Research Networking Programmes

Short Visit Grant ☐ or Exchange Visit Grant ☒

(please tick the relevant box)

Scientific Report

Scientific report (one single document in WORD or PDF file) should be submitted online within one month of the event. It should not exceed eight A4 pages.

Proposal Title: Forcing arguments for Gowers's Theorem and properties of P_{FIN}

Application Reference N°: 4375

1) Purpose of the visit

The main purpose of this visit was to work on the line of what we did regarding Ramsey's and Hindman's theorems, we wanted to seek a partial order suitable for a proof of Gowers' Theorem on the existence of homogeneous sets for partitions of FIN_k.

On the other hand, we wanted to continue studying P_{FIN}, by answering such natural questions as:
(1) What are the cardinal invariants associated with P_{FIN}?
(2) Does P_{FIN} add Sacks, Silver, or Laver reals?
(3) Is there an extrinsic characterization of the generic block sequences of P_{FIN}, in a way similar to Mathias reals being characterized in terms of almost disjoint families of the ground model?
(4) Is it true, again as in the case of Mathias forcing, that every condensation of a P_{FIN}-generic block sequence is also a generic block sequence?

2) Description of the work carried out during the visit

My work was within the combinatorial set theory, specifically in infinite Ramsey Theory, which deals with partitions of infinite sets. In my PhD thesis I gave proofs of Ramsey's and Hindman's theorems which rely on forcing arguments. The analysis of the partial order arising in the proof of Hindman's Theorem, which we denote by P_{FIN}, was the main object of the research. In my visit I was working on proving properties of P_{FIN} and finding a forcing proof of Gowers' Theorem.

I worked on my first article, A forcing notion related to Hindman's Theorem. This article contains the main results of my thesis and professor Brendle and I improved one result. This improvement is about a property of the forcing P_{FIN}, which can be stated as follows: P_{FIN} satisfies the Laver property, this implies that P_{FIN} adds neither Cohen nor Random reals.

I worked on a second article and I proved that the generic object added by P_{FIN} has a similar property as the Mathias real, stated as follows: A block sequence D^* is P_{FIN} generic if and only if for all MAD_{FIN} families A in the ground model there exists an element x in A such that D^* is an almost condensation of x, where a subset of infinite block sequences A is a maximal almost disjoint FIN family, denoted by MAD_{FIN}, if for any different elements D and D' in A , the intersection of FU(D) with FU(D') is finite and A is maximal with this property.

This last statement implies that every condensation of a P_{FIN}-generic block sequence is also a P_{FIN}-generic block sequence.

Similar to the definition of a Ramsey set, we define the notion of a Hindman set and we found a forcing proof of the fact that analytic sets are Hindman, using the characterization of the generic object added by P_{FIN} cited above.

At the moment we are working on the Hindman property in higher levels of the projection hierarchy like the \Sigma^1_2 and the \Delta^1_2 levels.

We are also carring out research connected with the last chapter of my thesis that was about M_2, a kind of product of two copies of Mathias forcing, and its relation with P_{FIN}. I proved in my thesis that M_2 is a projection of P_{FIN}.

Finally, we are working on defining a partial order that forces the homogeneous object in Gowers' Theorem.

Talks and conferences:

-Talk in the Seminar of the Group of Logic, Statistics and Informatics of the Graduate School of System Informatics, Kobe University (13/08/2013). Title: Forcing Arguments in Infinite Ramsey Theory.

-Talk in the International Conference on Topology and Geometry at Shimane University, in Matsue, Japan (02/09/2013). Title: A forcing notion related to Hindman's Theorem.

-Talk in the workshop Reflection principles and set theory of large cardinals at RIMS Kyoto, Japan (11/09/2013). Title: Hindman's Theorem and analytic sets.

-Talk in the 13th Asian Logic Conference in Guangzhou, China (17/09/2013). Title: A forcing notion related to Hindman's Theorem.

3) Description of the main results obtained

-We improved the following theorem from my thesis: P_{FIN} does not add Cohen reals. We prove that P_{FIN} has the Laver property. Hence P_{FIN} adds neither Cohen nor Random reals. In fact, every new real is contained in a closed measure zero set of the ground model.

-We obtained a characterization of the generic block sequences of P_{FIN}, similarly to Mathias real, which can be stated as follows: D^* is P_{FIN} generic if and only if for all MAD_{FIN} families A in the ground model there exists an element x in A such that D^* is an almost condensation of x.

-We gave a forcing proof of the ¡Hindman property for analytic sets, using the characterization of the generic object added by P_{FIN}.

4) Future collaboration with host institution (if applicable)

5)* Projected publications / articles resulting or to result from the grant *(ESF must be acknowledged in publications resulting from the grantee's work in relation with the grant)

Projected publications:
-A forcing notion related to Hindman's Theorem (Article).

-Hindman's Theorem and analytic sets (Article).

-Forcing proofs of Ramsey's and Hindman's Theorems (Survey in RIMS proceedings).

6) Other comments (if any)