SCIENTIFIC REPORT

HIROAKI MINAMI

- Purpose of the visit

The aim of the visit is to participate Young set theory workshop 2011 which took place at Königswinter near Bonn in German from 21 to 25 on March with Sunday 20 for arrivals.

- Description of the work carried out during the visit

During the Young set theory workshop 2011, I participated tutorials and discussions and I present a poster in the poster session. In discussion session, I gave a topic about maximal almost disjoint family inextensible to F_σ ideal, which is mentioned in my poster, and discuss with Dilip Raghavan, Dániel T. Soukup, Jonathan L. Verner and Juris Steprans.

- Description of the main results obtained modify In [1], Laflamme shows that CH implies the existence of a mad family inextensible to F_σ-ideal. By modifying the proof, we get the following results.

Proposition 0.1. *If* $\mathfrak{p} = \mathsf{cov}(\mathcal{M})$ *or* $\mathfrak{p} = \mathsf{cof}\left(\{\mathcal{I} : \mathcal{I} \text{ is an } F_\sigma\text{-ideal}\}, \leq_K\right)$, *then there exists a mad family inextensible to* F_σ *ideal.*

Also we discuss about the existence of a mad family extensible to F_σ-ideal.

In [2] it is shown that $\mathfrak{a} < \mathsf{cov}^*(\mathcal{I})$ for all F_σ-ideals, there exists a mad family inextensible to F_σ-ideal and I construct a model with $\mathfrak{a} < \mathsf{cov}^*(\mathcal{I})$ for all F_σ-ideals. In this model, $\mathfrak{a} = \omega_1$. So it is natural to ask the following question.

Question 0.2. *Are there model with* $\mathfrak{a} > \omega_1$ *and exists a mad family inextensible to* F_σ*-ideal?*

Concerning to this problem, Juris Steprans suggests the following forcing notion.

Definition 1. $\mathbb{P}(\omega_2)$ is a c.c.c forcing notion defined by

$$p = \langle F_p, \sigma_p, n_p \rangle \in \mathbb{P}(\omega_2) \text{ if } F_p \in [\omega_2]^{<\omega}, \sigma_p : F_p \to 2^{n_p} \text{ and } n_p \in \omega$$

ordered by $p \leq q$ if $F_p \supset F_q$, $\sigma_p \supset \sigma_q$, $n_p \geq n_q$ and

$$\forall \alpha \in F_q \left(|\{n \in n_p \setminus n_q : \sigma_p(\alpha)(n) = 1\}| \leq 1\right).$$

Using this forcing notion, I and Dilip Raghavan showed the following.

Proposition 1.1. *Suppose $\mathfrak{c} > \omega_1$ in V. Let G be a $\mathbb{P}(\omega_2)$-generic over V. Then $\{A_\alpha : A_\alpha = \bigcup\{\sigma_p(\alpha) : p \in G \wedge \alpha < \omega_2\}\}$ is a mad family inextensible to F_σ-ideal in $V[G]$. So there exists a mad family inextensible to F_σ-ideal with cardinality ω_2 in $V[G]$.*

I am trying to show this forcing notion doesn't add small mad family, i.e., $\mathfrak{a} = \omega_2$. This work will be related to the problem of Spectrum of mad families.

- Projected publications/articles resulting or to result from your grant

I develop result given during visit and try to make my note [2] extending to a paper.

2. INTRODUCTION

REFERENCES

[1] Claude Laflamme. Zapping small filters. *Proceedings of the American Mathematical Society*, 114(2):535–544, 1992.
[2] Hiroaki Minami. Mad family inextensible to F_σ-ideal. note.

KURT GÖDEL RESEARCH CENTER FOR MATHEMATICAL LOGIC AT THE UNIVERSITY OF VIENNA, WÄHRINGER STRASSE 25 A-1090 WIEN
E-mail address: minami@kurt.scitec.kobe-u.ac.jp