Vincenzo Dimonte's Scientific Report for an ESF Short Visit Grant

June 22, 2009

Purpose of the visit

I've applied for this grant with the purpose of being able to attend the ESI Workshop on large cardinals and descriptive set theory in Vienna. As a PhD student, this was obviously an invaluable experience for me to remain well-informed of the current developments and to have an idea of what the "hot topics" are, for better sketching a future personal research program. More important, however, is the fact that this could be an unique occasion for me to speak with the people that have made the most outstanding discoveries on my research field, since there are very few publications regarding it and otherwise it would have been difficult for me to know more on the subject.

Description of the work carried out during the visit

I've followed most of the scheduled talks of the first week, total time five hours per day. For the first three days I've been able to speak about half an hour per day with Prof. Hugh W. Woodin, the most important researcher in my field and probably the only one able to help me in my PhD thesis, since it's based mostly on his unpublished work. During these short sessions, we talked about the gaps in my thesis and possible future developments (see results below). On the last day, I've been able to give a talk on my results.

Description of the main results obtained

My work is based on Woodin's work on Higher Determinacy Axioms. These are a new kind of Large Cardinal Hypothesis, the largest so far, that are very similar to Determinacy Axioms but are expressed as elementary embeddings between large inner models. These Axioms are very well hierarchized, and are really near the inconsistency, so there is the possibility to find (after almost forty years) a new result of inconsistency that would cast some light on the structure of the universe. A key object in his analysis is the E^0_{α} -sequence, defined similarly to the sequence of the minimum model of $AD_{\mathbb{R}}$, and these new Axioms can be expressed as "There exists an elementary embedding from $L(E^0_{\alpha})$ to itself"

The analogy from Determinacy doesn't seem to work if the elementary embeddings are not proper, i.e. if the fixed point are not cofinal in the Θ of the model. So it's natural to ask if there are elementary embeddings that are not proper. Before coming to Vienna I've already proved that there is an α and an elementary embedding $j : L(E_{\alpha}^{0}) \prec L(E_{\alpha}^{0})$ that is not proper. After my sessions with Woodin, I am now able to prove that there is a β such that every elementary embedding $j : L(E_{\beta}^{0}) \prec L(E_{\beta}^{0})$ is not proper, and that there exists an elementary embedding $j : L(E_{\alpha}^{0}) \prec L(E_{\alpha}^{0})$ that is proper (so α is not β). I have also a more complete comprehension of these new Axioms, that can lead me shortly to new results (I think that there are few elementary embeddings from $L(E_{\beta}^{0})$ to itself, and a lot from $L(E_{\alpha}^{0})$ to itself).

Projected publications/articles resulting or to result from my grant

There cannot be a publication/article that fully result from the grant. However, the results that I've obtained will be in my PhD thesis and in the eventual (if there will be the occasion) article related.