INFTY - Short Visit Grant - 5424

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1 purpose

This grant supported the visit of Chris Laskowski, University of Maryland and John Baldwin, University of Illinois to Vienna to collaborate with Sy Friedman, Martin Koerwien, and Tapani Hyttinnen, Helsinki who was supported on other funds. Baldwin was in Vienna Feb 14-28.

The goal was to investigate interactions of model theory and set theory. In particular, we studied the interactions of descriptive set theory and the model theory of the infinitary language $L_{\omega_1,\omega}$. This work was carried out via extensive group discussions each day and review of partial drafts written by various participants.

2 results

Here is one strategy for establishing Vaught's conjecture that there is no sentence of $L_{\omega_1,\omega}$ that has exactly $\aleph_1$ countable models. Hjorth, using descriptive set theoretic results of Mackey and others, has established that if there is a counterexample then there is one that has no model in $\aleph_2$. On the other hand, unpublished results of Harrington show that every counterexample has models with arbitrarily large Scott ranks below $\aleph_2$. This supports the notion that one might construct a model of an arbitrary counterexample that has cardinality $\aleph_2$. The resulting contradiction would yield the conjecture.

Our work showed that this approach involves three misconceptions of the role of Descriptive Set Theory in this approach to the problem and emphasizes the importance of more model theoretic approach. These misconceptions are a) that descriptive set theory plays a central role in finding models with absolute indiscernibles, b) that the existence of a model in $\aleph_2$ rather than the embeddability relation in $\aleph_1$ is key and c) that complexity without embeddability conditions is a sufficient tool.

In studying problem a) we extracted from earlier work of Hjorth and Laskowski-Shelah a scheme for using an expanded language to study Fraissé constructions and gave a direct model theoretic proof of Hjorth's result mentioned above.

In studying problem b) we gave a detailed analysis of the three known 'fundamental' examples of complete sentences of $L_{\omega_1,\omega}$ that characterize ω_1 (have a model in $\aleph_1$ but no larger). In fact, every model in $\aleph_1$ is maximal. We identified a combinatorial property which accounts for two of the examples. And we conducted a detailed examination of the countable models of these examples. This examination was motivated by the conjecture: If every model in $\aleph_1$ of ϕ is maximal then ϕ has $2^{\aleph_1}$ models in $\aleph_1$. We showed that two different approaches of Shelah yield this result for each the three examples. But we did not establish the general result while in Vienna.

In studying problem c) we provided a new and more direct proof of Harrington's theorem discussed above. In conjunction with b) our results show that if this kind of Scott rank analysis is to succeed in attacking Scott's conjecture it must involve more interaction with the amalgamation properties of countable models.

Overall, this work suggests a refocusing of efforts on Vaught's conjecture to consider the embedding relation on countable models as the complexity of models. This is also suggests that more partial results (analogous to those in first order model theory) might be obtained by restricting attentions to classes that satisfy suitable model theoretic properties.

3 Future Collaborations

Two definite future collaborations are planned. Koerwien will visit the University of Illinois in Chicago to continue work on this project in April and Baldwin will visit Vienna in July, 2013. Collaboration continues by email.

4 Projected Publications

There is already roughly 12 page draft of a paper, Three Red Herrings around Vaught's conjecture.

Further work may evolve from the conjectures in section b).