INFTY Exchange Grant 3188
Final Report

David Schrittesser

September 2, 2011

Purpose.

The purpose of my visit to Barcelona was to learn about formulations of supercompactness in terms of second order reflection phenomena from Prof. Bagaria and asses the possibilty of obtaining a formulation that would be amenable to the type of argument given by Viale and Weiß in [VW]. Another goal was to communicate work done in my thesis and investigate possible ways to extend that research.

Work done.

During the course of my visit, Prof. Bagaria and I worked through several formulations of supercompactness and their relation to the new results in Viale and Weiss work ([VW]), in particular, the generalized tree property ITP. We discussed at length their proof that ITP(ω_2) is implied by PFA and essentially, as a first step came to a different proof of their result that PFA, under certain specific conditions, implies the consistency of the existence of a supercompact cardinal. This different proof used the reflection principle formulation of supercompactness.

We then embarked on a search of a proper forcing that would directly witness, under PFA and possibly additional hypotheses, that the reflection property holds. This search has proven to be very hard and is still ongoing. If we manage to find this forcing this would result in a publication, but further work is needed.

In the course of this research I gave a talk at the logic seminar briefly describing Viale and Weiss work.

I also continued to work on a joint project with Prof. Saharon Shelah, in which we prove that the above mentioned ITP(ω_2) is consistent with arbitrarily large continuum. This work has greatly profited from my advanced understanding of the combinatorial consequences of supercompactness and from discussion with Prof. Bagaria. Thus I was able to generalize the proof from my joint work with Prof. Shelah slightly, obtaining that ω_2 can be replaced by any successor cardinal. I also gave two talks on this topic in the logic seminar.

Another talk I gave in the logic seminar was about the research done in my thesis, after which Neus Castells asked the following question:

Question 1. *Is there a model where ω_1 is accessible to reals, all Σ^1_3 sets have the Baire property and all Δ^1_2 sets are Lebesgue-measurable?*

To my knowledge, the question is open, and a solution may yield important tools useful in generalizing results from my thesis to higher levels of the projective hierarchy.

Results.

Theorem 1 (with S. Shelah)**.** *Adding any number of Cohen reals to a model of* PFA *preserves* ITP(ω_2)*.*

Theorem 2. *For $\nu < \kappa$, where the latter is a supercompact cardinal, we can force* ITP(κ) *and $\kappa = \nu^+$ while at the same time $2^\omega > \kappa$.*

The search for a forcing witnessing a second order reflection property strong enough to imply supercompactness was partially succesful insofar as it became clear how such a forcing is implicit in Viale and Weiss work. To find a forcing which does the same but allows to improve the conditions under which a model for supercompactness can be found has shown to be very difficult and has to and will be studied further.

Publications.

Both results will be published in due course, with reference to the INFTY grant, as [SS]. Any future result about the reflection property would lead to an additional publication.

Further Collaborations.

Prof. Bagaria and I will share all further progress on the essential question left open at the end of my visit, that of finding a forcing that directly adds an embedding that preserves a second order property of the structure.

I have suggested a collaboration to Neus Castells, concerning generalizations of work in my thesis in the context of the question she has proposed.

References

[SS] D. Schrittesser and S. Shelah, ITP(ω_2) *is consistent with large continuum*, in preparation.

[VW] M. Viale and C. Weiß, *On the consistency strength of the proper forcing axiom*, to appear in Advances in Mathematics.