

Scientific Report - INSTANS Short Visit Grant 2284

Purpose of visit

The aim of the visit was to initiate a collaboration on q -difference equations and their application in quantum integrable systems and correlations functions. Employing representation theory one can construct operators, called collectively Baxter's Q -operator, which lead to a set of q -difference equations whose solutions allow one to compute the spectra of quantum spin-chain Hamiltonians such as the XXZ or XXX model. The purpose of the visit was to investigate the possibility of collaborations on numerically solving these equations and compare the results with known techniques such as the Bethe ansatz. The ultimate target is to apply the findings in a more efficient computation of correlation functions.

Work carried out during visit

During the visit several discussions took place on the mathematical background and derivation of the equations based on a set of preliminary lecture notes (C. Korff, Lectures on Q , unpublished). The q -difference equations for the XXZ model at zero temperature were discussed in detail and how to compute from its solutions the energy and transfer matrix eigenvalues. A Mathematica Programme was written which solves the q -difference equations numerically in terms of elementary symmetric polynomials in the Bethe roots. This programme will serve as starting point for more extensive numerical investigations. The host explained the connection with the Bethe ansatz equations and the string hypothesis and how the latter can be used for "state counting". This connection will prove important to match the solutions from the q -difference equations with those from the Bethe ansatz. These discussions between visitor and host were also attended by a master student who expressed interest in pursuing a PhD in this area.

During the visit a group seminar was presented on a slightly different but related area, the non-Hermitian XX model and PT-symmetry: <http://arxiv.org/abs/0803.4500> . The possible applications in investigating Logarithmic Conformal Field Theories attracted also interest from members of the string theory group. Also here joint projects were discussed with the host.

Future collaborations with host institution

The visit was also used to identify several concrete joint projects for grant applications. It is planned to apply in the near future for funding from the INSTANS project, the Engineering and Physical Research Council (UK) and the Royal Society to support further visits, PhD positions and possibly a postdoc working on q -difference equations and the numerical computation of correlation functions. If successful the collaboration is projected to last around 4-5 years.

Projected publications

The first joint work will be on the $q=1$ limit of the XXZ model, the isotropic XXX spin-chain. We expect first results in the near future with a possible resulting publication within one year. If the mentioned grant applications are successful several additional publications are expected.

Comments

The subsistence costs provided according to the ESF guidelines (EUR 85/day) proved insufficient to cover the accommodation costs (EUR 115/day) for hotels in Amsterdam. The additional funding needed will be covered by a grant from the Royal Society (UK).