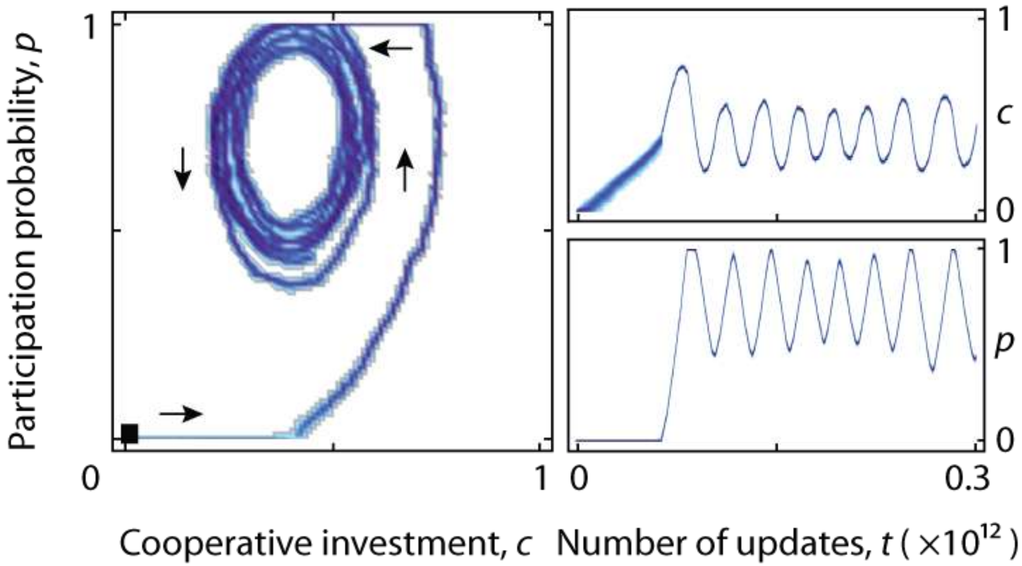
Intermediate cooperation with dimorphism of full and no participation



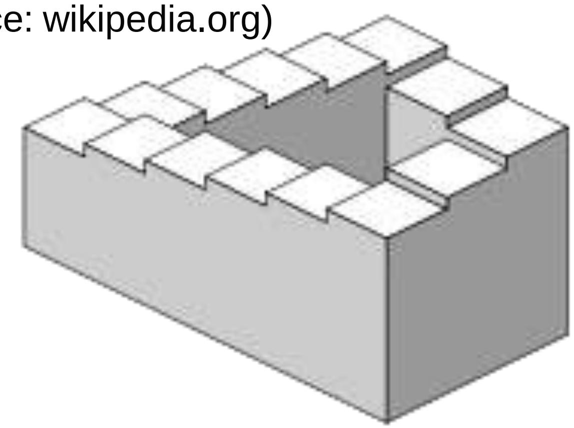


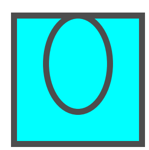


“Red Queen” oscillations
of cooperation and participation levels

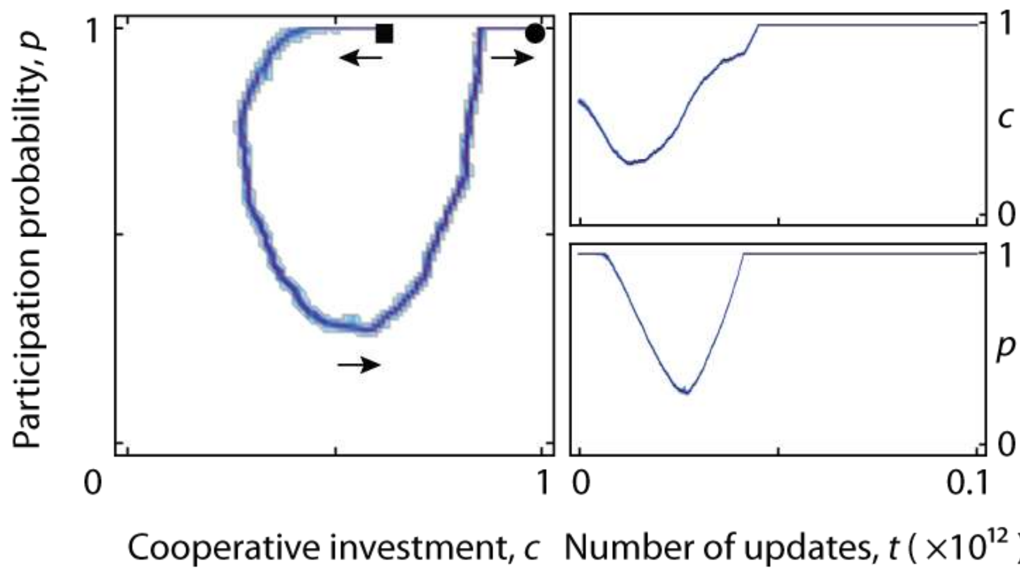


Climbing Penrose stairs
(source: wikipedia.org)

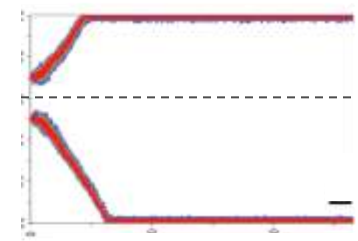


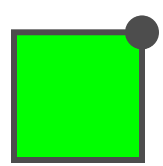


Full cooperation with full participation:
an extra-dimensional bypass of cooperation-defection divide

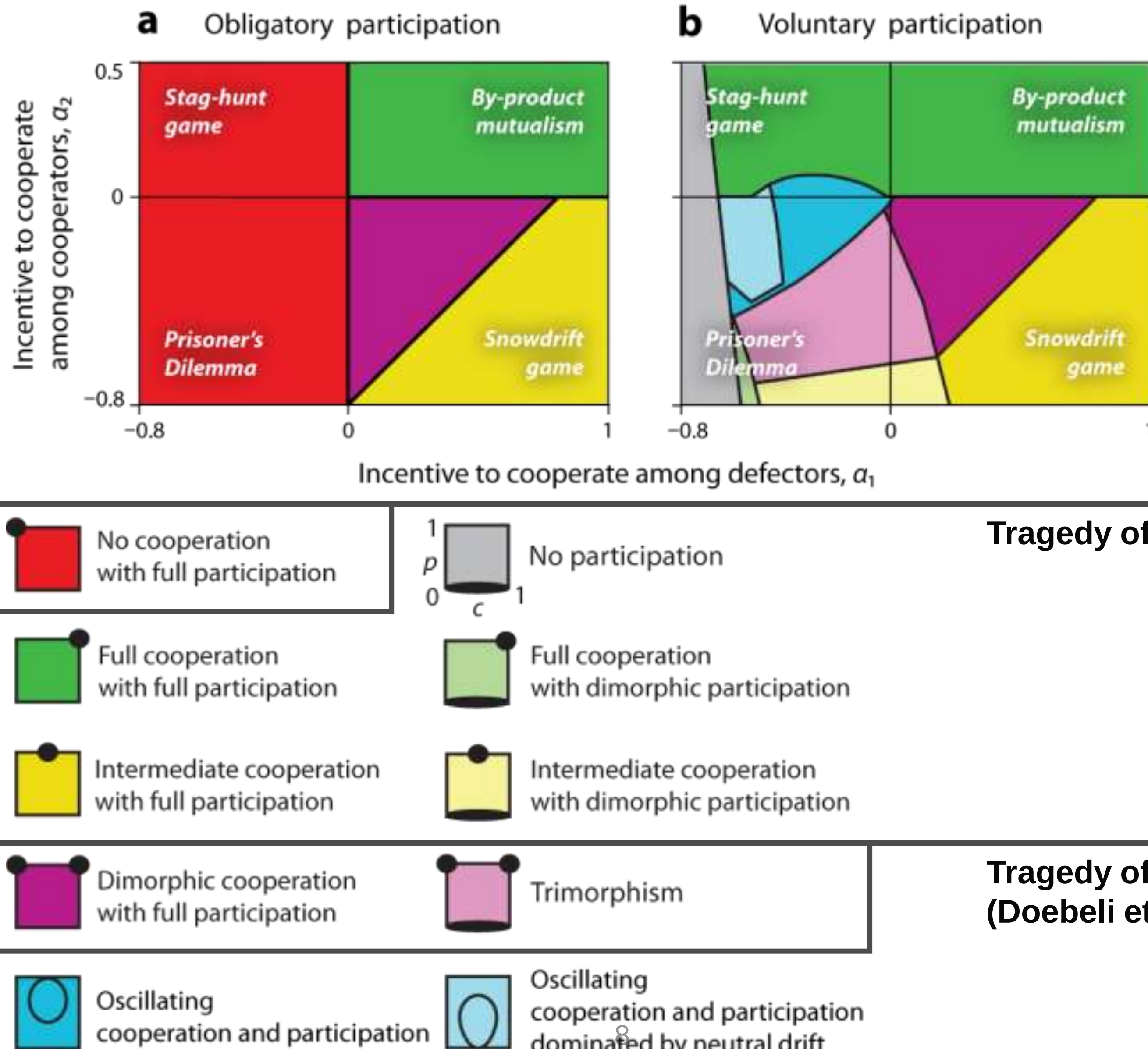


If p is fixed as 1
(obligatory participation),





Full Classification of the evolutionary fate of populations



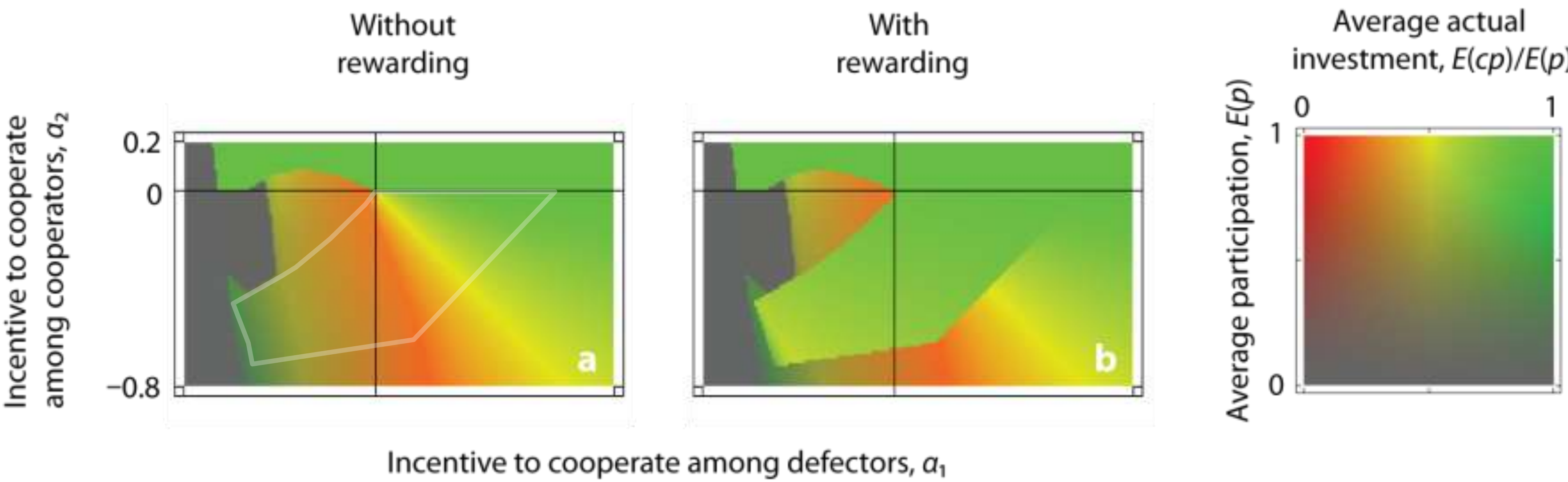
Parameters
$N = 5$,
$\gamma_1 = 1$,
$\gamma_2 = -0.1$,
$\eta_2 = 0.025$,
$\sigma = 1$

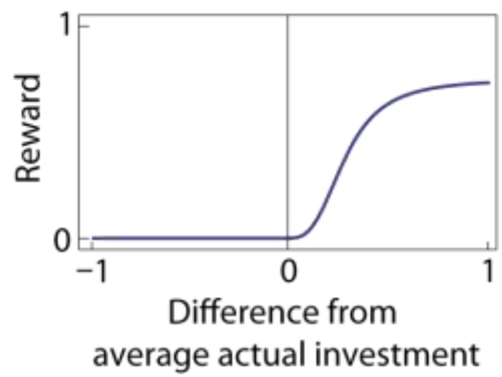
Initial conditions
a: $c = 0$
b: $(c,p) = (0,0)$

Tragedy of the Commons

Tragedy of the Commune (Doebeli et al., 2004)

Tragedy of the Commune can be beneficial





Summary

- Voluntary participation can thus help cooperation also in continuous-investment games

- The three pure strategies of full cooperation, full defection, and non-participation (e.g., Hauert et al., 2002) naturally emerge in our framework through gradual evolution of cooperation and participation

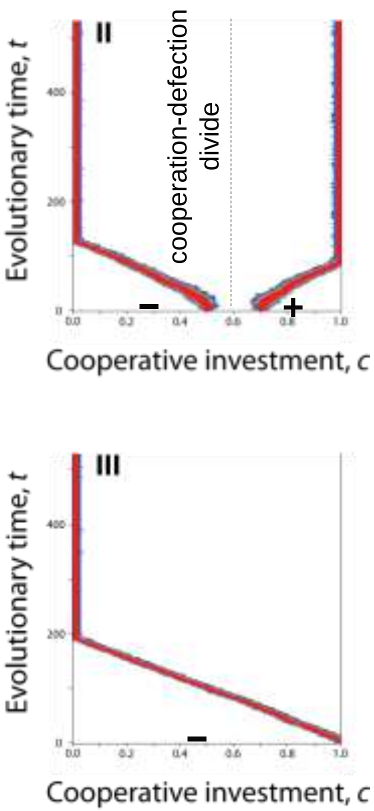
- Such strategy diversification is not restricted to *Snowdrift*-like games (as was previously shown in Doebeli et al., 2004), but can also occur in *Prisoner's Dilemma*-like games

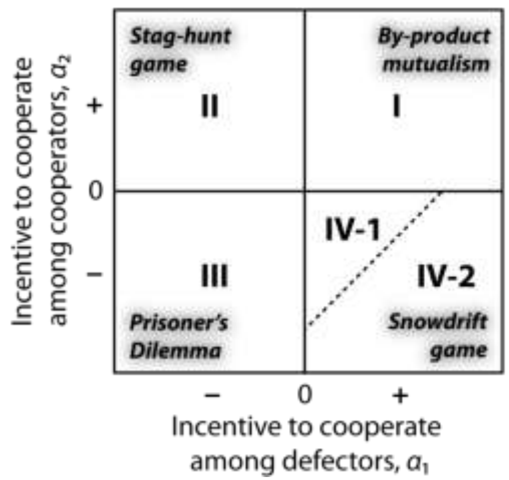
- Importantly, however, outcomes cannot always be understood in such simple terms as a mixture of traditional discrete strategies. Examples: "Red Queen" oscillations, extra-dimensional bypass of cooperation-defection divide, etc

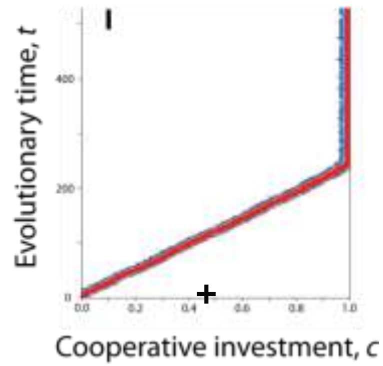
- Evolutionary branching may cause the "Tragedy of the Commune", but can also act as a powerful catalyst of cooperation-facilitating mechanisms

Full coverage of basic game dynamics (Doebeli *et al.*, 2004)

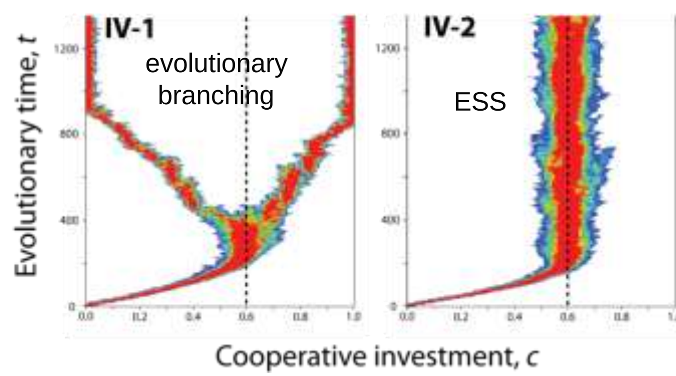
- Direction of gradual evolution of c on $p = 1$, $D_c(c_x) := \left.\dfrac{\partial S_{\boldsymbol{x}}(\boldsymbol{y})}{\partial c_y}\right|_{\boldsymbol{y}=\boldsymbol{x}=(c_x,1)} = 2(\beta_2 - \gamma_2)\,c_x + \dfrac{\beta_1}{N} - \gamma_1$
- The signs of $(\alpha_1, \alpha_2) := \big(D_c(0), D_c(1)\big)$ determine the global dynamics for obligatory participation



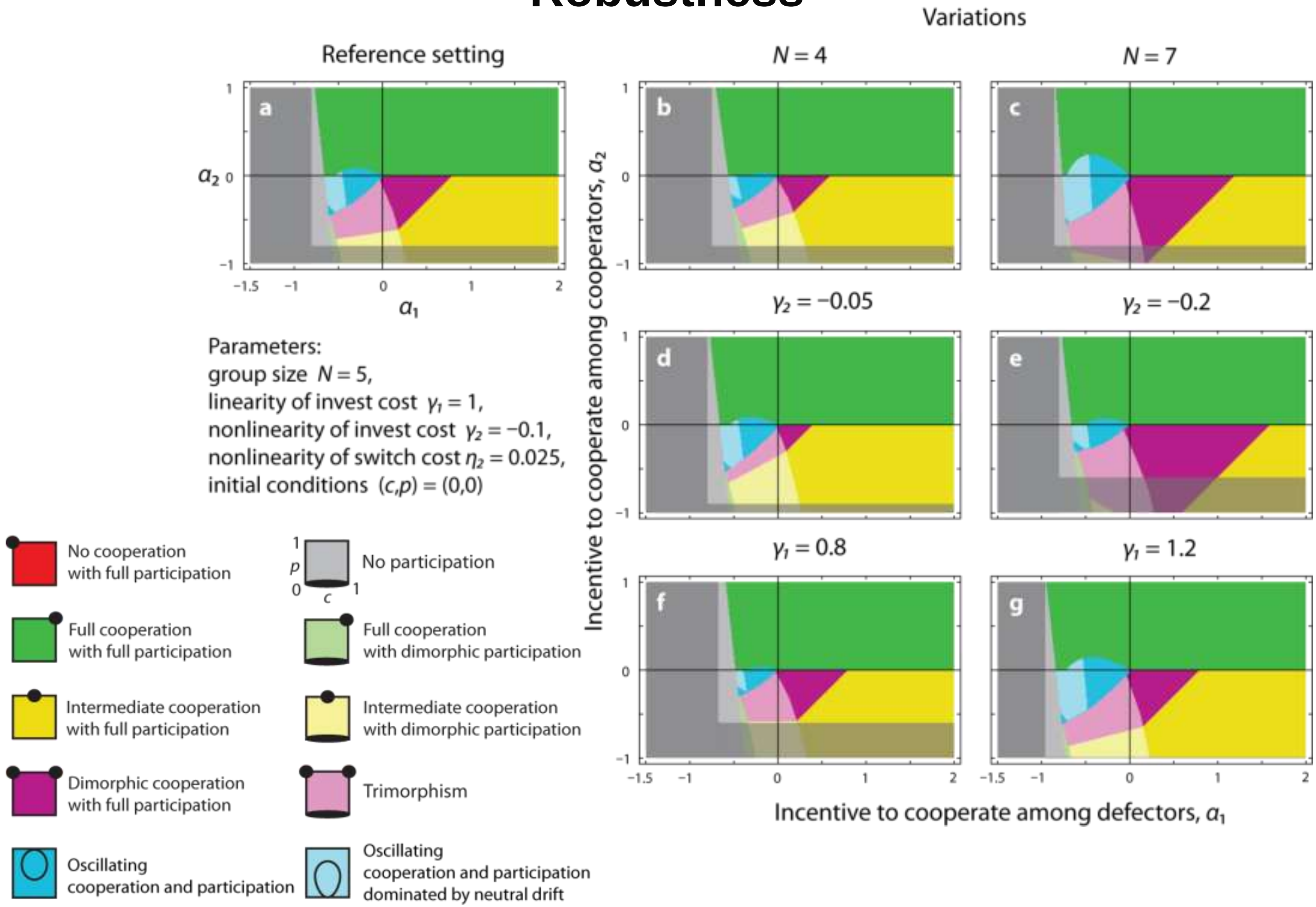




The local dynamics at singular strategies $\boldsymbol{x}^*$ depends on the curvature of $S_{\boldsymbol{x}^*}(\boldsymbol{y})$



Robustness



Variations
Reference setting
$N = 4$
$N = 7$
a
b
c
α_2
α_1
Incentive to cooperate among cooperators, α_2
$\gamma_2 = -0.05$
$\gamma_2 = -0.2$
d
e
$\gamma_1 = 0.8$
$\gamma_1 = 1.2$
f
g
Incentive to cooperate among defectors, α_1
Parameters:
group size $N = 5$,
linearity of invest cost $\gamma_1 = 1$,
nonlinearity of invest cost $\gamma_2 = -0.1$,
nonlinearity of switch cost $\eta_2 = 0.025$,
initial conditions $(c,p) = (0,0)$
No cooperation with full participation
No participation
Full cooperation with full participation
Full cooperation with dimorphic participation
Intermediate cooperation with full participation
Intermediate cooperation with dimorphic participation
Dimorphic cooperation with full participation
Trimorphism
Oscillating cooperation and participation
Oscillating cooperation and participation dominated by neutral drift

Robustness (continued)