Final Report

May 16, 2012

One of my research interests aimed at generalizing the reduction procedure to the formalism introduced by Lu [3]. In a recent work [1], I indeed proved that, given a Poisson action of a Poisson Lie group on a Poisson manifold with equivariant momentum map $\mu : M \to G^*$, it is possible to define a G-invariant foliation of M/G by Poisson leaves which yields a generalization of the symplectic reduction procedure. Moreover, in this most generaI context, I obtained results about uniqueness of a momentum map up to infinitesimal deformations.

During my stay (3 weeks) at the Universitat Politecnica de Barcelona I developed a research project with Prof. Eva Miranda focused on the rigidity of Poisson actions of Poisson Lie groups and momentum maps up to gauge equivalence. We generalized a previous result of Eva Miranda with Philippe Monnier and Nguyen Tien Zung that considers the case when the Poisson structure on the Lie group G is trivial. In my PhD thesis [1] I proved that the infinitesimal generator of a Poisson Lie group action with equivariant momentum map can be always written as a linear combination of Hamiltonian vector fields

$$\xi_M = \sum_i c_i(\xi)\{H_i, \cdot\}$$

where $H_i = y_i \circ \mu$ and y_i are the transversal coordinates obtained by the splitting theorem [5].

We used this property to recover the results obtained in [4] on the infinitesimal rigidity of Hamiltonian actions. I calculated the Chevalley Eilenberg cohomolgy in the general case, collaborating with Romero Solha (PhD student under the supevision of Prof. Eva Miranda), and the rigidity of the momentum map has been proved. We proved that when the Poisson Lie group is semisimple the moduli space of Poisson actions is discrete. We applied this construction to the theory in integrability of Poisson Lie actions [2]. Given a momentum map associated to a Poisson action, it is known that it can be lifted to the correspondent symplectic groupoid. The concept of "close" momentum maps can be easily extended to the lifted one.

I partecipated to the group meetings and I gave a minicourse for the "Teen seminars of geometry" organized by Eva miranda, Jaume Amoros and Miguel Rodriguez Olmos. The minicourse was focused on Poisson Lie groups, Lie bialgebras and Dirac structures. It was structured in 2 lectures (2+2 hours) and dedicated essentially to

Master and PhD students interested in Poisson geometry.

I partecipated to discussions with Eva Miranda and Romero Solha about his PhD thesis, in order to learn the basic theory of geometrical quantization and we compared it with deformation quantization.

Prof. Francisco Presas invited me to give a talk for the Geometry seminar at the ICMAT (Instituto de Ciencias Matematicas) about my PhD research, called "Momentum map and Reduction in Poisson geometry and deformation quantization". With Prof. Francisco Presas we discussed open questions related to my PhD thesis, in particular related to the definition of a quantum reduction.

References

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[4] Miranda, E. and Tien Zung, N. and Monnier, P. *Rigidity of Hamiltonian actions on poisson manifolds*. Advances in mathematics, **229** (2012).

[5] Weinstein , A. *The local structure of Poisson Manifolds*. Journal of Differential Geometry, **18** (1983).