

The purpose of my short visit to TU Delft was to establish a collaboration with K.P. Hart and to explore new possible approaches to the Katowice problem.

The so called Katowice problem is a question whether the statement (K): “Boolean algebras  $P(\omega)/fin$  and  $P(\omega_1)/fin$  are isomorphic” is consistent with ZFC. So far, models of ZFC + various non-trivial consequences of (K) have been constructed, but the basic question remains unsolved.

Recently K.P. Hart proved that (K) implies that there exists a nontrivial (not generated by almost permutation on  $\omega$ ) Boolean automorphism of  $P(\omega)/fin$ . During my visit in Delft we were utilizing approach used in this proof. We were investigating further consequences of (K) on the structure of automorphism group of  $P(\omega)/fin$  in the hope of getting closer to solving Katowice problem.

The other topic of our collaboration was investigation of properties of the cardinal invariant  $\mathfrak{f}$  defined in [1]. A free sequence in  $P(\omega)/fin$  is a sequence  $\{A_\alpha \in P(\omega) : \alpha \in \gamma\}$  such that for each  $\beta \leq \gamma$  and for each  $F \in [\beta]^{<\omega}$ ,  $G \in [\gamma \setminus \beta]^{<\omega}$  the intersection  $\bigcap_{\alpha \in F} A_\alpha \cap \bigcap_{\alpha \in G} (\omega \setminus A_\alpha)$  is infinite. A free sequence is maximal if it is not a proper initial segment of any other free sequence and cardinal  $\mathfrak{f}$  is minimal cardinality of a maximal free sequence.

Main result of our collaboration is better understanding of behaviour of possible isomorphism between  $P(\omega)/fin$  and  $P(\omega_1)/fin$  and automorphisms on  $P(\omega)/fin$  it induces. This approach gives us some new hope for possible solution of Katowice problem.

Regarding cardinal  $\mathfrak{f}$ , we were able to observe that results already obtained in [1] are sufficient to establish consistency of  $\mathfrak{u} = \mathfrak{f} < \mathfrak{g} = \mathfrak{s}$  (in the Blass-Shelah model).

We have also conjectured that consistency of  $\mathfrak{f} < \mathfrak{u}$  should be possible to establish using the Shelah’s model for  $\mathfrak{i} < \mathfrak{u}$  and that it is even likely that  $\mathfrak{i} = \aleph_1$  implies  $\mathfrak{f} = \aleph_1$  (if not  $\mathfrak{f} \leq \mathfrak{i}$  in general). However, proper proofs of these conjectures still remain to be done.

It is certain that my collaboration with K.P. Hart established during this visit will continue. We will try progress towards solving the Katowice problem and we also have some conjectures about  $\mathfrak{f}$  to prove.

Due to the unspecific nature of our results it is hard to predict a concrete form of their publication. These results will serve as a basis for ongoing research and as such will be included in future publications.

[1] D. Monk, *Minimum sized maximal free sequences in a Boolean algebra (DRAFT)*