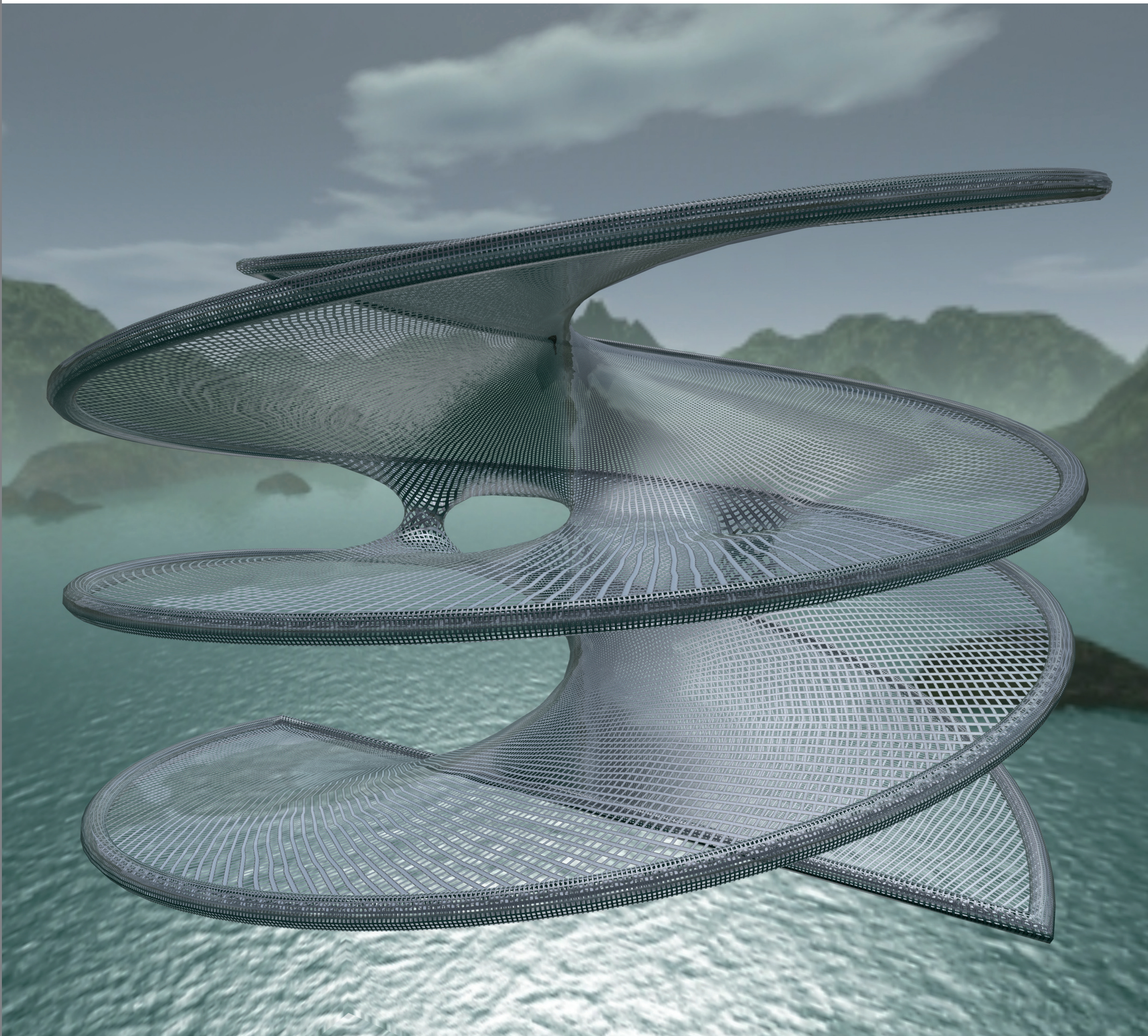
EUROPEAN SCIENCE FOUNDATION

SETTING SCIENCE AGENDAS FOR EUROPE

RESEARCH NETWORKING PROGRAMME

METHODS OF INTEGRABLE SYSTEMS, GEOMETRY, APPLIED MATHEMATICS (MISGAM)

Standing Committee for Physical and Engineering Sciences (PESC)



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The European Science Foundation (ESF) was established in 1974 to create a common European platform for cross-border cooperation in all aspects of scientific research.

With its emphasis on a multidisciplinary and pan-European approach, the Foundation provides the leadership necessary to open new frontiers in European science.

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Headquartered in Strasbourg with offices in Brussels, the ESF's membership comprises 75 national funding agencies, research performing agencies and academies from 30 European nations.

The Foundation's independence allows the ESF to objectively represent the priorities of all these members.

Cover picture : Helicoid with two handles; minimal surface constructed using the methods of the theory of integrable systems.

A.Bobenko, U.Pinkall and M.Schmies

Introduction

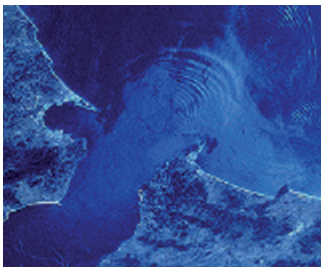
Integrable systems and random matrices nowadays arise in various problems of pure and applied mathematics and statistics.
Integrable systems with finite degrees of freedom have been the object of intense investigation by mathematicians and physicists for many centuries because of their applications in mechanics and geometry. The trajectories of integrable mechanical systems display a regular behaviour as opposed to a chaotic one.

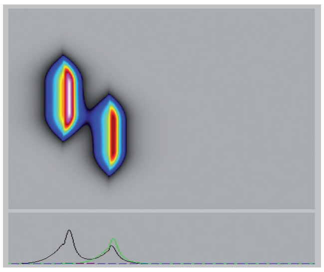
The discovery of integrable behaviour in classical and quantum physical systems with infinitely many degrees of freedom is one of the most exciting developments of Mathematical Physics in the second half of the 20th century. Such systems arise in fluid dynamics, classical and quantum field theory, statistical physics, optical communications. Algebraic and differential geometry play a prominent role in the mathematical foundations of the theory of integrability.

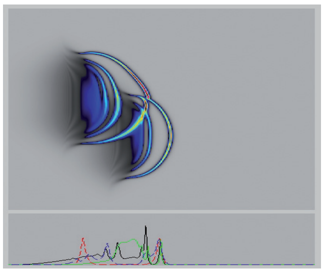
The theory of random matrices originates from the study of energy levels of heavy nuclei. It proved to be efficient also in statistics, combinatorics and in studying growth processes. Recent developments in random matrix theory are intimately connected with the theory of integrable systems.
The aim of the Methods of Integrable Systems, Geometry, Applied Mathematics (MISGAM) Programme is to join the efforts of European researchers actively involved in different aspects of the theory of integrable systems and random matrices. The mathematical techniques developed have to be transformed into working tools for Applied Mathematics. A significant boost to this domain of research in Europe will certainly contribute to the spread of these new ideas in a large number of related fields.

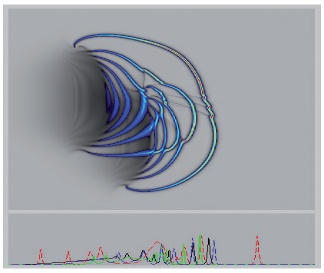
MISGAM unifies the European efforts on this exciting interdisciplinary project creating a fruitful training ground for young researchers.

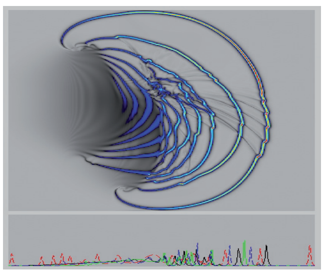
The running period of the ESF MISGAM Research Networking Programme is for five years from July 2004 – June 2009.











The figures show numerical simulations of the evolution and interaction of singular solutions of the EPDiff equation from DD Holm and MF Staley. EPDiff is short for Euler-Poincare equation on the diffeomorphisms. EPDiff may be derived as a shallow water equation, which applies to the propagation and interaction of nonlinear internal waves.
The surface effects of these internal waves are observed in Nature, for example, on the Ocean surface, such as at the Strait of Gibraltar shown in the figure above. In the figures, two straight segments are initialized moving rightward.
The one behind has twice the speed of the one ahead, and the two segments are offset in the vertical direction.
The segments each break into curved strips of a certain width and these undergo overtaking collisions.
The segments retain their integrity as they expand until the overtaking collision occurs. Upon overtaking a slower segment, the faster segment transfers its momentum to the slower one ahead and a remarkable "reconnection" or "melding" of the segments may occur.

Scientific Background

Integrable systems have been studied since the 18th century by the giants of geometry and mechanics, like Euler, Hamilton, Jacobi and Lagrange, who revealed the deep symplectic and algebro-geometric nature of these problems. Around 1895 Korteweg and de Vries (KdV) described shallow water waves by means of a celebrated nonlinear differential equation which is shown to have soliton solutions and has many of the features similar to those of the 18th century integrable mechanical problems. In the late 1960s it was discovered that the Schrödinger equation whose potential is given by a solution of the KdV equation has a spectrum that remains invariant in time. It means that the theory of KdV flows makes part of the spectral theory of the Schrödinger equation known to be of fundamental importance for Quantum Mechanics.

These discoveries stimulated an outburst of ideas which marked the beginning of the modern theory of integrable systems. This led to the discovery of the Hamiltonian nature of the KdV equation and its generalisations like the Kadomtsev-Petviashvili equation to algebro-geometric solutions and the Lie-theoretic description to the tau-function representation of solutions, just to name a few.

Geometry and integrable systems grew even closer after the discovery that the celebrated KdV equation plays a fundamental role in the matrix and topological models of two-dimensional quantum gravity. Namely, the partition function of 2D gravity turns out to coincide with the tau-function of a particular solution to the KdV hierarchy. More recent investigations, stimulated and motivated by the theory of Gromov - Witten invariants of compact symplectic manifolds, suggest that the constitutive rules of a wide class of integrable partial differentials equations are encoded in certain topological characteristics of moduli spaces of algebraic curves and their mappings. It is a challenge to uncover and explain this novel relationship between the theory of integrable systems and geometry and to make these new ideas work in applied mathematics.

Random matrix models play an important role in the following intimately related problems.

- The distribution of the eigenvalues of a random matrix, having a certain symmetry condition to guarantee the reality of the spectrum, is given by matrix integrals, which are related to solutions of integrable systems.
- The problem of the statistics of the length of the longest increasing sequence in random permutation or random words (Ulam's problem) has recently been reduced to questions on random matrices.
- Integrals over groups and symmetric spaces lead to a variety of interesting matrix models, which satisfy non-linear integrable differential equations. As a striking feature, they also satisfy Virasoro constraints and the coefficients of the expansions are combinatorial or topological quantities.
- The sample covariance matrix of a Gaussian population is used to estimate the true covariance matrix of that population and the study of the statistical distribution of the largest eigenvalues of the sample covariance matrix requires random matrix technology and has led to useful results in statistics where the sizes of the sample and the population are of the same order. This is actively being applied to medical statistics, financial mathematics and communication technology.

The four problems above and their time-perturbations are all solutions to integrable equations or lattices. Matrix integrals point the way to new integrable systems, and also to the formulation of new combinatorial and probabilistic problems.

Further progress in integrable theory, as well as possible applications to real physical problems, will strongly depend on understanding the asymptotic integrability of classical and quantum systems depending on a parameter. The Whitham averaging technique proved to be an efficient tool not only in the asymptotic theory of nonlinear waves but also in quantum field theory and in the theory of random matrices. Geometric and analytic foundations of the averaging methods rely upon the study of behaviour of trajectories of integrable systems in the neighbourhood of a finite dimensional invariant manifold. The results of this study are used in applying dispersive integrable limits to weak turbulence.

Surface with constant mean curvature
and rotationally symmetric metric described
by a solution of a Painlevé equation.
A.Bobenko and U.Eitner

Research Topics

Deformation theory of integrable systems and Frobenius manifolds

The modern theory of integrable systems began with the discovery that certain remarkable nonlinear evolutionary Partial Differential Equations (PDEs) are integrable, such as the Korteweg - de Vries (KdV) equation, nonlinear Schrödinger equation, Toda lattice, etc. Nevertheless the classification of systems of integrable PDEs remains an open problem.

The perturbative approach to the classification problem proved to be successful in classifying systems of spatially one-dimensional integrable evolutionary PDEs admitting the hydrodynamic limit. The moduli space of the PDEs is closely related with the moduli space of semisimple Frobenius manifolds. The families of PDEs classified in this way possess properties inspired by 2D topological field theory. The bihamiltonian structure of the integrable PDEs constructed in this way can be considered as a deformation of classical W-algebras associated to simply-laced simple Lie algebras. We study the full symmetry algebra of these PDEs in order to produce deformations of quantum W-algebras. We also plan to develop an alternative construction of the families of integrable PDEs in terms of a suitable class of infinite-dimensional Lie algebras.

The geometry of integrable Hamiltonian PDEs possesses much of the richness of finite dimensional integrable systems, together with other features coming from classical analysis of partial differential equations. In this framework, a satisfactory Hamilton - Jacobi theory for integrable evolutionary PDEs, as an effective tool to actually find solutions to these PDEs, is still under construction. The interest in such a theory has grown in relevance in connection with the so-called symplectic field theory. This theory suggests a new link between Gromov - Witten invariants of symplectic manifolds and integrable hierarchies. The potential of these invariants can be calculated as the value of the Hamilton - Jacobi generating functional in certain points of the Lagrangian subspace.

Integrable systems, singularity theory, and deformation theory of algebraic varieties

There are three completely different sources of Frobenius manifolds:
- Hierarchies of integrable partial differential equations;
- Quantum cohomology;
- Singularity theory.

The main questions are: how much of the rich structure of the theory of integrable partial differential equations and of the theory of Gromov - Witten invariants can be constructed for the Frobenius manifolds in singularity theory? What is the role of the tt* geometry in the theory of integrable partial differential equations as well as in the theory of Gromov - Witten invariants? More generally, the interplay between integrable systems and generalisations of variations of Hodge structures attached to families of non-commutative objects, such as A-infinity algebras or categories, is under investigation. Furthermore a development of a general approach to the problem of relationships between integrable systems with deformation theory of algebraic varieties and singularities is currently in progress.

Quantisation and integrable systems

The comparison of semiclassical techniques with methods based on the theory of Lie superalgebras and with approaches based on deformation quantisation leads to two classes of problems.

The first one concerns quantum many particles and/or spin systems. A current area of research is the study of the deformed quantum Calogero - Moser (CM) problems in relation with Lie superalgebras and symmetric superspaces. Preliminary results suggest that these issues are related to the solutions of the WDVV equations. We are deepening this line of research to clarify to what extent deformations of the quantum CM model can provide new solutions to the WDVV associativity equations. The second class of problems focuses on the quantisation of integrable systems and also comprises Topological Field Theory (TFT). We are mainly studying the problems in TFT associated to the quantisation of Poisson diffeomorphisms and of more general Poisson maps.

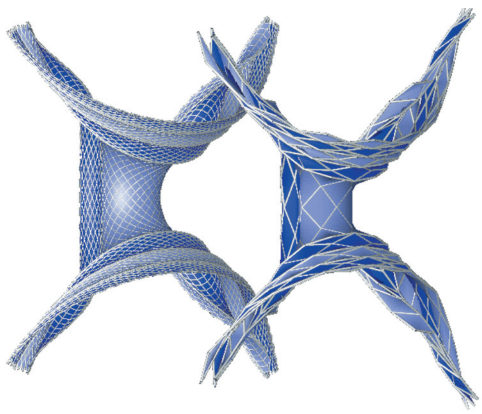
Applications of integrable systems to geometry of surfaces and to its discrete generalisations

Differential equations describing interesting special classes of surfaces (as well as classical parametrisations of general surfaces) are integrable, and, conversely, many important integrable systems admit a differential-geometric interpretation. In our project we use this phenomenon to develop new geometric methods of the integrability theory as well as to obtain new interesting geometries and dynamical systems.
The problem of an “intelligent discretisation” is central to our research. We study this problem in frames of discrete differential geometry, which aims to develop discrete equivalents of the geometric notions and methods of differential geometry. Current interest in this field derives not only from its importance in pure mathematics but also from its relevance for computer graphics. An important example are polyhedral surfaces approximating smooth surfaces.

One may suggest many different reasonable discretisations with the same smooth limit. Which one is the best? From the theoretical point of view the best discretisation is the one which preserves all fundamental properties of the smooth theory. Such a discretisation clarifies the structures of the smooth theory and possesses important connections to other fields of mathematics. On the other hand, for applications the crucial point is the approximation: the best discretisation is supposed to possess distinguished convergence properties and should represent a smooth shape by a discrete shape with just few elements. Although these theoretical and applied criteria for the best discretization are completely different, in many cases “theoretical” discretisations turn out to possess remarkable approximation properties and are very useful for applications. We are working on both theoretical and applied aspects of discrete differential geometry.

Recently the roots of integrable differential geometry were identified in the multidimensional consistency of discrete nets (the so-called consistency approach). This led to a new (geometric) understanding of integrability itself. First, we adhere to the viewpoint that the central role in this theory belongs to discrete integrable systems. Further, and more important, this leads to a constructive and almost algorithmic definition of integrability of discrete equations as their multidimensional consistency. We focus on the development of the consistency approach for integrable systems on (discrete) manifolds. In particular we classify discrete integrable equations in dimensions two and three and address the question whether there are discrete integrable equations in more than three dimensions.

Discrete surface with constant negative Gaussian curvature and two straight asymptotc lines. The corresponding smooth surface is described by a Painlevé solution of the sine-Gordon equation. In the discrete case one has a discrete Painlevé and a discrete sine-Gordon equation.
T.Hoffmann and A.Bobenko



Random matrices, random permutations, matrix models and applications

The study of matrix integrals and random matrices remains an extremely important domain of research, which bridges several areas, probability, geometry, combinatorics and mathematical physics. This interaction has already proved to be fruitful. Indeed, it has lead to the solution of the famous Ulam's problem in combinatorics: what is the distribution of the length of the longest increasing subsequence in large random permutations?

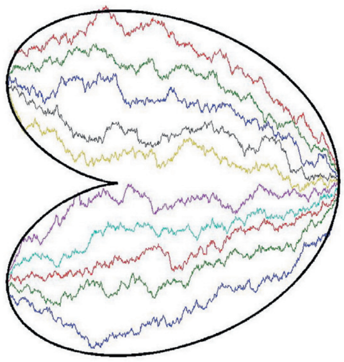
Unitary matrix integrals play a big role. Similarly, integrals over symmetric spaces, like Grassmannians, lead to a variety of important matrix models, which satisfy nonlinear integrable differential equations and Virasoro constraints. Furthermore, the computation in the large N limit of a class of matrix integrals with polynomial potential over complex N x N matrices is also both well known and surprisingly instructive and rich in applications: the result beyond the leading large N limit contains information on the number of maps drawn on Riemann surfaces of a given topology.

A relatively new development in the theory of random matrices concerns normal matrix models. The new feature with respect to the Hermitian matrix models is that the eigenvalues lie in the complex plane rather than on the real axis, which gives rise to geometrically interesting situations. We study this model from a rigorous analytical point of view, beyond the realm of formal power series used in the existing literature, introducing suitable regularisations of the (ill-defined) integrals over matrices.

A related class of matrix integrals (in a certain sense the holomorphic square roots of normal matrix integrals) have recently made their appearance in the string literature, and are expected to be important objects in supersymmetric gauge theory and strings. Here a formal saddle point calculation suggests that the eigenvalues are asymptotically distributed along certain curves. The determination of these curves and the investigation of their possible relation with integrable models is another part of this project. Again, a rigorous definition of the integrals over matrices is a prerequisite.

The pictures shows ten nonintersecting Brownian paths with two different starting points and one common end point. At any intermediate time the positions of the paths are distributed like the eigenvalues of a Gaussian unitary random matrix with external source. As the number of paths increases the paths fill out a two-dimensional region whose boundary is indicated in black. This random model has a phase transition. For small time there are two well-separated groups of paths that come together at a critical time and then continue as one group. At the critical time the boundary has a cusp singularity.

A. Kuijlaars



Analytic and geometric approaches to asymptotic integrability

The weak dispersion expansion of integrable systems is a problem of increasing importance. The so-called method of hydrodynamic reductions discovered in the study of the dispersionless limits reveals a deep relationship with topics in complex analysis. The connections between these ideas and the known geometrical theory of integrable hierarchies need to be explored further.

The use of Riemann - Hilbert problems to describe solutions of dispersionless limits of integrable systems leads to an interesting twistor formulation of dispersionless integrable systems. The twistor formalism will be used to investigate relevant sets of additional symmetries and to find new classes of solutions of these equations.

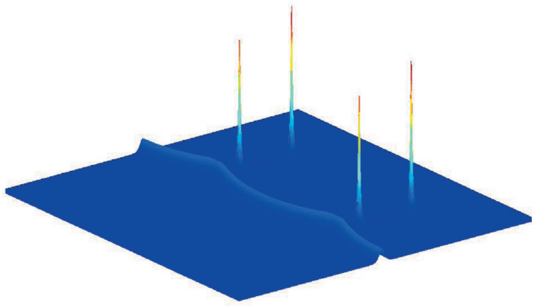
Conformal maps find interesting applications to quadrature domains. These are special classes of two-dimensional domains parametrised by compact Riemann surfaces. It is proposed to explore further connections between integrable systems theory and these special classes of domains with emphasis on potential applications to free surface flows in fluid dynamics.

Other specific aspects of fluid dynamics constitute an important part of the research project. We study measure-valued solutions for a family of evolutionary partial differential equations in one, two and three spatial dimensions.

Concerning the borderline between integrable and non-integrable behaviour in evolutionary partial differential equations, semiclassical problems for non-integrable equations are somehow reduced to integrable problems. Among our plans is the clarification of this issue. We also study numerically the properties of the partial differential equations obtained by a truncation of the perturbative expansions of the integrable systems.

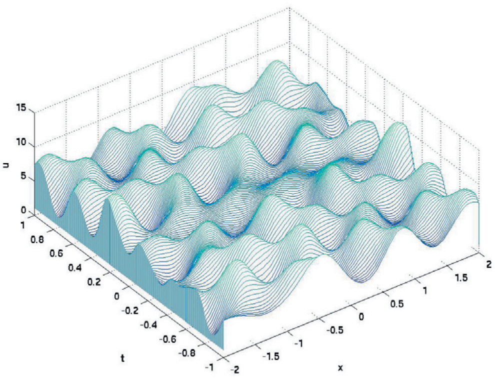
Soliton solutions to the Korteweg de Vries equation provide an infinite extended solutions to the Kadomtsev Petviashvili (KP) equation which are called line solitons. In the case of the so called KP1 equation, these line solitons are unstable. The figure shows the evolution of a perturbed line soliton of the KP1 equation. Instabilities are developed in the form of peaks.

C. Klein, C. Sparber and P. Markowich



Multiperiodic solution
of the Korteweg de Vries equation

J. Frauendiener and C. Klein



European Context

There are a great number of collaborations between European researchers in geometry and integrable systems, random matrices, and their applications. However, for the most part these have been established on an individual basis and not as part of any formal European network. The MISGAM programme aims at stimulating the exchange of ideas, ranging from theory to applications, and at improving cross-fertilisation of researchers and students across boundaries. The topics involved are highly related, and at the same time have an impact on many different areas of geometry, combinatorics, probability, statistics and physics. Nowadays, European scientists play an important and leading role in these fields, however they are scattered all over Europe. Many interesting questions, both fundamental and applied, are ready to be tackled. With this in mind, we propose a very broad network to provide ample opportunities of interaction and growth. The MISGAM programme is providing the necessary impetus to improve communications between researchers, not only in different countries, but also in different areas of science. Yet the project is sufficiently focused in order to serve as a fruitful training ground for young researchers.

Activities

The activities of the MISGAM programme are:

Short visits grants

A visitor exchange programme is active for visits of up to two weeks with an electronic open call through all the year. Processing time for short visit grants is roughly three weeks. Applications can be submitted on the web-page.

Exchange grants

A visitor exchange programme is active for visits for a period between two weeks and six months with electronic open call through all the year. Processing time for exchange grants is roughly six weeks.

Workshops/Conferences

Each year, few thematic workshops or conferences and brainstorm meetings (5-10 participants) will be organised.

Applications should be submitted on the web page, http://www.esf.org/misgam. Proposal for workshops and conferences will be processed once a year during the Steering Committee Meeting. Proposal for a brainstorm meeting should be submitted at least four months in advance of the event date.

Website

There are two websites where all the relevant information can be found: *http://misgam.sissa.it* and *http://www.esf.org/misgam.*

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For the latest information on this Research Networking Programme consult the MISGAM website: www.esf.org/misgam

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