Report on the 1 month Infty funded exchange visit to Munich in May, 2011 Reference Number 3483

Peter Aczel

June 28, 2011

Purpose of visit

The purpose of my visit to Munich was to develop a better understanding of Homotopy Type Theory (HoTT), mainly by interacting with Steve Awodey, one of the founders of the subject, who was also visiting Munich. The hope was that this would lead to me being able to make a contribution, in the longer term, to the very intense research activity that has been going on, particularly in the last few months. Last February I took part in a small Oberwolfach meeting on the subject, where most of the key researchers were present. The discussions with Steve were intended to increase my understanding of HoTT.

Description of work carried out

I had fairly intensive, almost daily discussions with Steve on a variety of aspects of HoTT. Some of the topics we discussed were

1. the new notion of higher dimesional inductive definitions in type theory, how they should be understood semantically, and the examples of the circle, intensional bracket types for resizing etc,

2. the h-levels of propositions and sets and their properties,

3. my ideas concerning logic-enriched type theory and their possible application to HoTT.

I gave a talk at the formal host for my visit to Munich, Hannes Leitgeb's Munich Center for Mathematical Philosophy. The title was "Conceptual frameworks for classical and constructive set theory". I also gave a talk at a small workshop, "Computations, Logic, Algebra, and Categories (CLAC)", organised by Peter Schuster. My title was "On Voevodsky's Univalence axiom".

A major feature of the work on HoTT, initiated by Voevodsky, has been the use of the computer systems of Coq and Agda to develop machine checked definitions and proofs in HoTT. This has turned out to be very fruitful and is perhaps the first time that such computer systems have played a serious role at the forefront of the development of new abstract mathmatics. In the past I had some experience with Coq and Agda, but I have not been familiar with the recent versions that are being exploited by the HoTT community. This has been a problem for me in fully understanding the definitions and results that the HoTT community has been developing, mostly using Coq. I had hoped that I might be able to benefit from some expertise on Coq and Agda in Munich. In fact I was able to get help one afternoon from the computer Scientist, Dr Andreas Abel from the LMU Department of Theoretical Computer Science, who implemented Agda on my laptop and gave me a tutorial on Agda.

Main Results obtained

No obviously significant mathematical results were obtained. The discussions with Steve were very useful in deepening my understanding of HoTT.

I became convinced that HoTT, with the Univalence axiom, gives a new perspective to ideas about identity and structuralism in mathematics. I am in the process of formulating a Structure Identity Principle (SIP), which roughly states that isomorphic structures are structurally identical. This may appear tautological, but need not. The Univalence axiom seems to imply the strong version of SIP where 'structurally identical types' are defined to be those types that are identical in the sense of Martin-Löf's propositional identity types, which play the role of path spaces in HoTT. I hope to talk on this topic at the forthcoming Edinburgh European 3rd Set Theory Conference.

Over the last few months most of the work on developing Coq seems to have been done using the Coq system. While this is a novel and significant way to do mathematical research there is also a need to present the definitions and theorems in the standard style of research mathematics. Unfortunately this has not been happening. I think that the key reason for this has been that developing mathematics informally in type theory is a rather new kind of activity, which has not yet matured into the smooth, often ambiguous, style we are used to in standard mathematical practise. So I fear that researchers have been reluctant to make the effort to develop an informal style for type theory and prefer the safety of Coq, in spite of the proof scripts being unreadable. I hope to be able to make a contribution here, as I have been familiar with dependent type theory, as a setting for the representation of mathematics, since the early 1970s, and struggled in the past with the issues concerning a good informal style.

Future Collaboration with Host Institution

I hope to visit Munich again for the month of June next year, when Steve Awodey is again expected to be visiting. We hope that this may become a regular interaction.

Projected publications/articles

I am in the process of writing notes, that will eventually be made available on my web site, on informal HoTT and on the Univalence axiom. It is too early to say what kind of journal or conference publication will appear as a result of my visit to Munich last month.