Regular and chaotic properties of the Bose Hubbard model with a static field

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1 Introduction

Ultracold bosons are usually described by the Bose Hubbard model [1]. Adding a static force can cause Bloch oscillations of bosons on the lattice [2]. When the other two control parameters of the model, i.e. the tunnelling amplitude and the onsite interaction strength have comparable values, the energy spectrum as a function of the force acquires chaotic properties. The level spacing distribution is given by the Wigner-Dyson statistics, in contrast to the regular case described by the Poissonian distribution [3]. As an immediate consequence the Bloch oscillations are destroyed [4]. A visual signature of quantum chaos is presence of wide and overlapping avoided crossings in the energy spectrum. For a wide range of the static field values, a distinct group of energy levels is, however, noticeable. They have approximately constant slopes and exhibit only narrow avoided crossings with other levels, being recovered even after passing a region dense with avoided level crossings.

During my stay at the group of prof. Buchleitner in Freiburg I studied these levels visualizing the corresponding eigenvectors, I showed underlying particle localization that is responsible for their stability, and proved their clear dynamical distinction from the surrounding chaotic sea [5]. Moreover, my results indicate a possibility of establishing an interesting link between our system of small particle filling fractions with larger systems described by mean field equations such as the discrete nonlinear Schrödinger equation [6]. The system I was considering was one dimensional, and with the filling fraction of the order of one half. For moderate particle numbers exact diagonalization of the Bose Hubbard hamiltonian was available by means of numerical algorithms.

We are planning to publish our results within the next few months. The manuscript is now in preparation.

2 Regular levels of the Bose Hubbard hamiltonian with the static field

As the first part of my research project I analyzed eigenvectors of the Bose Hubbard hamiltonian with the static field. The eigenstates corresponding to the straight lines in the spectrum have been identified as states with approximately all particles occupying single lattice sites. The most robust level corresponds to the localization of particles in the lowest lattice well. Simultaneously, since it has the highest negative slope, it crosses the highest number of other, "chaotic" levels. The upper straight lines exhibit slightly lower localization, which was indicated by smaller overlap values with the respective Fock basis states. The described behaviour comes together with a high localization in the basis. The constant slope values are approximately given by derivatives of the hamiltonian with respect to the static field.

The origin of the localization could have been a particular choice of boundary conditions. I assumed open boundary conditions, i.e. a hard wall potential outside the periodic optical lattice. Actually closed boundary conditions do not make sense here, since the hamiltonian with the static field is not translationally invariant. I will address this issue in the following sections.

We expect the described property of small Bose Hubbard systems to be preserved also in cases of much higher filling fractions, i.e. where the exact diagonalization is not available due to an enormous size of the Hilbert space.

3 Translationally invariant hamiltonian

In order to address the question concerning the role of the boundary conditions I studied a transformed hamiltonian. In contrast to the original hamiltonian, its interaction representation with respect to the static field term is translationally invariant [3]. Then it is possible to apply both open and closed (periodic) boundary conditions. The closed boundary conditions mean, however, that the transformed hamiltonian corresponds to a different system. That is, we are considering now an optical lattice on a ring with time dependent phase shifts additionally describing the tunnelling between adjacent lattice sites.

Application of the closed boundary conditions makes it possible to introduce a new quantum number, so called quasimomentum [3]. The Hilbert space factorizes then to subspaces with different values of the quasimomentum. There is no coupling between the subspaces, therefore there are no avoided crossings between energy levels corresponding to different quasimomenta. This significantly reduces a computational effort required. What is more important, an intuitive definition of the momentum and the occupation space is then applicable. We decided to take advantage of these properties, loosing on the other hand the clear connection between the original and the transformed system. We will finally clarify this issue in our publication.

The transformed hamiltonian is explicitly time dependent and periodic in time. The main part of my project was to find solutions of the Schrödinger equation by means of the Floquet ansatz [7]. By using a Fourier expansion and an appropriate cut-off of the basis I have diagonalized the Floquet operator. I have analyzed Floquet quasienergy spectra and corresponding eigenvectors. I have identified quasienergy levels having similar properties as the regular levels of the untransformed hamiltonian. As a variable control parameter I used the tunnelling amplitude or the inverse of the static field value, taking care that the other two parameters ensure the chaotic properties of the spectrum. The "regular" eigenvectors were now superpositions of N particles sitting in all lattice sites, where N is the total number of particles in the system.

I compared the Floquet method of finding the time evolution operator with a method implemented and used in the group of prof. Buchleitner earlier. The flexibility guaranteed by the two completely different methods is extremely important especially in the chaotic domain of the problem.

Varying the system size I found as a generic property that the quesienergy spectra corresponding to different quasimomenta are the same. Indeed, we proved existence of a unitary transformation between the blocks of the time dependent hamiltonian. As a consequence the respective Floquet eigenvectors are related by a matrix multiplication.

4 Atom dynamics

The presence of regular energy and quasienergy levels surrounded by the chaotic sea suggests that an underlying classical phase space should have a mixed structure with regular islands surrounded by chaos. Consequently, eigenstates belonging to those two classes should exhibit completely different kinds of dynamics. I have studied the time evolution of the Floquet eigenstates in the momentum and in the occupation space. The former is defined by a single quasimomentum subspace, whereas the latter corresponds to eigenstates being superpositions over all quasimomenta. Also vicinities of avoided crossings between regular and chaotic levels have been analyzed.

The mean momentum over one period of motion for the "regular" eigenstates reveals clear oscillations, when calculated in a single quasimomentum subspace. The period of these oscillations is a fraction of the Bloch period that defines the time periodicity of the hamiltonian. In contrast, the time dependence of the mean momentum of chaotic eigenstates is much more complex.

To study the dynamics in the real space I have calculated mean occupation numbers of individual wells of the lattice over one Bloch period. Here limiting to only one quasimomentum would imply always the same number of particles in each well, due to the quasimomentum subspaces definition. The two sorts of eigenstates are again easily distinguishable. The regular dynamics amounts to correlated oscillations of particles in different lattice wells. In addition, there are cases of particle localization in one of the wells, which suggests a link between the regular eigenstates in the chaotic sea and the phenomenon of discrete breathers [6]. Since discrete breathers are present in nonlinear systems, where the nonlinearity can occur as a consequence of interparticle interactions, they would correspond to a mean field limit of our system. The possibility that it is the underlying mixed structure of the phase space that causes the breather behaviour we find very promising.

5 Further remarks

I find my stay in Freiburg extremely stimulating and fruitful. Apart from my main research project, I had an opportunity to take part in many discussions and seminars devoted to ultra-cold atoms, classical and quantum chaos, information theory and many other subjects. I appreciate discussions with guests and members of the group, concerning also technical aspects of my project, as well as projects that I had finished earlier in Kraków. An outcome of this interaction are a few interesting research ideas, investigated hopefully in future with participance also of other members of the two groups. I am sure that my exchange grant will foster the cooperation, not to mention an opportunity to finish my PhD next year as a joint binational PhD project with dr hab. Krzysztof Sacha and prof. Andreas Buchleitner as my supervisors on the Polish and the German side, respectively.

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