Scientific report INFTY grant No. 3908

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During the twelve-day visit to Kobe University, Japan, the researchers (Yurii Khomskii and prof. Jörg Brendle) conducted work on three inter-related topics.

1. *Polarized partitions on the projective hierarchy.* Polarized partition properties are generalizations of the classical Ramsey property which came under attention recently in works of DiPrisco, Todorčević and Zapletal, among others. The authors studied this property for $\mathbf{\Delta}^1_2$ and $\mathbf{\Sigma}^1_2$ sets of reals and obtained several interesting result relating this property to the classical Ramsey property, eventually different reals, unbounded reals and splitting reals. Although most of the work on these questions had already been carried out prior to the INFTY visit, several important details were filled in and completed during this visit too, and the work was concluded to procude a joint publication, entitled "Polarized partition properties on the second level of the projective hierarchy", to be submitted soon to the Annals of Pure and Applied Logic.

2. $\aleph_1$*-perfect mad families.* Work on this topic was the main purpose of the visit. The researchers studied maximal almost disjoint (mad) families of subsets of the natural numbers, with the additional property that they can be seen as unions of $\aleph_1$ perfect sets. The innovation and interest in studying these "$\aleph_1$-perfect mad families" lies in their indestructibility properties by forcing. Whereas in previous results, indestructibility of mad families usually talked about the literal preservation of the mad family (as a subset of $[\omega]^\omega$), the new situation allows the individual perfect sets to be re-interpreted in the forcing extensions. This, for instance, makes it possible to add dominating reals by forcing while preserving a mad family with a $\mathbf{\Sigma}^1_2$-definition, thus answering an open question from [3, Question 16] about the consistency of $\mathfrak{b} > \omega_1$ plus the existence of a $\mathbf{\Sigma}^1_2$ mad family.

   The researchers also defined the cardinal invariant number $\mathfrak{a}_B := \min\{\kappa \mid$ there is a mad family which is a union of κ Borel sets$\}$, and proved several non-trivial results about it, among them the consistency of $\mathfrak{a}_B < \mathfrak{a}$ (where $\mathfrak{a}$ denotes the *almost disjointness number*, i.e., the minimal size of *any* mad family.) Many questions about this cardinal number remain unknown.

   During this visit several new results were obtained. Some of the open questions were not solved successfully but were discussed and future research on them has been planned. This work will result in at least one publication "$\aleph_1$-perfect mad families", possibly followed by other publications after more work has been done.

3. *Laver-, E_0, Silver-quasigenerics and splitting reals.* Several questions for further research were discussed, related to the open questions in [1, 2] and those in the paper from section 1 above (polarized partitions), concerning

the relationship between Laver-quasigeneric reals, Silver-quasigeneric reals, E_0-quasigeneric reals and splitting reals. The conjecture that Laver forcing does not add any of these reals was put forward and studied. No results have been obtained yet but there are some concrete plans for further research.

The researchers intend to continue their collaboration and work on topics 2 and 3 from above, in the near future. Possible visits include a visit of prof. Brendle to Amsterdam in March/April 2011, and possible repeat visits of Yurii Khomskii to Kobe University.

References

[1] Jörg Brendle, Lorenz Halbeisen, Benedikt Löwe, *Silver measurability and its relation to other regularity properties*, Mathematical Proceedings of the Cambridge Philosophical Society 138 (2005), pp. 135-149.

[2] Jörg Brendle, Benedikt Löwe, *Eventually different functions and inaccessible cardinals*, to appear in: Journal of the Mathematical Society of Japan 63 (2011).

[3] Sy-David Friedman, Lyubomir Zdomskyy, *Projective mad families*, Annals of Pure and Applied Logic 161 (2010), pp. 1581–1587.