SCIENTIFIC REPORT

ESF Exploratory Workshop on

Dissipative Systems: Entropy Methods, Classical and Quantum Probability

Vienna (Austria), 31 October – 3 November 2010

Convened by:
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1. Executive summary

This workshop was intended to bring together people working in research fields related to classical and quantum evolution problems. The meeting aimed at identifying mathematical contact points and stimulating joint research between three different communities: classical probability theory, quantum probability theory, and partial differential equations (PDE).

The workshop brought together both key representatives and young researchers from these three communities in one place, with the aim to initialize a broad and continuous cooperation. On the one hand, the participants gained a concise view on the state of the art in the respective other two fields, which provides a new perspective on the own work, indicates new research directions and helps to avoid "double discoveries" in the future. On the other hand, the meeting aimed at the exchange of mathematical toolboxes: using tools of another community, a mixture of tools, or just a problem reformulation in a "foreign" mathematical framework frequently leads to genuinely new results. At the very least, it provided a better understanding of the underlying mathematical structures of the problem and thus gives rise to extensions and generalizations of existing results.

The invited scientists of the three fields (classical/quantum probability and PDE analysis) share a common core interest, which is to capture the long-time behavior of (many-particle) systems, be it a system of particles with linear, nonlinear or quantum-mechanical interactions. Moreover, techniques from (nonlinear) functional analysis, operator theory and measure theory are fundamental in all three disciplines. Entropy methods, which are a powerful analytic tool to describe the equilibration behavior, provide a natural link between the fields. To give an example, entropies are employed to prove concentration inequalities on curved manifolds in classical probability (e.g. Bakry), to estimate the rate of equilibration for a chemotaxis system in the PDE context (e.g. Carlen), and to establish hypercontractivity of the quantum Ornstein-Uhlenbeck semigroup in quantum probability (e.g. Carbone).

The workshop was organized as follows. The talks have been collected in morning and afternoon sessions. Each session consisted of 3-5 talks and included a keynote presentation to introduce into the topic. The sessions were focused on one of the fields Classical Probability, Quantum Probability, or PDE Theory. Extended coffee and lunch breaks provided space for discussions. During the round-table discussion

at the end of the workshop, points of contact were identified and follow-up activities have been discussed (see below).

2. Scientific Content

The sessions on **Classical Probability** have been concerned with various topics, including the discussion of moment identities and quasi nilpotence of iterated anticipative stochastic integrals (Privault), the computation of multiple convolutions and a comparison of classical and free convolutions (Götze), stochastic completeness for jump processes (Grigoryan), the characterization of complementarity (Petz), and the ergodicity of dissipative dynamics in large interacting systems (Zegarlinski). Questions also of interest in PDE theory have been raised by the presentations of Sturm, Guillin, and Bakry. Sturm presented an overview of optimal transportation theory and the relation to gradient flows, in particular the Otto calculus, characterizing the heat equation as a gradient flow with respect to the Wasserstein distance on the space of probability measures. He also presented extensions on Wiener and Alexandrov spaces and indicated relations to Ricci flows. The talk of Guillin was concerned with the long-time behavior of kinetic Fokker-Planck equations using the logarithmic Sobolev inequality, the Wasserstein distance, and hypocoercivity properties. Bakry highlighted the relation between hyper- and ultracontractivity obtained through the Nash inequality.

The functional inequalities mentioned above (logarithmic Sobolev, Nash) were also used by researchers in **PDE Theory** in order to prove the long-time behavior of solutions. The main techniques are entropy methods which are closely related to the methods employed in probability theory. The long-time decay of solutions was discussed by Canizo (in the context of fragmentation-drift equations), by Dolbeault (also studying the relation between Gagliardo-Nirenberg, Poincaré, and logarithmic Sobolev inequalities as well as between Lieb-Thirring and Gagliardo-Nirenberg inequalities), and by Matthes (for Fokker-Planck type equations, for instance the linearized fast-diffusion equation). In particular, Matthes underlined relations of his results with those obtained by Dolbeault and Bakry (and Emery). The talk of Neumann on Wigner-Fokker-Planck models (PDE Theory) can be related to the presentation of Guillin on Vlasov-Fokker-Planck equations (Classical Probability). Markowich and Milisic (PDE Theory) spoke on topics related to quantum theory. The talk of Milisic was on quantum fluid models,

whereas Markowich gave two interpretations of quantum mechanics, using the Wigner and Bohmian measures. A Newtonian interpretation of the Schrödinger equation was given by von Renesse (Classical Probability) using the second-order calculus of Lott.

Neumann (PDE Theory) employed semigroup theory to prove existence results. Semigroup theory is an important tool also in **Quantum Probability**. For instance, Carbone established hypercontractivity of quantum semigroups and explored the question under which conditions hypercontractivity is equivalent to the existence of a logarithmic Sobolev inequality (see above). Lindsay demonstrated a method for obtaining dilations of minimal quantum dynamical semigroups overcoming domain constraints imposed by quantum stochastic differential equations by a holomorphic assumption on an associated contraction semigroup. Certain Wigner-Fokker-Planck-type models can be written in Lindblad form (see the talk of Neumann). This form has been examined by Rebolledo (also studying the existence of steady states and the convergence to equilibrium) and Vacchini (who discussed generalizations of the Lindblad structure of quantum master equations for describing time evolutions of quantum systems with strong memory effects). Rebolledo characterized the equilibrium as zero-entropy production states. Entropy production, on the other hand, is an important concept for entropy methods (PDE Theory) in order to prove, for instance, the long-time decay of solutions. The presentations proved that some topics are of common interest in at least two of the three communities of Classical Probability (CP), PDE Theory (PDE), and Quantum Probability (QP):

- CP, QP, and PDE: long-time asymptotics using functional inequalities (logarithmic Sobolev,
- Nash, Gagliardo-Nirenberg) and proof of new functional inequalities;
- QP and PDE: steady-state analysis of Fokker-Planck-type equations and quantum Markov semigroups;
- CP and PDE: comparison of the probability approach with the entropy-method approach;
- CP and QP: coupling of quantum systems to the environment (heat bath, phonon fields, other dissipative effects); open quantum systems.

3. Assessment of the Results

The meeting was meant to provide an initial spark for lasting collaborations between the three involved communities. The participants have been selected to include representatives from various European countries, including three researchers from Eastern Europe. Therefore, the resulting network of scientific interactions is of genuine European scale. The participants intend to use the dynamics resulting from the workshop for joint activities. These activities are arranged into two action types: research in small groups of 3-4 persons from different communities and "larger" activities involving all three communities. In particular, the following questions will be examined within the next months:

- Formulate the Wigner-Fokker-Planck equation in operator language, study nonlinear quantum Markov semigroups, and extend the Bakry-Emery method to the quantum case (PDE Theory: Arnold, Neumann; Quantum Probability: Fagnola, Rebolledo).
- Develop logarithmic Sobolev inequalities for quantum Ornstein-Uhlenbeck processes (PDE Theory: Arnold; Quantum Probability: Carbone, Fagnola).
- Compare the Wigner, Bohmian, fluid dynamical, and Newtonian-type picture of quantum mechanics and formulate dissipative quantum systems as a Newton law with friction (PDE Theory: Jüngel, Markowich; Classical Probability: von Renesse).
- Analyze quantum coagulation models and the long-time behavior of their solutions (PDE Theory: Canizo; Quantum Probability: Belavkin).
- Model quantum systems coupled to phonon fields and examine their properties (PDE Theory: Neumann; Classical Probability: von Renesse; Quantum Probability: Accardi).
- Develop Lieb-Thirring and interpolation inequalities to obtain lower bounds for the energy functional (PDE Theory: Arnold, Dolbeault) and study the corresponding gradient flows (PDE Theory: Dolbeault, Matthes).

These tasks will be solved by mutual visists financed by the follow-up activity program of the ESF of by individual grants of the participants. The participants agreed to continue the promising path of joint activities by setting up a workshop

series on "Dissipative Systems: Entropy Methods, Classical and Quantum Probability". Subject to an appropriate financement, the next workshop will be held end of 2011 in Pavia (Italy), local organizer: Raffaella Carbone. Since the future financement of the ESF seems to be uncertain, other options for a follow-up workshop would be to apply for a funding at the *International Centre for Mathematical Sciences* in Edinburgh or for a mini-workshop at the *Mathematisches Forschungsinstitut Oberwolfach*.

Long-term perspectives are to apply for a research program at the *Newton Institute for Mathematical Sciences* in Cambridge. The Newton Institute aims to bring together mathematical scientists from UK universities and leading experts from overseas for concentrated research on specialised topics in all branches of the mathematical sciences. Furthermore, an application to a Marie-Curie Initial Training Network would be an option.

4. PROGRAMME

Sunday, October 30, 2010

Afternoon   *Arrival*

Monday, November 1st, 2010

08.30-09.00	**Registration**
09.00-09.10	**Welcome by Convenor** **Ansgar Jüngel** (Vienna University of Technology, Vienna, Austria)
09.10-09.30	**Presentation of the European Science Foundation (ESF)** **Mats Gyllenberg** (ESF Standing Committee for Physical and Engineering Sciences (PESC))
09.30-12.30	**Morning Session: Classical Probability**
09.30-10.15	**Keynote Lecture "Quasi-invariance and mixing of Poisson point measures under interacting transformations"** **Nicolas Privault** (Nanyang Technological University, Singapore)
10.20-10.50	**"Asymptotic approximations in free probability"** **Friedrich Götze** (Universität Bielefeld, Germany)
10.55-11.25	*Coffee Break*
11.25-11.55	**"Optimal transportation, gradient flows and geometry"** **Karl-Theodor Sturm** (Universität Bonn, Germany)
12.00-12.30	**"On stochastic completeness of jump processes"** **Alexander Grigoryan** (Universität Bielefeld, Germany)
12.30-14.30	*Lunch Break and Discussions*

14.30-18:10 **Afternoon Session: Quantum Probability**

14.30-15.15 **Keynote Lecture "The equilibrium of open system dynamics: an analysis of the Markov chain"**
Rolando Rebolledo (Pontificia Universidad de Chile, Chile)

15.20-15.50 **"From classical to quantum hypercontractivity for Ornstein-Uhlenbeck semigroups"**
Raffaella Carbone (Università di Pavia, Italy)

15.55-16.25 *Coffee Break*

16.25-16.55 **"Non-Markovian quantum dissipation"**
Bassano Vacchini (Università di Milano, Italy)

17.00-17.30 **"An optimal transport perspective on the Schrödinger equation"**
Max von Renesse (Technische Universität Berlin, Germany)

17.35-18.05 **"Stochastic Schrödinger equations"**
Carlos Mora (Universidad de Concepción, Chile)

Tuesday, November 2nd, 2010

09.00-12.40 **Morning Session: PDE Theory**

09.00-09.45 **Keynote Lecture "Functional inequalities, thick tails and asymptotics for the critical mass Patlak-Keller-Segel model"**
Eric Carlen (Rutgers University, USA)

09.50-10.20 **"Entropy methods for fragmentation-drift equations"**
José Alfredo Canizo (Universitat Autonoma di Barcelona, Spain)

10.25-10.55 *Coffee Break*

10.55-11.25 **"Fast diffusion equations: matching large time asymptotics by relative entropy methods"**
Jean Dolbeault (Université Paris Dauphine, France)

11.30-12.00 **"Proving convex Sobolev inequalities by an algebraic entropy method"**
Daniel Matthes (Technische Universität Wien, Austria)

12.05-12.50 **Keynote Lecture "On Wigner and Bohmian measures"**
Peter Markowich (Cambridge University, United Kingdom)

12.55-14.30 *Lunch Break and Discussions*

14.30-18.10 **Afternoon Session: Classical Probability**

14.30-15.15 **Keynote Lecture "Efficient quantum tomography and complementarity"**
Denes Petz (Budapest University of Technology, Hungary)

15.20-15.50 **"How hot can a heat bath get?"**
Martin Hairer (University of Warwick, United Kingdom)

15.55-16.25 *Coffee Break*

16.25-16.55 **"On long-time behaviour of some kinetic Vlasov-Fokker-Planck type equation"**
Arnaud Guillin (Université de Provence, France)

17.00-17.30 **"Ergodicity of dissipative dynamics in large interacting systems"**
Boguslaw Zegarlinski (Imperial College, United Kingdom)

17.35-18.05 **"Around Nash inequalities"**
Dominique Bakry (Université Paul Sabatier, France)

19.30-22.00 *Conference Dinner*

Wednesday, November 3, 2010

09.00-12.20	**Morning Session: Quantum Probability**
09.00-09.45	**Keynote Lecture "Equivalence of dynamical detailed balance and local KMS condition for non equailibrium states"** **Luigi Accardi** (Università di Roma, Italy)
09.50-10.20	**"Dilatation of minimal quantum dynamical semigroups"** **Martin Lindsay** (Lancaster University, United Kingdom)
10.25-10.55	*Coffee Break*
10.55-11.40	**"Quantum dissipative dynamics and generalized master equations"** **Viacheslav Belavkin** (University of Nottingham, United Kingdom)
11.45-12.15	**"On the new approach of quantum probability"** **Wladyslav Majewski** (Gdansk University, Poland)
12.20-14.10	*Lunch Break and Discussions*
14.10-16.50	**Afternoon Session: PDE Theory**
14.10-14.40	**"Semiconductor modeling and nonlinear higher-order PDEs"** **Josipa-Pina Milisic** (University of Zagreb, Croatia)
14.45-15.15	**"Quantum Fokker-Planck models: long time properties of solutions"** **Lukas Neumann** (Universität Innsbruck, Austria)
15.20-16.50	*Discussion of follow-up activities*
16.50	*End of Workshop and Departure*

5. List of Participants

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6. Statistical Information

Age structure: 19 of the 28 participants have been senior scientists, 9 of 28 participants have been junior scientists. By a junior scientist, we understand a researcher who does not have already a (full) professor position and who is not older than about 40 years. Thus, 32 % of the participants have been junior scientists.

Gender repartition: In Mathematics, the number of female tenured professors is still very low (2003: 9.7 % full professors in USA, 2009: 10 % associate or full professors in Germany). In addition, there are very few female mathematicians working in the fields of classical or quantum probability. From the 28 participants, only two have been female (7 %), both being junior scientists.

Countries of origin: The following table shows the country repartition. 24 of 28 participants came from Europe, 4 of 28 from oversea (only three participants from oversea have been partially financed by the ESF grant). Three participants came from Eastern European countries (Croatia, Hungary, Poland).

Country	Number of participants	Percentage
Austria	3	11
Croatia	1	4
France	3	11
Germany	5	18
Hungary	1	4
Italy	3	11
Poland	1	4
Spain	1	4
United Kingdom	6	21
Chile	2	7
Singapore	1	4
United States	1	4