SCIENTIFIC REPORT ESF INFTY SHORT VISIT (REF: 5544): *HUMAN PERCEPTION OF INFINITY: FORMAL LANGUAGES AND COGNITIVE EXPERIMENTS*

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1. Purpose of the visit

Mankind has always been fascinated by the concept of infinity. In particular, a large body of research in the last century has been inspired by how infinite objects can be manipulated and produced through finitist methods. However, the human cognitive capabilities to perceive and compute sensory objects embodying the concept of potential infinity are neither clear nor well studied. The proposed research constituted an experimental approach to understanding how the concept and perception of infinity are grounded in human cognition.

In my experiments, I tested whether human participants could learn a formal language without being explicitly taught its generating grammar. The question is whether an infinite language can be induced only by exposure to a finite subset of strings. This crucial point is extremely relevant to both computational learning theory and infinity in human language. Every human who learns a language is exposed to a finite set of mostly correct sentences, while Gold's theorem shows that some recursive formal languages are unlearnable from only positive evidence. The paradox is how humans, hearing a finite subset of sentences and unaware of explicit rules, can theoretically still produce a countably infinite set of correct utterances.

This proposed collaboration with an expert in the field was intended to enable a detailed data analysis and its insertion into a quantitative framework, unveiling the human implicit conceptualization of infinite formal languages. Preliminary results from my experiment suggested that the likelihood that a grammatical string is perceived as such decreases with the number of induction steps away from the exposure string set ("induction" strings are perceived as less grammatical than the training ones but more grammatical than strings not in AB^nA). The proposed cognitive-computational approach was intended to provide an experimental complement to the research done in the consortium.

2. Description of the work carried out during the visit

Before the visit took place, I read relevant papers and started the statistical analysis of my human data. While in Amsterdam:

- I had frequent meetings with the host. During the first meeting, we discussed a working plan and specific ideas.
- I worked at the statistical analysis of human data using frequentist methods (Anova, etc.).
- I started developing, together with the host, a Bayesian model selection framework.
- I gave two talks about, among others, the intended topic of research for this short visit. The talks were useful in obtaining feedback from other scientists. The ESF RNP INFTY was duly acknowledged in both cases.
- I met a PhD student with shared interests and we discussed possible collaborations. The current plan is to do a pilot experiment testing human perception and generalization of the recursive Lindenmayer grammars.

3. Description of the main results obtained

3.1. **Methods.** Strings were mapped to auditory stimuli, and a finite subset of well-formed "words", belonging to an infinite formal language, was used to train and test participants. In particular, I used the grammar AB^nA to expose participants to strings with $1 \leq n \leq 3$ and then test them with $n \geq 4$, probing whether the finite training subset of stimuli can induce the infinite language in the participant's mind. A hypothesis is that the higher the number of induction steps, the lower the likelihood that a grammatical string is perceived as such.

3.1.1. *Pitch classes.* The stimuli were designed to evaluate perception and generalization of the generative rule AB^nA, indicating any pattern having one A at the beginning, one at the end, and any number of Bs in between. Any other pattern is a violation of the rule. To experimentally test this rule, habituation and test stimuli were auditory patterns generated by concatenating pure sine wave tones (Phase T) or human syllables (Phase S) belonging to two pitch classes. Apart from this feature (syllables vs. tones) the two phases were identical, containing a habituation part and two tests.

3.1.2. *Phase T: Pure tones.* As and Bs were mapped to two tone classes, high (H) and low (L), consisting of 44 elements each. Tone frequencies in class L were randomly, uniformly sampled from an interval centered at 2kHz; likewise class H, whose mean frequency was 11kHz. Which specific tone was chosen within a class was randomly determined.

During the habituation, participants were exposed to sounds starting and ending with a low tone and containing one to three tones in between. Specifically, participants were habituated to a set of three patterns: LHL, LHHL and LHHHL. These are the shortest possible strings that can be generated by the AB*A rule.

3.1.3. *Phase T: Test 1.* In this first test, test stimuli were evenly distributed between two main classes: stimuli consistent with the training rule and those violating the rule. Consistent stimuli were, in turn, evenly distributed between two subclasses, repetition and extension stimuli, corresponding to increasing pattern generalization levels. Extension stimuli featured a previously unheard number of low tones, generalizing the rule by induction over n (e.g. LHHHHL). Violation stimuli had different lengths and were evenly distributed between those lacking the first low tone or the last one (e.g. LHH, or HHL).

3.1.4. *Phase T: Test 2.* Test 2 consisted of exactly the same patterns as test 1. This time, however, the mapping between low and high tones was inverted, so that As corresponded to high tones and Bs to low frequencies (e.g. HLLLH). Crucially, no further habituation was done between test 1 and test 2.

3.1.5. *Phase S: Human syllables.* As and Bs were mapped to two syllable classes, high (H) and low (L), consisting of 44 elements each. The L class consisted of syllables uttered with a male voice. Likewise, class H contained 44 syllables spoken with a female voice. Excluding this difference, habituation and tests in Phase S were structurally identical to those in Phase T.

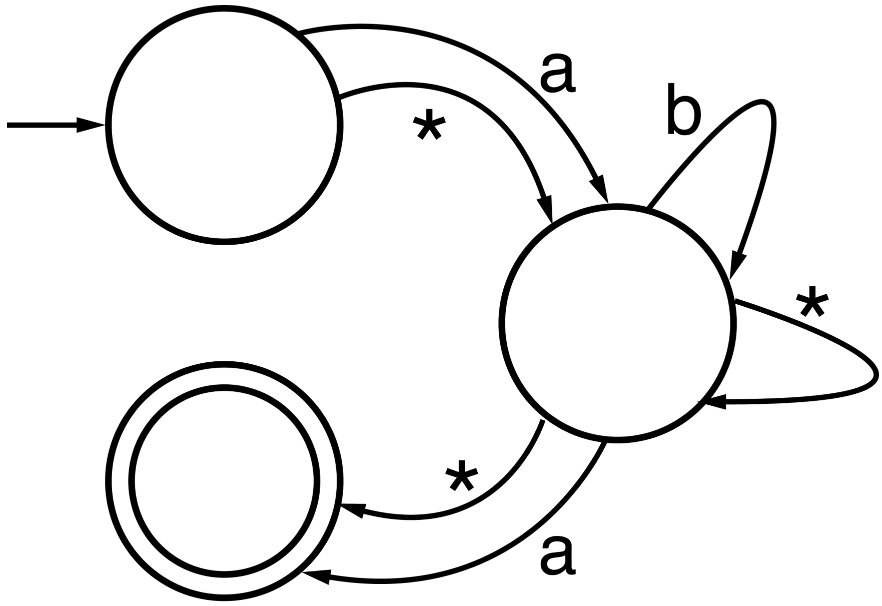
3.1.6. *Participants.* 20 (10 Male, 10 Female) participants between 18-30 years old were involved in the experiment. 10 participants (5 M, 5 F) took Phase T before S, while 10 first Phase S and then T.

3.2. **Frequentist analysis.** (Additional details on request) A detailed analysis confirmed that "induction" strings are indeed perceived as less grammatical than training stimuli, although significantly more grammatical than strings not contained in the target grammar AB^nA. We also found an effect of stimulus quality on performance: participants tested with stimuli composed of spoken syllables perform better than those tested with pure sine-wave tones. In addition, familiarity with the physical-auditory properties of the stimuli plays a role in generalization to more abstract patterns.

3.3. **Bayesian model selection.** Learning of the AB^nA pattern could be "simulated", while participants would be using simpler rules in order to discriminate between grammatical and ungrammatical strings. The assumption here is that the exposure phase will "induce" a grammar in the participants' minds, and AB^nA is only one of the possible induced formal languages. Developing one of the host's ideas, I am working at a model selection approach to find, for each participant, the maximum likelihood finite state machine, induced by the habituation, which could have generated the participant's choices in the experiment. After assigning error probabilities to the transitions in the figure below, the log-likelihood of each hypothetically induced grammar will be calculated for each participants.

I am currntly refining the theoretical framework for the model selection approach sketched above. Once completed, I will write an ad-hoc script in Python which I will use to perform the data analysis.

3.4. **Ongoing work.** Considering the length of the visit (12 days) and the common intention to develop two complementary paradigms, these are still object of ongoing work. It is certainly my intention to keep working on, and complete, the project within a few weeks' time.



4. Future directions of the project and collaboration with host institution

Both the host and I expressed a strong will to keep collaborating on this and other projects. I will continue working at the data analysis and model selection approach. I will prepare an interim report, which I will send to the host in order to receive scientific input and feedback. I will write a journal manuscript with the results of the short visit and subsequent collaborative work.

5. Projected publications / articles resulting or to result from the grant

- Submission of a manuscript to a conference (to be published in its proceedings), and possibly to a journal, encompassing both cognitive aspects, modeling, theoretical implications and future experimental development.
- The applicability of the quantitative and conceptual framework developed with the host is in principle not limited to the specific data set I collected and described here. The model could hence be applied in the future to other datasets (which either were collected by me or other colleagues of mine), meaning that in the long run ESF INFTY would be acknowledged in a number of publications.