ESF (POLATOM) Travel grant final report

This short report details the work done by Erik van der Wurff during his visit in the research group led by dr. Thomas Pohl at the Max Planck Insitute for Complex Systems in Dresden, Germany. The visit extended from October 1st 2014 to March 1st 2015. An ESF travel grant was awarded for this visit.

Grant recipient:	Erik van der Wurff
E-mail:	erikvanderwurff@gmail.com
Telephone:	+31(0)636190433
Current position:	PhD student (expected starting date: 1st of April)
Supervisor(s):	prof. dr. ir. Henk Stoof and/or dr. Rembert Duine
Institution:	Utrecht University Leuvenlaan 4 3584 CE Utrecht the Netherlands

Host(s):	dr. Thomas Pohl & dr. Rick van Bijnen
E-mail:	tpohl@pks.mpg.de & rvb@pks.mpg.de
Telephone:	+49(0)3518712211 & 017646195160
Host institution:	Max Planck Institute for the Physics of Complex Systems Noethnitzer Str. 38 01187 Dresden Germany

Project topic:	Effective theory of quantum light propagation in strongly-interacting EIT media
Project duration:	5 months (originally 3 months)
Grant amount:	€5040 (including €240 for travel costs)

1 Purpose of the visit

The purpose of the grantee's visit was threefold:

- Firstly, the scientific goal of the visit was to use the grantees experience in statistical field theory to find an effective Hamiltonian to describe the propagation of nonclassical light in a medium, subject to conditions of electromagnetic induced transparancy (EIT). This will be described in more detail below.
- Secondly, the visit was intended for personal development of the grant recipient. This has been achieved by providing the grant recipient with valuable working experience in a slightly different scientific field (as compared to his original field), in a different working climate in a foreign country.
- Thirdly, the visit was intended to initiate a new collaboration between the Institute for Theoretical Physics in Utrecht and the Max Planck Institute for the Physics of Complex Systems in Dresden. Besides the stay of the grantee, this has been achieved by the visit of prof. dr. ir. Henk Stoof on Monday the 26th of January. During this visit, professor Stoof has spoken at the colloquium of the institute. Furthermore, he has had discussions with various members of the institute.

2 Work carried out during the visit

The research carried out during the visit combines two concepts in physics: electromagnetic induced transparancy (EIT) and Rydberg atoms. We start by briefly introducing the former.

EIT is the phenomenon that under specific conditions, a beam of electromagnetic radiation tuned resonantly to an atomic transition[1] can travel through a cloud of atoms without being absorbed significantly. These conditions can be reached by considering a three-level excitation scheme for a gas of atoms with a ground state $|1\rangle$, an intermediate excited state $|2\rangle$ and an even higher excited state $|3\rangle$. The transitions $|1\rangle \leftrightarrow |3\rangle \leftrightarrow |2\rangle$ are allowed, wheres the transitions $|1\rangle \leftrightarrow |2\rangle$ are dipole forbidden. If one simply couples the states $|1\rangle$ and $|3\rangle$ with a probe laser, then some atoms might be excited to the $|3\rangle$-state, but due to the inevitable spontaneous decay from this state to the $|2\rangle$-state and subsequent photon scattering, the electromagnetic radiation will not be transmitted through the cloud of atoms. However, if one also couples the excited states $|2\rangle$ and $|3\rangle$ with a so-called control laser, then it turns out that a significant amount of electromagnetic radiation is transmitted through the cloud of atoms, when the control field is much stronger than the probe field. The most important condition for this effect to occur, is that

[1] Note that one can always detune the beam of electromagnetic radiation from any atomic resonance to obtain full transparancy. EIT is special because the same result can be reached while keeping the beam resonant with an atomic transition.

the sum of the detunings of the probe and control laser equals zero, which is called a condition of *two-photon resonance*.

Qualitatively, one can explain EIT by noting that due to destructive quantum interference of the $|1\rangle \rightarrow |3\rangle$ and the $|1\rangle \rightarrow |3\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ excitation paths [1], the intermediate state is not populated and absorption does not occur. More precisely, the single-particle Hamiltonian describing the system has an eigenstate which is a superposition of only the ground state and the highest excited state. This state is called the *dark state* [2].

One of the most important consequence of EIT is that the real part of the complex linear susceptibility $\chi(\omega)$ of the atomic cloud becomes very steep as a function of the laser frequency ω centered around the two-photon resonance. As the group velocity v_g of the light pulse is related to χ as $v_g \propto (\mathrm{dRe}[\chi]/\mathrm{d}\omega)^{-1}$, one can slow down the light pulse by simply tuning the EIT conditions such that the derivative of $\mathrm{Re}[\chi]$ becomes very large. Indeed, in a series of spectacular experiments, the group velocity of a light pulse was reduced from the speed of light to a mere 17 m/s [3] and subsequently even brought to a complete stop [4].

To make things even more interesting, one can tune the probe laser to an atomic transition between the ground state and a Rydberg state $|r\rangle$. The latter is a highly excited state with a very high principal quantum number $n \gg 1$. An atom in such a state has extreme properties. Most notoriously, the interaction strength between two of such atoms is of the long-range Van der Waals type ($\sim 1/r^6$) and scales with n^{11}. When a photon probe pulse is sent into a setup where the probe laser is tuned to such a Rydberg state, the dark state can be associated with a quasiparticle called a *dark state polariton*. This quasiparticle is a superposition of a photon and a collective Rydberg excitation distributed over many atoms. However, when a second photon now enters the medium, it can no longer bring the atoms in the dark eigenstate, due to Van der Waals interaction energy between the Rydberg levels that shift the two-photon frequency out of resonance. The EIT-condition is broken and the intermediate atomic level will be populated, leading to a loss of the photon. Thus, depending on the presence of the first photon, the incoming second photon is transmitted or scattered. This means that the Rydberg medium functions as a photon-photon gate.

Creating such a gate is of great importance in the field of quantum computing. Hence, understanding what happens when one sends in multiple photons from either only one side or from different sides into such a Rydberg medium is vital. Therefore, the ultimate goal of our research was to generate a more solid theoretical understanding of this complex, quantum many-body problem. Due to the interactions between different Rydberg atoms, the multi-particle Hamiltonian is no longer diagonalized by dark state polaritons. However, we do know the eigenstates of the Hamiltonian including interactions, but without the coupling between the photons and Rydberg state. These eigenstates all correspond to a fixed total number of polaritons. Hence, we performed perturbation theory in this coupling term by means of Van Vleck perturbation theory [5]. The resulting effective Hamiltonian then describes the effective dynamics for one fixed total number of polaritons.

3 Main results obtained

We started by calculating the effective Hamiltonian for a single polariton propagating in a medium under EIT conditions. This we did by performing Van Vleck perturbation theory in the coupling between the photonic part and excitation part of the polariton. The associated coupling constant β can be assumed small, as it scales inversely with the number of atoms in the gas in appropriate units. The perturbation in this coupling term induces virtual processes, which contribute to the effective Hamiltonian. One such virtual process is annihilating a photon in a certain mode, subsequently creating an excitation and finally recreating the photon in (possibly) a different mode. This is a second order process in the coupling constant, which results in a new quadratic, off-diagonal photon term in the photonic effective Hamiltonian. Similar reasoning holds for the excitation part of the polariton. Deriving the equations of motion from these effective Hamiltonians, we were able to show that these coincide with well-known results from literature.

As a second application of the Van Vleck approach we considered the interaction of a propagating polariton with a stored excitation. Such a stationary excitation can be created by ramping down the group velocity of a polariton, as was discussed in the previous section. The stored excitation acts as a background potential of the Van der Waals type $V(z) \sim 1/z^6$ for the propagating polariton. Hence, it is represented by a position-dependent quadratic term in the Hamiltonian. Both analytical and numerical solutions to this problem have been found [6]. The most important feature of these solutions is that the EIT resonance condition is broken in the region where the potential is not negligible. As the polariton enters this region, it will become a photon again, i.e. it looses its excitation component because the electronic transition is not resonant anymore. However, when the polariton comes out on the other side of the potential, the EIT resonance condition is restored and it will return to a superposition of a photonic and an excitation component.

It is exactly these features we set out to reproduce by using Van Vleck perturbation theory in the coupling between the photonic and excitation part of the polariton. This induces the same virtual processes as was the case for a polariton simply propagating under EIT conditions. However, due to the presence of the background potential, the energy differences necessary to calculate the contribution of the virtual processes become position dependent. Ultimately we find that the effective Hamiltonian governing the excitation field $\mathcal{S}(z)$ is given by

$$\begin{aligned}\hat{H}_{\text{eff}}^{\text{exc}} = &\int \mathrm{d}z \big[V(z) + \beta^2\alpha\big]\mathcal{S}^\dagger(z)\mathcal{S}(z)\\ &- \frac{i\beta^2\alpha^2}{2}\int \mathrm{d}z \int \mathrm{d}z'\ \mathrm{sign}(z-z')\Big[e^{-i[G^2-V(z)](z-z')} + e^{-i[G^2-V(z')](z-z')}\Big]\mathcal{S}^\dagger(z)\mathcal{S}(z'),\end{aligned}$$

where α is an additional energy scale and the term on the first line was already present in the original Hamiltonian. The nonlinear term on the second line is due to the effective treatment. It describes long-range hopping of excitations, which resembles results obtained using different methods [7]. Intuitively its physical meaning is clear: an excitation that is part of a polariton can cross the region in space where the potential is not negligible by transforming into a virtual photon and return back to its original state again on the other side of the potential barrier. Hence, the nonlinear part describes photon-mediated long-range hopping. The hopping is necessarily of long range, as the Van der Waals potential itself is of long range.

Thus, qualitatively we seem to have produced the expected physical behavior. However, the equation of motion resulting from the derived effective Hamiltonian is nonlinear and cannot be solved exactly. In order to produce some quantitative results, we resorted to a numerical simulation of a polariton, governed by the effective Hamiltonian. This work is still under way. However, when this simple situation can be simulated successfully, then an extension to more particles under more difficult conditions should be within direct reach.

4 Future collaborations with host institution

During the visit of prof.dr.ir. Henk Stoof at the institute, the intention to (further) collaborate with group leaders dr. Sebastian Wüster, dr. Thomas Pohl and dr. Jörg Götte was made clear. Furthermore, it is the intention to invite dr. Pohl and/or members of his group for colloquia at the Institute for Theoretical Physics at Utrecht University in the near future.

5 Projected publications resulting from grant

Upon filling in the last details of the described work, a publication is expected in a journal such as Physical Review A.

References

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