Project Report - QUDEDIS Exchange Grant
- Ultra Cold Atoms in Disordered Potentials -

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Receiving Institute:

Theoretical Quantum Optics Group

ICFO - The Institute of Photonic Sciences

Host:

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I. DESCRIPTION OF THE WORK CARRIED OUT AND THE RESULTS OBTAINED

The goal of this project was to theoretically investigate ultra-cold gases in disordered potentials. One aim was to study the effects of the introduction of a random field in a system with continuous symmetry. A two-component Bose-Einstein condensate with a random coupling between the components served as a model system for our investigations. We have observed the emergence of order in this system, if the random field breaks the continuous symmetry.

In a second line of research, we have numerically investigated the effects of disorder on Bloch oscillations of ultra-cold gases in optical lattices. Special attention was paid to the important interplay of disorder and interactions. We have obtained numerous numerical results on this phenomenon and by this discovered an exciting regime, in which interactions and disorder strongly compete with each other. The complete understanding of this regime requires a further careful analytical investigation.

In the following we describe our results on random field induced order (section II.) and on the dynamics of Bloch oscillations in disordered lattices potentials (section III.).

The results of this work will be published in two papers, which are currently in preparation.

II. RANDOM FIELD INDUCED ORDER IN A TWO-COMPONENT BOSE-EINSTEIN CONDENSATE

Systems with continuous symmetry do not exhibit long range order in two dimensions for any finite temperature. This is a consequence of the famous Mermin-Wagner-Hohenberg theorem [1]. Amazingly, the introduction of a random field can lead to the appearance of order, if the field breaks the continuous symmetry.

We have studied this effect with a two-component Bose-Einstein condensate with random coupling between the components. This system can be experimentally realized by two internal atomic states and a random Raman coupling between them. The dynamics of the condensate wavefunctions Φ and Ψ is determined by the coupled equations

$$i\hbar\partial_t\Psi = \left[-\frac{\hbar^2}{2m_1}\nabla^2 + V_1 + g_{11}\left|\Psi\right|^2 + g_{12}\left|\Phi\right|^2\right]\Psi + \frac{\Omega}{2}\Psi$$

$$i\hbar\partial_t\Phi \; = \; \left[-\frac{\hbar^2}{2m_2}\nabla^2 + V_2 + g_{22}\left|\Phi\right|^2 + g_{12}\left|\Psi\right|^2\right]\Phi + \frac{\Omega^*}{2}\Phi, \tag{1}$$

where V_i, m_i and g_{ij} denote the respective external trapping potentials, atomic masses and interaction coupling constants. The Raman coupling of the two compenents is governed by the field $\Omega(\mathbf{r})$ and is in the equations above assumed to be resonant. In the absence of the coupling field the system obeys a continuous symmetry, which is generated by translations of the global phase difference of the two condensate wavefunctions.

We have numerically propagated initial condensate wavefunctions according to the coupled equations (1) in imaginary time to determine the ground state of the system. Figure 1 shows the ground state density profile of one component and Figure 2 the corresponding phase difference between the two condensate wavefunctions. The phase difference is given by the initial conditions (as long as the individual phases are homogenous), because the ground state energy is independent of the (flat) phase difference.

The introduction of an arbitrarily small coupling field Ω with spatial disordered profile and zero meanvalue changes this picture drastically. We have investigated the effect of an incommensurate sinusoidal field configuration $V = A_1 \sin(\vec{k_1}\vec{r}) + A_2 \sin(\vec{k_2}\vec{r})$, which can be experimentally realized by a so-called optical super lattice potential. As shown in Figure 2, the phase difference of the two condensate wavefunctions is in this case fixed to $\pi/2$.

This effect can be understood by considering the energy functional of the coupled system:

$$\begin{aligned} E = \int d^3r & \left[\frac{\hbar^2}{2m_1}\left|\nabla\Psi\right| + \frac{\hbar^2}{2m_2}\left|\nabla\Phi\right| + V_1\left|\Psi\right|^2 + V_2\left|\Phi\right|^2 \right. \\ & \left. + g_{11}\left|\Psi\right|^4 + g_{22}\left|\Phi\right|^4 + g_{12}\left|\Psi\right|^2\left|\Phi\right|^2 + \Omega\Phi\Psi^* + \Omega^*\Phi^*\Psi\right]. \end{aligned} \tag{2}$$

The actual density is only barely affected by the random field, so that the ground state of the system is determined by the interplay of the kinetic terms and the terms containing the random field. The latter can be rewritten by inserting $\Psi = \left|\Psi\right| e^{i\psi}$ and $\Psi = \left|\Phi\right| e^{i\phi}$:

$$E_{rf} = \int d^3r \, 2\,\Omega(\mathbf{r}) \; \left|\mathbf{\Phi}\right| \left|\mathbf{\Psi}\right| \cos(\phi - \psi) \tag{3}$$

for real Ω. This term is minimized, if the sign of $\cos(\phi - \psi)$ is opposite to the sign of $\Omega(\mathbf{r})$. Since $< \Omega >= 0$ and $\Omega(\mathbf{r})$ is spatially rapidly varying, this can be achieved only at the cost of kinetic energy. The total energy is in this case minimized by $< \cos(\phi - \psi) >= 0$ and very small fluctuations around this value.

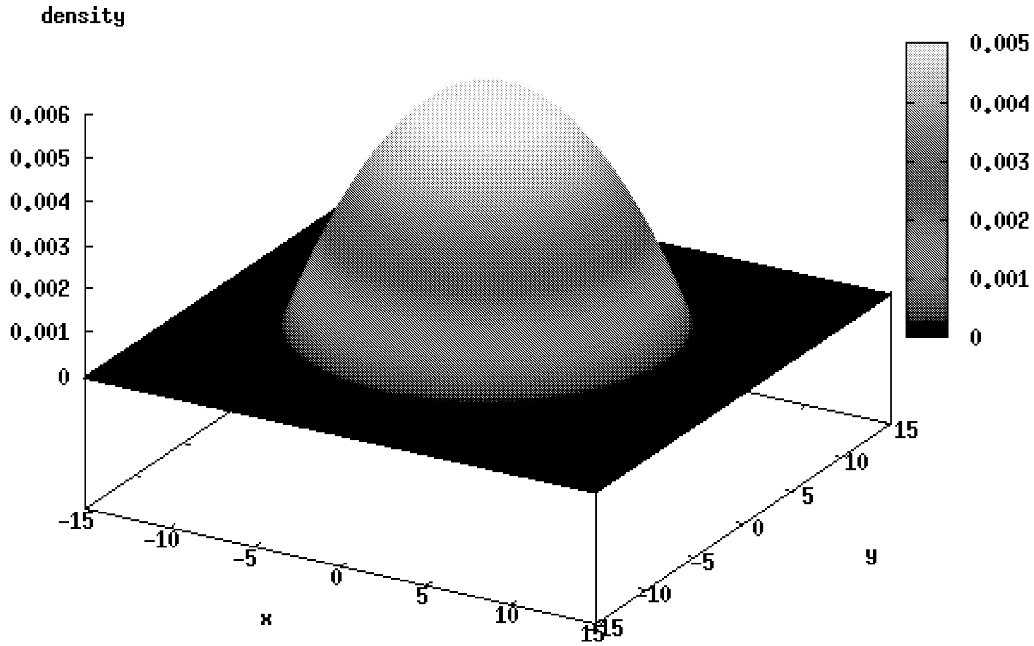

FIG. 1: Density profile of one condensed component in a symmetric harmonic trap in 2D. All numbers are given in units of the trap harmonic oscillator length.

III. BLOCH OSCILLATIONS IN DISORDERED LATTICES POTENTIALS

In a second line of research, we have investigated the use of ultra-cold gases to study the dynamics of Bloch oscillations in disordered periodic potentials. Bloch oscillations are a striking quantum mechanical effect that occur for the dynamics of a particle in periodic potentials. Under the influence of a constant force the particle is supposed to undergo a coherent oscillatory motion instead of being linearly accelerated [2]. The experimental observation of Bloch oscillations has already been achieved in ultra-cold gases [3] by the help of accelerated optical lattices. Optical lattices form perfect periodic potentials which are almost free from any kind of imperfections. Due to this pureness is the life-time of Bloch oscillations in these systems by orders of magnitudes larger then for electrons in solid state samples. This makes ultra-cold gases very promising candidates to experimentally study

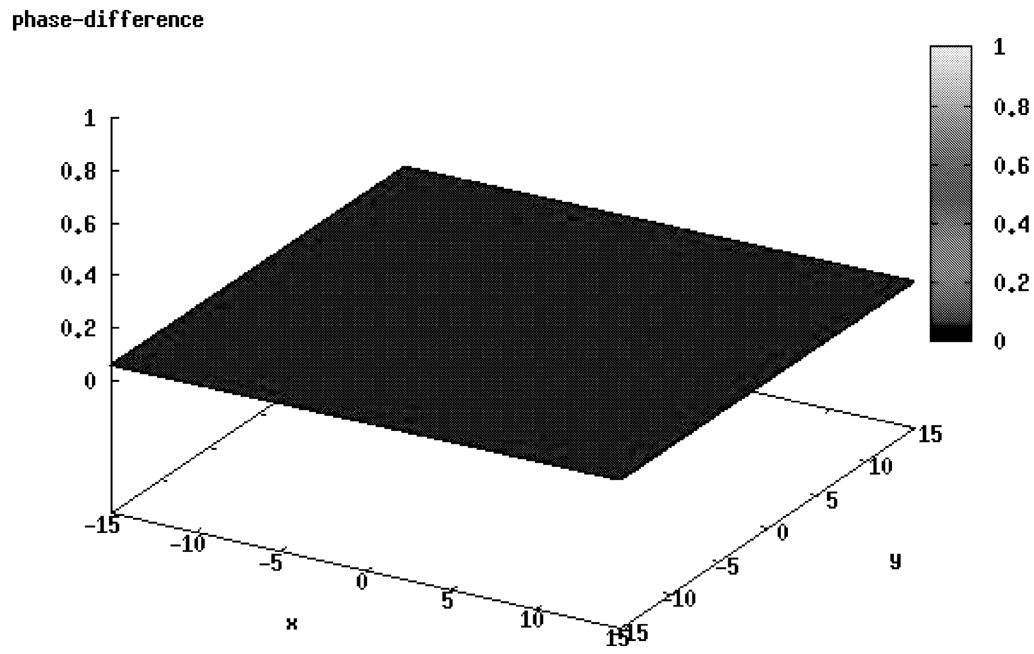

FIG. 2: Phase difference between the condensate wavefunctions in units of π in the absence of the random field. The spatial coordinates are given in units of the harmonic oscillator length of the trap.

damping mechanisms and dynamics by introducing controllable disorder to the system. Our goal was to set the theoretical background for those investigations. This work was carried out in close collaboration with the Prof. Luis Santos at Institute of Theoretical Physics at Leibniz University Hannover and shall serve as a direct guide to the experiments, which are currently performed in the group of Prof. Ertmer/ Prof. Arlt at Institute of Quantum Optics at Leibniz University Hannover.

We have numerically analyzed the time-evolution of Bloch oscillations in presence of disorder and interactions. Consider first a single particle wave packet in 1D, which is situated in an optical lattice. When the wave packet is accelerated by a constant force, it performs coherent Bloch-oscillations with unrestricted life-time. This picture changes, when disorder and interactions are introduced. We have numerically calculated the time evolution of

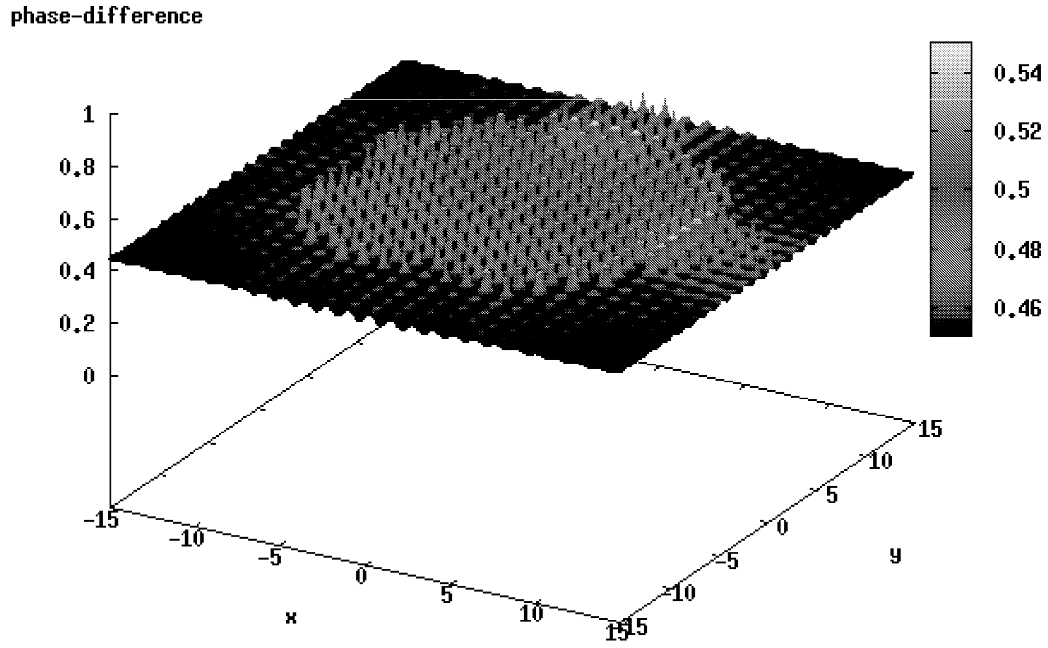

FIG. 3: Phase difference between the condensate wavefunctions in units of π in the presence of the random field. The spatial coordinates are given in units of the corresponding harmonic oscillator length.

the wave function Φ for different initial conditions and we have depicted the mean value of position $< z >= \int dz\, \Phi^* \, z \, \Phi$ as a function of time. Figure 4 shows the results for a single particle in a disorder potential V_Δ. This disorder potential corresponds to a typical configuration of the experiment in Hannover and has a typical correlation length $L \approx 10\mu m$. It is evident, that the Bloch oscillations are damped due to the presence of disorder.

Figure 5 shows the results for condensates with weak and strong nonlinearity. In both cases a significant damping of the oscillation amplitude can be observed, which is caused by the presence of the disorder potential. Amazingly, the nonlinearity can either enhance or diminish the damping in respect to the single particle case, depending on the strength of the nonlinearity.

We could identify two different mechanisms, which are responsible for this interesting

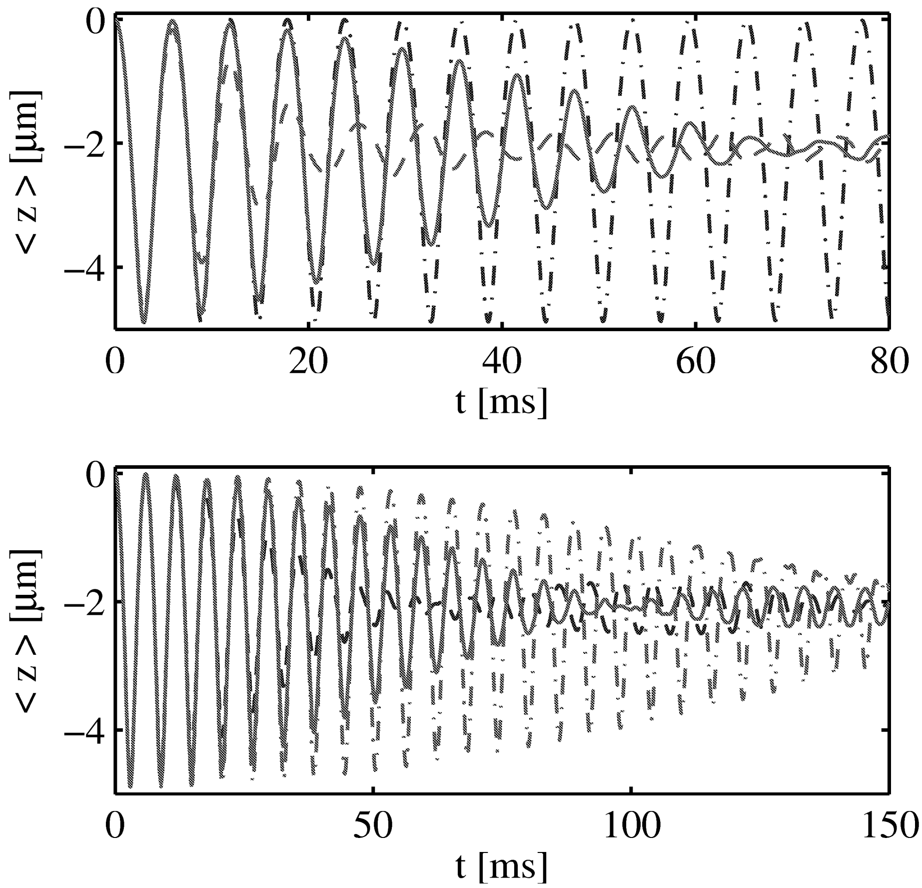

FIG. 4: Expectation value of the axial position for a single particle in 1D and various disorder configurations. The upper panel shows the result for a typical disorder realization of the Hannover experiment with $L \approx 10\,\mu m$. The three curves correspond to $V_\Delta = 0.02\,E_r$ (solid), $V_\Delta = 0.06\,E_r$ (dashed) and $V_\Delta = 0$ (dashed dotted). In the lower panel the results for a disorder with $L \approx 1\,\mu m$ (dashed), an incommensurate (solid) and a commensurate super lattice (dashed dotted) are shown. The depths of the additional potentials are $0.02\,E_r$ respectively. The depth of the main lattice is $2\,E_r$ in each case.

behavior. For the case of strong nonlinearity, the damping is significantly increased due the presence of the so-called dynamical instability [4]. In the cause of the Bloch oscillations, the quasi-momentum of the condensate wave function scans the whole Brillouin-zone. For large values of the quasi-momentum, the condensate becomes dynamical unstable and small excitations $\delta\Phi$ can grow exponentially in time [5]. This causes a rapid dephasing of the condensate wavefunction on different lattice sites and leads to a strong damping of the Bloch oscillations. In the regime of weak nonlinearity, interactions act against the dephasing of

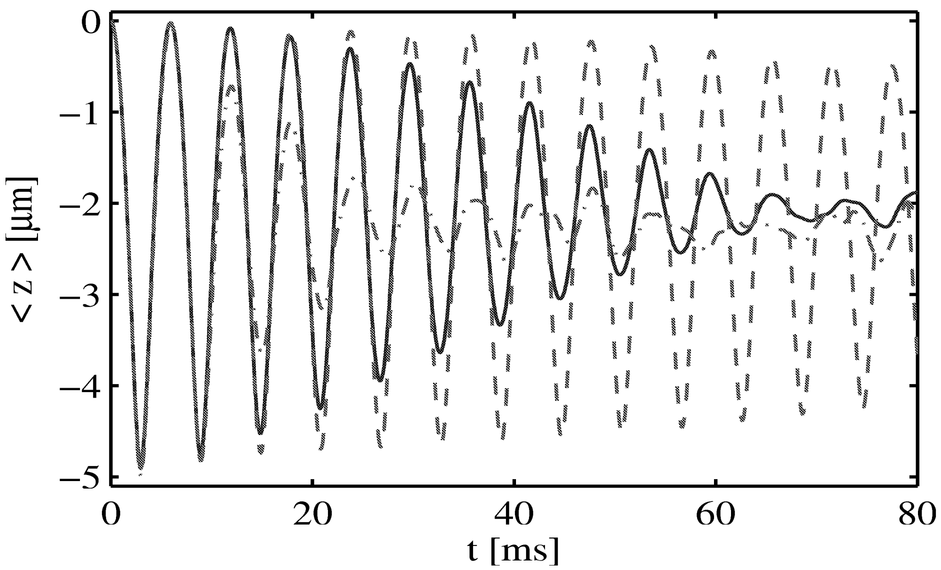

FIG. 5: Expectation value of the axial position obtained for a typical disorder configuration of the Hannover experiment with $V_\Delta = 0.02\,E_r$. The curves correspond to the following 1D configurations: $N = 0$, $\omega = 2\pi \times 3\,Hz$ (solid), $N = 350$, $\omega = 2\pi \times 7\,Hz$ (dashed), and $N = 1300$, $\omega = 2\pi \times 12\,Hz$ (dashed-dotted). The trap frequencies where adjusted to match the sizes of the clouds in the disorder potential. The 1D coupling constant was obtained by integrating over the radial degree of freedom in trap with $\omega_\perp = 2\pi \times 200\,Hz$.

the wavefunction, which is introduced by the disorder potential. This leads to a significant reduction of the damping rates as shown in Figure 5. However, the full understanding of this phenomenon requires a further analysis.

Therefore, we have employed a perturbative approach to get analytical insight into this intriguing dynamics. We have treated the disorder potential $V_\Delta(z)$ and the mean field interactions $g\,|\Phi|^2$ as a perturbation to the Wannier-Stark states and energies. Assuming that the population of the Wannier-Stark states does not change in time, we get results which are in the case of weak disorder and absence of interactions in good agreement with our numerics. However, the interacting case requires a further analysis. In fact, we have numerically checked, that the assumption of unchanged Wannier-Stark population does not hold for realistic values of the nonlinearity. This further analytical investigation will be the subject of our future work.

In our numerical analysis, we have also investigated the corresponding 3D problem. In this case, the dynamical instability leads to a complex radial dynamics, which significantly

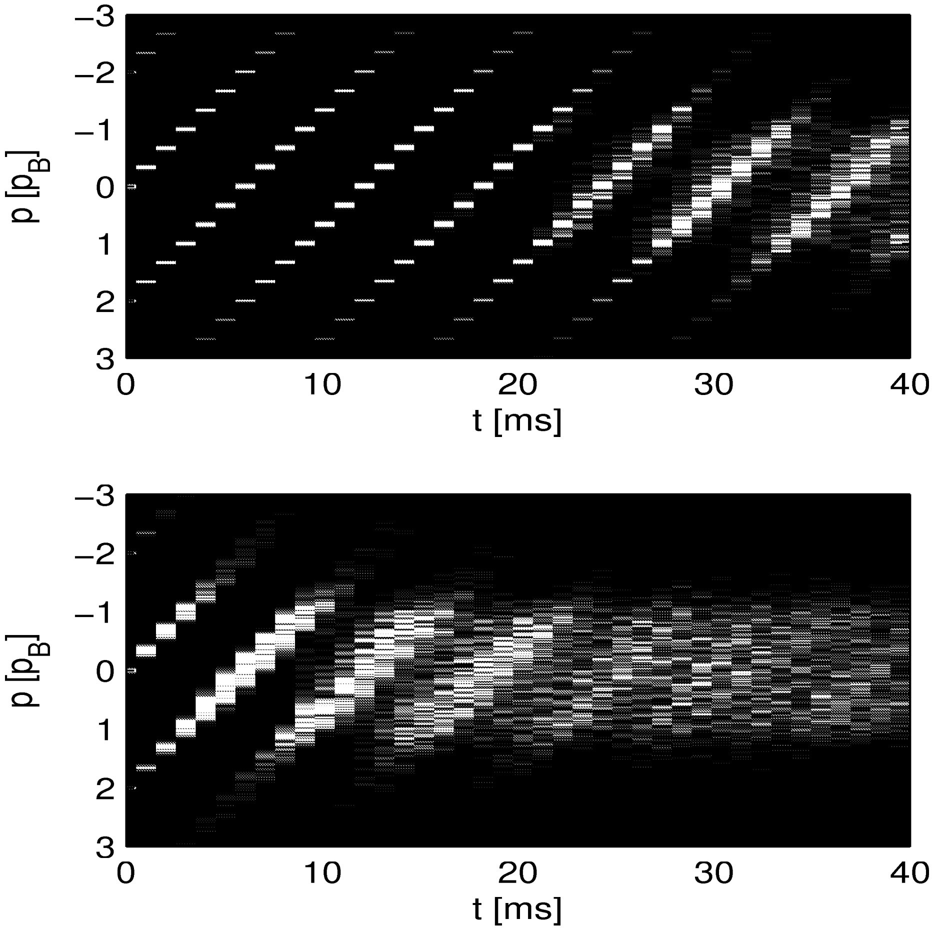

FIG. 6: Gray-scale plot of the momentum distribution, calculated every $1\,ms$ during the Bloch oscillations. The upper panel shows the non-disordered case, the lower panel shows the result for a disorder depth of $0.06\,E_r$. The pictures correspond to the 3D trap configuration (see text) and a typical disorder realization of the Hannover experiment.

alters the evolution. However, the basic features of the problem are unchanged in respect to the 1D case. This is particular important for the experiments in Hannover, as they will be carried out in a 3D regime. We have also paid special attention to the question of the observability of the damping. It can be experimentally derived from a measurement of the momentum spectrum of the gas by a time-of-flight analysis. In Figure 6 we display the momentum spectrum at various stages during the Bloch oscillations. In the absence of disorder, the quasi-momentum scans the Brillouin-zone according to the acceleration theorem. Consequently, the population of the different momentum components changes, resulting in a coherent oscillations of the mean momentum. However, the spectrum still consists of several sharp peaks, which are separated from each other by the lattice momentum $p_B = 2\hbar k$. This picture dramatically changes as disorder is introduced to the system. The initial sharp momentum components are consecutively broadened, eventually providing an irregular occupation of momenta. A broadening of the momentum components is also introduced by interactions, as shown in Figure 6. However, a very clear influence of disorder can be observed, which allows for a distinction between the two scenarios after expansion. In the non-damped case, the density distribution consists of several small clouds, that correspond to the different momentum components. They are well separated from each other in position space by $\Delta x = 2p_B\tau/m$, where τ denotes the expansion time. In the disordered case, the density subsequently forms a broadened distribution after expansion, clearly indicating the onset of damping. We have furthermore computed that the expansion also leads to clearly distinguishable center of mass positions in the damped and undamped case. This is caused by the differences in the momentum expectation values. Let us conclude by remarking, that uur numerical findings have been prepared for publication.

IV. FUTURE COLLABORATION

In the main parts of this work, the collaboration between the researcher and the host shall be continued. The investigation of random field induced order will be extended to the case of three dimensions and then prepared for publication. The research on Bloch oscillations in disordered lattices shall be extended by an analytical treatment of the intriguing problem in the presence of interactions. The numerical part of the research has been already prepared for publication. This work directly serves as a guide to the experiments performed at the Institute of Quantum Optics, Leibniz University Hannover. Thus a direct benefit for the researchers home institution is given.

V. FURTHER COMMENTS

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