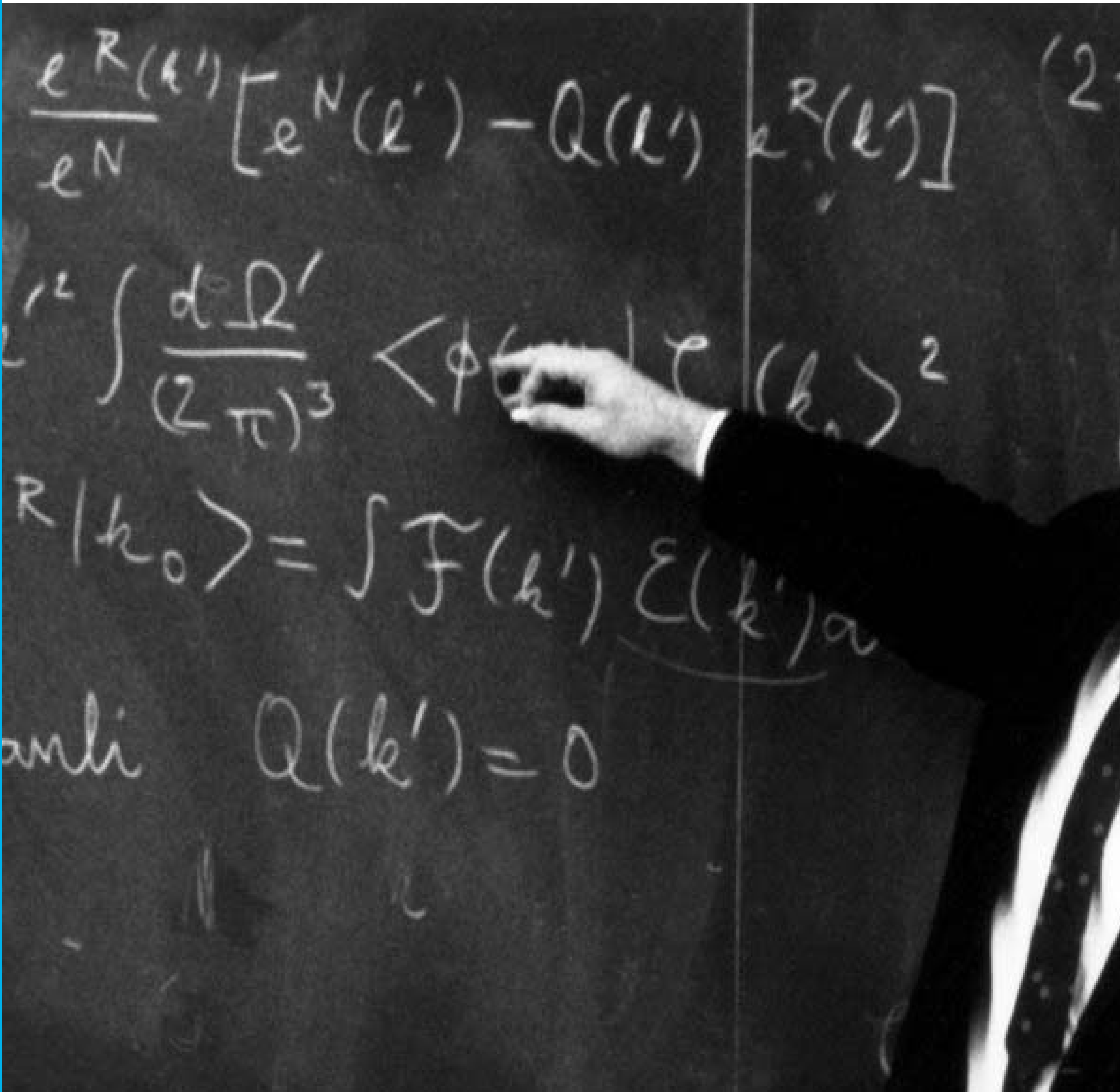
EUROPEAN
SCIENCE
FOUNDATION

Advanced Mathematical Methods for Finance (AMaMeF)

An ESF Standing Committee for Physical and Engineering Sciences (PESC) Research Networking Programme



The application of advanced mathematical methods to problems in finance has made impressive progress in the last decade.

Stochastic analysis, control theory, differential equations, and numerical methods have been used extensively for financial modelling, analysis and software development. In particular, techniques from various fields must be put together to improve numerical approaches and cope with their computational complexity.

The use of sophisticated financial instruments, particularly with the current widespread use of derivatives, makes the development of such mathematical tools of essential importance for financial institutions of all types, including investment and commercial banks, fund management and insurance companies, hedge funds, regulating bodies, corporate and state treasuries.

The AMaMeF programme aims at the development and application of advanced mathematical tools in finance with two main overall goals.

The first goal is the creation and reinforcement of relationships among European research teams in the fields of stochastic analysis, control theory, differential equations and other mathematical disciplines, with the purpose of undertaking and carrying out highly innovative interdisciplinary and interactive research in mathematical finance and its applications.

The second goal is the cultivation and maintenance of strong and mutually reinforcing links with the financial industry, with a view to enhancing the impact and influence of mathematical research, and to providing an effective scientific response to the many new European financial challenges.

The running period of the ESF AMaMeF research networking programme is for five years from April 2005 to March 2010.

Cover:
Hans Bethe at the blackboard at Cornell University in 1967.
Mathematics opens new research directions in Finance

Introduction

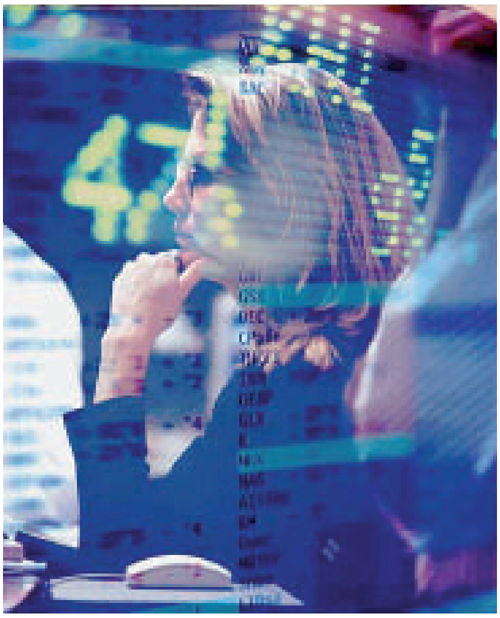
AMaMeF is addressed to the following main research areas, with a strong focus on interdisciplinary interactions: 1) mathematical foundations of financial analysis, 2) numerical methods in finance, 3) applications to financial modelling.

1.

The modern key framework required for the study of the dynamics and statistical properties of asset prices is stochastic analysis. The European mathematical community is strong. One of the main goals of the programme is to build upon this strength and to consolidate exisiting efforts. In particular AMaMeF is engaged in investigating many aspects of the mathematical foundations and the modelling of financial dynamics using existing tools of stochastic analysis and providing new ones as required. For example, a primary use of stochastic analysis in financial matters is in asset pricing. Conventionally, the asset price processes are assumed to be continuous and driven by Brownian motion. The Black-Scholes model (1973) was introduced in this context. However, there is now great interest in modelling asset prices also as discontinuous processes. These allow more flexibility in the sources of randomness driving the dynamics, resulting in better data fitting and more refined financial analysis. In order to proceed in this approach new tools are necessary and AMaMeF is intensively engaged in this direction of research.

2.

Numerical methods play a crucial role for the applications of mathematical tools to finance, especially for those applications involving a very large number of variables. This leads to challenging problems for which techniques from various fields (ranging from stochastic analysis and differential equations to Monte Carlo methods) must be put together to improve numerical approaches and cope with their computational complexity. AMaMeF will be mostly engaged in improving numerical methods for pricing and hedging products and their implementation, calibration, sensitivity and risk analysis.



Women have an increasing role in Financial Mathematics

3.

AMaMeF aims to improve the applications to financial modelling. There are many topics in this area that are to be pursued collaboratively within the scope of the programme. Among the most significant areas of applications are default/credit risk, modelling asymmetric information on pricing models, optimal portfolio management, risk-sensitive optimal control, public debt management. An Advisory Board, consisting of industry-based experts in mathematical finance and its applications in a practical context, has the role of advising and assisting the Steering Committee of AMaMeF in ensuring the appropriate and timely dissemination of the results and the knowledge arising from the programme at a full European level.

Scientific objectives

Improving the general understanding of the more mathematically demanding aspects of financial modelling is a matter of the utmost importance for future European financial policies, both for public and private institutions. At present there are many valuable research activities in Europe which lack a unifying framework. The main issues of the AMaMeF programme are at two levels:

– The mathematical level: bringing together complementary research groups over Europe. The programme will bring together theoretical and numerical researchers to improve their interaction, multidisciplinarity, and training. AMaMeF will also provide a unifying platform for research and exchange of knowledge.
– The applied level: it is a key requirement that new theoretical and computational research be undertaken with a clear appreciation of the needs of the practitioner community. AMaMeF will promote interaction with this community and diffusion of methods and results.

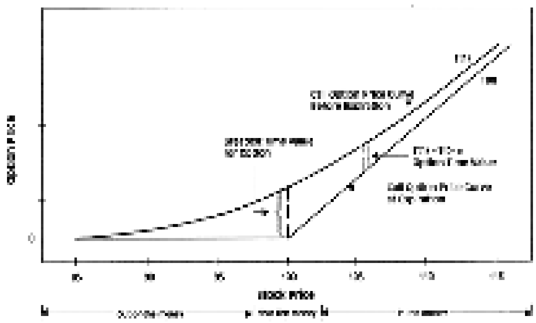
The AMaMeF programme addresses the following main research areas, with a strong focus on interdisciplinary interactions:

Mathematical foundations of financial analysis

Stochastic analysis and calculus.
One of the key tools required for the study of the dynamics and statistical properties of asset prices is stochastic analysis, which is a powerful modern synthesis of classical mathematical analysis and probability theory. Some of the areas that will be pursued are as follows:

a) ***The use of Lévy processes as a basis for the formulation of realistic, testable models for asset price dynamics.*** The goal is to establish new mathematical tools appropriate for addressing hedging issues in markets where asset prices undergo discontinuities. These issues constitute one of the main long-term concerns in the banking sector, and it is our intention that significant new mathematical input will be made in this area. This is a topic where significant research efforts are already underway in a number of the groups represented in this programme.

b) ***Mathematical foundations of long-term asset liability management.*** The goal is to provide the mathematical foundations for a general framework appropriate for the treatment of long-term asset liability management problems. Parsimony will be taken as a guiding criterion in the formulation of modelling methodologies to ensure the resulting models can be statistically tested using market data.



Call option price curve vs. Stock price (source: IAC-CNR)

c) ***Fractional Brownian motion and related processes.*** This is an area that is being widely investigated as a basis for asset price dynamics. Since the fractional Brownian motion is neither a Markov process nor a semimartingale, many of the usual tools of stochastic anaysis are no longer directly applicable for studying these processes. The development of alternative methods based, for example, on spectral theory, is therefore of particular importance, and this is one of the intentions of the programme.

d) ***Functional central limit theorems for semimartingales.*** The goal in this investigation is to obtain a deeper understanding of the random process most commonly used as a basis for modelling asset prices.

e) ***Modelling asymmetric or imperfect dynamical information***, is another major topic of investigation, addressed with several methods as filtration enlargement, forward integral, additional utility, or Bayesian dynamical modelling.

f) ***Rare events and risk management.*** Essential for the use of stochastic processes within risk management is their extreme behaviour. This governs the measurement of risk in finance and insurance. The development of stochastic tools within the framework of large deviations and extreme value theory is a priority in this connection that complements the probabilistic and statistical properties of financial models.

Stochastic control and differential equations.
Stochastic control theory has been widely developed in recent years for diffusions, jump-diffusions, and Lévy processes: both probabilistic and differential equations methods were successfully employed. A major field of research in mathematical finance is the construction of hedging strategies that are optimal in terms of some criterion of risk-minimisation.

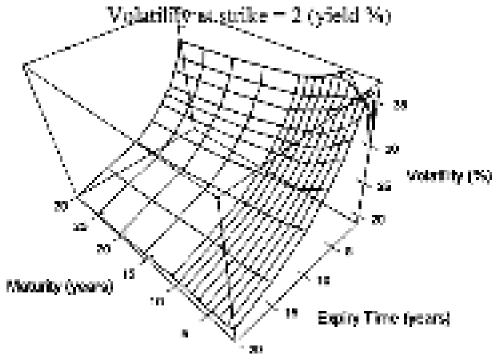
a) ***Risk-sensitive dynamic asset management theory.*** A large variety of applied control problems rely on the maximisation of a multiobjective functional. Risk-sensitive dynamic asset management theory has been introduced to tackle such problems. Nevertheless the complete approach to the subject, including the case of partial observation of factors and/or transaction costs, is an open and challenging problem.

b) ***Realistic modelling methodologies.*** Improvements and insights are needed to deal with realistic modelling, including singular control, impulse control, measures of risk, and irreversible/reversible investments. Solutions will require combinations of new ideas with techniques from fields such as infinite-dimensional convex analysis, robust statistics, backward stochastic differential equations, non-linear partial differential equations, viscosity solutions, quasi-variational inequalities.

c) ***Infinite-dimensional Hamilton-Jacobi-Bellman equations.*** Such equations arise in the stochastic control of evolution equations in Banach (Hilbert) spaces, with applications in the Musiela-type models of interest rate dynamics. Optimal control of the Duncan-Mortensen-Zakai equation will also be treated.

d) ***American options.*** There are important and difficult problems related to American options pricing and hedging. Various mathematical tools can be involved: probabilistic optimal stopping theory, reflected backward stochastic differential equations, viscosity solutions, variational inequalities, free boundary problems.

e) ***Parabolic integro-differential equations.*** The use of Lévy processes has led to a wealth of new problems related to the associated parabolic integro-differential equations and inequalities. Moreover option pricing models based on pure jump processes lead to degenerate cases, recently considered on the theoretical side by members of the programme using the viscosity solution approach and the variational formulation.



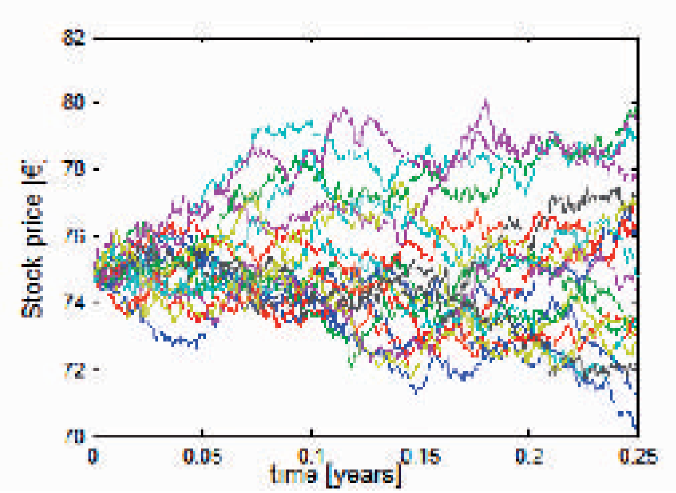
Volatility surface plotted against Expiry time (in years) and Maturity (in years)
(source: IAC-CNR)

Numerical methods in finance

Numerical methods are crucial to the applications of mathematical tools to financial modelling. The main topics of interest of the programme are related to numerical methods for pricing and hedging products and their implementation, calibration, sensitivity and risk analysis.

Numerical methods for pricing and hedging.
Encompasses Monte Carlo methods, Malliavin calculus, probabilistic numerical methods and the study of numerical schemes for partial differential equations. Moreover, the increasing interest in financial modelling with jump processes has led to a wealth of new theoretical and numerical problems related to jump-diffusion, Lévy processes and the associated integro-differential equations and inequalities. The case of jump-diffusion models with finite intensity has recently been extended to models based on Lévy processes and non-linear problems. Future work will be addressed to the development of efficient, adaptive pricing algorithms for contracts on Markov processes with unbounded jump intensity and also for stochastic volatility models. Various non-linear problems and efficient multidimensional algorithms still remain to be explored and the high dimensional state spaces of many financial problems lead to challenging questions for which techniques from various fields (differential equations, Monte Carlo methods, stochastic analysis) can be put together to improve numerical approaches.

Stopping time problems.
A sizeable fraction of traded option contracts involve American or Bermudan exercise, and for this reason optimal stopping continues to be a key area of



Monte Carlo simulation of stock price evolution
(Technischen Universität München)

research. There has been important recent progress, for example in the theory of 'universal signals' and in various approaches to solving optimal stopping by Monte Carlo methods.

Mathematical theory of market calibration analysis.

The problem of calibration, related to the so-called 'volatility smile', is a crucial issue in equity, FX and interest rates markets. Research in this field is in full development. One of the goals of the proposed programme is to draw together these various strands of theoretical and empirical work into a coherent mathematical framework for market calibration analysis, which can then be used in a banking context and for other risk management purposes.

Efficient computation of market parameter sensitivites.

The computation of 'greeks' is crucial to hedging and portfolio management. Many problems related to computational efficiency (in particular variance reduction) remain to be solved. Recent research has been concerned with the mathematical modelling of risk analysis. The main goal will be the establishment of highly efficient new computational algorithms for the calculation of market sensitivities for complex portfolios and highly structured trading positions. This will include computation of the Value at Risk, and extreme-value problem.

Applications to financial modelling

There are many topics in this area that are to be pursued collaboratively within the scope of the programme. Some especially significant examples are as follows.

a) ***Default/Credit risk.*** A major cause of concern in managing the credit risk of most financial institutions is the occurrence of a disproportionate number of joint defaults of different counterparts. Joint default events also have an important impact on the performance of derivative securities, whose payoff is linked to the loss of a whole portfolio of underlying bonds or loans such as collaterised debt obligations. Precise mathematical models for the loss in a portfolio of dependent credit risks are needed to adequately measure and price this risk. Progress in this area will be of great importance to the efficiency of financial institutions exposed to credit risk.

b) ***Statistical inference for Lévy processes.*** Lévy processes are currently extensively studied as a modelling device for stock prices and for stock price volatility. Non-parametric procedures to estimate the Lévy measure is one of the research topics. Progress in realistic mathematical modelling of financial markets, based on careful study of empirical data from the markets, will be an essential part of improved understanding and monitoring of the markets. Members of the programme group, representing comprehensive expertise in stochastics, statistics and econometrics, are contributing over a broad spectrum to this area.

c) ***Modelling asymmetric information on pricing models.*** The asymmetry of information on financial markets is a crucial source of risk. We deal with financial market models in which, both on a mesoscopic as well as on a microscopic level, asymmetry of information and its consequences for risk and utility of individual agents are treated.

d) ***Optimal portfolio management.*** Traditional portfolio management is based on mean-variance analysis. Both in academia and in the financial industry, however, there is a growing emphasis on a new class of monetary measures of risk which satisfy certain consistency requirements and reflect the fundamental uncertainty of the future asset price evolution. The analysis of the corresponding investment and optimal hedging strategies is a topical research issue.

e) ***Risk-sensitive optimal control.*** A large variety of applied control problems relies on the maximisation of a multiobjective functional. Since such multiobjective control problems are hard to solve, they are usually replaced by single-objective problems with constraints, which are also not easy to solve. Risk-sensitive dynamic asset management theory considers a portfolio optimisation problem in the case when a number of macroeconomic and financial factors are useful to forecast future assets returns. Many questions remain to be answered.

f) ***Public debt management.*** Among the activities requiring strategic investment planning is the problem of public debt management, which is a challenging and little-studied task. It differs from conventional asset allocation problems because of legal and policy constraints. The Growth and Stability Pact (GSP), subscribed by the countries of the European Union, introduces deficit stabilisation as a new objective of debt management. The choice of an issuing security policy to achieve optimal debt composition is in order.

Activities

Workshops and conferences

To promote interaction between scientists, and between scientists and practitioners, and also to provide a platform for the interchange and spread of knowledge, AMaMeF will organise workshops in specific topics of the programme and also general conferences (one or two per year).
AMaMeF will support with co-funding certain scientific activities arising in Europe that are related to the programme.

Training

AMaMeF will support the organisation of schools for students and young researchers where experts in the various fields of the programme will be invited to give courses.

Grants

– ***Short term visits*** for approximately a total of 25 weeks per year can be awarded to promote the mobility of researchers. This will provide the establishment or the strengthening of scientific links at the individual level. These grants are complementary to workshops and conferences in achieving the goals of the programme. These fellowships are for a period of no more than 15 days.
– ***Exchange grants*** for approximately a total of 25 weeks per year can be awarded to specific individual projects within the programme. These are for longer projects aiming to facilitate the transfer of knowledge and techniques relevant to the individual and the hosting research group. These fellowships are for a period from 15 days up to 6 months.

Further details about proposals and applications can be found at the website of AmaMeF:
www.iac.rm.cnr.it/amamef

More information about the programe and its activities, technical reports and an on-line newsletter can be found at this website.
Any private or public institution is welcome to take part in the activities of AMaMeF with due acknowledgement.

Funding

ESF research networking programmes are principally financed by the Foundation's Member Organisations on an *à la carte* basis. AMaMeF is supported by:
Fonds zur Förderung der wissenschaftlichen Forschung, Austria; Fonds voor Wetenschappelijk Onderzoek-Vlaanderen, Belgium; Forskningsrådet for Natur og Univers, Denmark; Suomen Akatemia/Finlands Akademi, Finland; Institut Europlace de Finance, France; Deutsche Forschungsgemeinschaft, Germany; Consiglio Nazionale delle Ricerche (Istituto per le Applicazioni del Calcolo "M. Picone" and Università LUISS – Guido Carli di Roma), Italy; Nederlandse Organisatie voor Wetenschappelijk Onderzoek, Netherlands; Norges Forskningsråd, Norway; Polska Akademia Nauk, Poland; Vetenskapsrådet, Sweden; Schweizerischer Nationalfonds zur Förderung der wissenschaftlichen Forschung/Fonds national suisse de la recherche scientifique, Switzerland; Scientific and Technical Research Council of Turkey, Turkey; Engineering and Physical Sciences Research Council, United Kingdom.

EUROPEAN
SCIENCE
FOUNDATION

European Science Foundation
1 quai Lezay-Marnésia, BP 90015
67080 Strasbourg cedex, France
Tel: +33 (0)3 88 76 71 00
Fax: +33 (0)3 88 37 05 32
www.esf.org

AMaMeF Steering Committee

Professor Bernt Øksendal (Chair)
Centre of Mathematics for Applications
Department of Mathematics
University of Oslo
PO Box 1053 Blindern
0316 Oslo • Norway
Tel: +47 228 55913
Fax: +47 228 54349
E-mail: oksendal@math.uio.no

Professor Giulia di Nunno
(Vice-Chair)
Centre of Mathematics for Applications
Department of Mathematics
University of Oslo
PO Box 1053 Blindern
0316 Oslo • Norway
Tel: +47 228 55854
Fax: +47 228 54349
E-mail: giulian@math.uio.no

Professor Ole Barndorff-Nielsen
Department of Mathematical Sciences
University of Aarhus
8000 Aarhus • Denmark
Tel: +45 89423521
Fax: +45 86131769
E-mail: oebn@imf.au.dk

Professor Tomas Björk
(from 2005 to 2007)
Department of Finance
Stockholm School of Economics
PO Box 6501
113 83 Stockholm • Sweden
Tel: +46 8 736 91 62
E-mail: Tomas.Bjork@hhs.se

Professor Mark Davis
Department of Mathematics
Imperial College London
London SW7 2AZ • United Kingdom
Tel: +44 20 7594 8486
Fax: +44 20 7594 1191
E-mail: mark.davis@imperial.ac.uk

Professor Lane Hughston
(Head of the Advisory Board)
Department of Mathematics
King's College London
Strand
London WC2R 2LS
United Kingdom
Tel: +44 20 7848 2855 (direct line)
Tel: +44 20 7848 2217 (general office)
Fax: +44 20 7848 2017
E-mail: lane.hughston@kcl.ac.uk

Professor Claudia Klüppelberg
Technische Universität München
Zentrum Mathematik
Lehrstuhl für Mathematische Statistik
Boltzmannstr. 3
85747 Garching • Germany
Tel: +49 089 289 17432
E-mail: cklu@ma.tum.de

Professor Hayri Körezlioglu
Institute of Applied Mathematics
Middle East Technical University
06531 Ankara • Turkey
Tel: +90 312 210 5606
E-mail: hayri@metu.edu.tr

Professor Damien Lamberton
Laboratoire d'Analyse et de Mathématiques Appliquées
Université de Marne-la-Vallée
Cité Descartes
5, Boulevard Descartes
Champs sur Marne
77454 Marne-la-Vallée cedex
France
Tel: +33 1 60 95 75 20
Fax: +33 1 60 95 75 45
E-mail: dlamb@math.univ-mlv.fr

Dr. Benedetto Piccoli
Istituto per le Applicazioni del Calcolo "M. Picone"
Consiglio Nazionale delle Ricerche
Viale del Policlinico 137
00161 Roma • Italy
Tel: +39 06 88470244
Fax: +39 06 4404306
E-mail: b.piccoli@iac.cnr.it

Professor Uwe Schmock
Financial and Actuarial Mathematics
Vienna University of Technology
Wiedner Hauptstraße 8-10/105-1
1040 Vienna • Austria
Tel: +43 1 58801 10520
Fax: +43 1 58801 10599
E-mail: schmock@fam.tuwien.ac.at

Professor Christoph Schwab
Seminar for Applied Mathematics
ETH Zurich
HG G58.1
8092 Zurich • Switzerland
Tel: +41 44 632 3595
Fax: +41 44 632 1104
E-mail: schwab@sam.math.ethz.ch

Professor Peter Spreij
Korteweg-de Vries Institute for Mathematics
Universiteit van Amsterdam
Plantage Muidergracht 24
1018 TV Amsterdam • Netherlands
Tel: +31 20 5256070
Fax: +31 20 5255101
E-mail: spreij@science.uva.nl

Professor Lukasz Stettner
Department of Probability Theory and Matematics of Finance
Institute of Mathematics of the Polish Academy of Sciences
Sniadeckich 8
PO Box 21
00-956 Warsaw • Poland
Tel: +48 22 5228 100
Fax: +48 22 629 39 97
E-mail: L.Stettner@impan.gov.pl

Professor Johan Tysk
(from 2008 to 2009)
Department of Mathematics
Uppsala University
PO Box 480
751 06 Uppsala • Sweden
Tel: +46 18 471 32 08
E-mail: Johan.Tysk@math.uu.se

Professor Esko Valkeila
Institute of Mathematics
Helsinki University of Technology
Box 1100 02015 TKK • Finland
Tel: +358 9 451 3028
Fax: +358 9 451 3016
E-mail: valkeila@cc.hut.fi

Professor Michèle Vanmaele
Department of Applied Mathematics and Computer Science
Ghent University
Krijgslaan 281, S9
9000 Gent • Belgium
Tel: +32 9 264 4895
Fax: +32 9 264 4995
E-mail: Michele.Vanmaele@UGent.be

Programme coordinator

Dr. Roberto Natalini
Istituto per le Applicazioni del Calcolo "M. Picone"
Consiglio Nazionale delle Ricerche
Viale del Policlinico 137
00161 Roma • Italy
Tel: +39 06 88470257
Fax: +39 06 4404306
E-mail: r.natalini@iac.cnr.it

The AMaMeF Advisory Board chaired by Professor Lane Hughston (Department of Mathematics, King's College London) is composed of nine members. They are all very well-known individuals in the mathematical finance community who work as practitioners in investment banks and other financial institutions. It is envisaged to recruit a few more individuals from financial institutions in the European capitals. The full list of members can be found at www.iac.rm.cnr.it/amamef

ESF Liaison

Dr. Patricia Arsene
Science

Ms. Carole Mabrouk
Administration

Physical and Engineering Sciences Unit (PESC)
European Science Foundation
1 quai Lezay-Marnésia
BP 90015
67080 Strasbourg cedex • France
Tel: +33 (0)3 88 76 71 69
Fax: +33 (0)3 88 37 05 32
E-mail: cmabrouk@esf.org

For the latest information on this research networking programme consult the AMaMeF websites:
www.iac.rm.cnr.it/amamef
and
www.esf.org/amamef

April 2006 – Print run: 1500