REPORT ON INFTY SHORT VISIT COLLABORATION: SELECTIVE COVERING PROPERTIES AND PRODUCTS

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The theory of selection principles synthesizes a number of classic branches of analysis and topology, studying various aspects and applications of the infinite. These branches include function spaces and convergence properties, the structure of the real line, and topological covering properties. As a coherent theory, it started with the 1996 papers of Scheepers et al. A number of fundamental problems, posed in the classical era of this field, by mathematicians like Menger, Hurewicz, Rothberger, and others, were cracked using this approach. The main goal of the proposed visit was to carry out a substantial part of a program to clarify the productive properties of classic and modern types of sets of real numbers and related spaces, within the framework of selection principles.

We used the whole duration of the supported visit to concentrate on the main goal. We also corresponded, via email, with Arnold Miller, who participates in this project. Some of the main outcomes of this effort can be summarized as follows (for brevity, we provide here only important special cases of the main theorems proved):

(1) The methods of an earlier joint paper of Zdomskyy and myself are applicable to scenarios beyond those mentioned in that paper. In particular, we proved that the product of every concentrated space and every $\mathsf{S}_1(\Gamma,\Gamma)$ space is $\mathsf{S}_1(\Gamma,\mathrm{O})$, thus strictly strengthening a result of Babinkostova and Scheepers concerning products of Luzin and Sierpiński sets of reals.

(2) In every model of $\mathfrak{u} < \mathfrak{g}$ (or semifilter trichotomy), every $\mathfrak{d}$-concentrated space is productively Menger.

(3) Every scale set is productively Menger, productively Hurewicz, and productively Scheepers. These results are proved using an extension of earlier methods of Bartoszyński and Shelah (2001), Bartoszyński and Tsaban (2006), and Tsaban (2011).

(4) If $\mathfrak{d} = \aleph_1$, then every productively Lindelöf space is productively Menger, productively Hurewicz, and productively Scheepers. This extends results of Tall and various collaborators in a recent series of papers of theirs.

(5) Every γ-space remains a γ-space in every extension of the universe by Cohen forcing, but Sierpiński sets do not remain Hurewicz in a Cohen extension. This refutes a recent conjecture of Shceepers and Tall.

We began writing a detailed joint paper reporting these findings. We hope to conclude its writing in at most two months.

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