

SYMPLECTIC AND CONTACT DYNAMICS WORKSHOP

MARCH 23-27, 2014

TEL AVIV UNIVERSITY, ISRAEL

Organizers:

Peter Albers (Universität Münster),
Yaron Ostrover (Tel Aviv University),
Leonid Polterovich (Tel Aviv University).

Summary and Scientific content:

This workshop brought together leading experts in symplectic geometry and topology, as well as promising students and young researchers (total number of 45 participants), in order to explore major developments in symplectic and contact geometry. The content of the workshop was a mixture of standard-format research talks, reporting the state of the art of the subject and presenting new important results in the field, together with in-depth mini-courses on recent developments that were given by Barney Bramham (Ruhr-Universität Bochum), Viktor L. Ginzburg (University of California, Santa Cruz), and Sheila Sandon (University of Strasbourg and CRM Montreal). In addition, there were 5 short presentations by Ph.D and post-doctoral students reporting on their recent work. Apart from the talks, there was enough time for informal discussions which were an integral part of the workshop.

List of Speakers:

Alberto Abbondandolo - Ruhr-Universität Bochum
Yeongjin Bae, Inst. for Basic Science, S. Korea
Barney Bramham - Ruhr-Universität Bochum (3-hour mini-course)
Lev Buhovski - Tel Aviv University
Francois Charette, ETH Zurich
Urs Frauenfelder - Seoul National University
Urs Fuchs, Universität Münster
Viktor L. Ginzburg - University of California, Santa Cruz (3-hour mini-course)
Basak Gurel - University of Central Florida
Helmut Hofer - Institute for Advanced Study, Princeton
Vincent Humiliere - Université Pierre et Marie Curie (Paris 6)
Michael Khanevsky, University of Chicago
Asaf Kislev, Tel-Aviv University
Will J. Merry - ETH Zurich
Sheila Sandon - University of Strasbourg and CRM Montreal (3-hour mini-course)
Felix Schlenk - Université de Neuchâtel
Sobhan Seyfaddini - Ecole Normale Supérieure

Jake Solomon - Hebrew University, Jerusalem
 Alfonso Sorrentino - Universit degli Studi Roma Tre

List of Participants (excluding the speakers):

Charel Antony, ETH Zrich
 Gabriele Benedetti, University of Cambridge
 Misha Bialy, Tel Aviv University
 Adi Dickstein, Tel Aviv University
 Alexandru Doicu, Augsburg University
 Michael Entov, Technion
 David Frenkel, University of Neuchatel
 Yaniv Ganor, Tel-Aviv University
 Doris Hein, Institute for Advanced Study, Princeton
 Kathrin Helmsauer, Augsburg University
 Felix Hensel, ETH Zrich
 Yochay Jerby, Tel-Aviv University
 Vika Kaminker, Tel-Aviv University
 Konstantin Kliakhandler, Tel-Aviv University
 Marco Mazzucchelli, ENS Lyon
 Matthias Meiwes, Universitt Mnster
 Kathrin Naef, ETH Zrich
 Lorenzo Rigolli, Ruhr-Universitt Bochum
 Daniel Rosen, Tel-Aviv University
 Karina Samvelyan, Tel-Aviv University
 Sobhan Seyfaddini, Ecole Normale Suprieure
 Egor Shelukhin, CRM Montreal
 Richard Siefring, Max Planck Institute, Leipzig
 Tobias Sodege, University College London
 Samuel Trautwein, ETH Zrich
 Igor Uljarevic, ETH Zrich
 Frol Zapolsky, Haifa University

Titles and Abstracts:

Alberto Abbondandolo: *A non-squeezing theorem on infinite-dimensional Hilbert spaces.*

Youngjin Bae: *Periodic orbits in virtual contact structures*

A virtual contact structure which naturally appears in the study of magnetic flows is a generalization of a contact structure. In this talk, I will show the existence of a periodic orbits in a virtual contact structure with an ovetwisted disk. More precisely, the Bishop family and the finite energy plane will be discussed which are due to Hofer.

Barney Bramham: *The disk map as a “playground”.*

Many of the difficulties encountered in Hamiltonian systems (such as coexistence of order and disorder) are present in dimension 2. Helmut Hofer has suggested studying low dimensional

Hamiltonian systems by combining information from multiple foliations from pseudoholomorphic curves (somehow!) in combination with techniques from dynamics. Since foliations necessarily fill up an entire space one may dream, for example, of detecting certain dynamical features even on sets of positive measure. The disk is a convenient “playground” to explore these ideas, where the influence of the geometry of the space is kept to a minimum. I will explain some results for zero entropy area preserving disk maps obtained using symplectic methods, and discuss some further questions and problems.

Lev Buhovski: *The gap between near commutativity and almost commutativity in symplectic category.*

The main subject of this talk is a question of Leonid Polterovich about a possibility of approximating a near Poisson-commutative pair of smooth functions by a Poisson-commutative pair. I will talk about its solutions in the case of dimension 2, and in the case of a higher dimension, and will explain how certain computation of the invariant pb_4 helped to solve the higher dimensional case.

Francois Charette: *Gromov width of monotone vs. non monotone Lagrangians.*

Recent work of Rizell shows that in general, it is hard to control the Gromov width of a Lagrangian, as opposed to the monotone case. I will present a result showing that non monotone orientable Lagrangian surfaces behave with respect to the width rather like monotone Lagrangians.

Urs Frauenfelder: *Real capacities and the moon*

Real capacities were recently introduced by Liu and Wang. They give rise to real systolic inequalities. In this talk I explain how real systolic inequalities are related to the analysis of the restricted three body problem.

Urs Fuchs: *Geometric Removal of Boundary Singularities and Doublings of Pseudoholomorphic Curves*

I will present two theorems on the removal of boundary singularities of pseudoholomorphic curves, one of them without any a priori finite area assumption. I will then explain how one can construct extrinsic and intrinsic doublings of pseudoholomorphic curves. These doublings turn out to be useful in the proof of removal of boundary singularities and other applications. This is based on joint work with Lizhen Qin

Viktor L. Ginzburg: *The Conley Conjecture and Beyond*

Basak Gürel : *Perfect Reeb flows and action-index relations*

In this talk we will discuss a recent work where we study non-degenerate Reeb flows arising from perfect contact forms, i.e., the forms with vanishing contact homology differential. The main results are upper bounds on the number of simple closed Reeb orbits for such forms on a variety of contact manifolds and certain action-index resonance relations for the standard contact sphere.

Helmut Hofer: *Problems and Questions in Symplectic Dynamics*

Symplectic Dynamics is new field (under construction), which aims at studying Hamiltonian systems by integrating ideas from symplectic geometry/topology and the field of dynamical

systems. The talk describes the relevant backgrounds and some of the questions, which one would like to study.

Vincent Humilière: *C^0 -rigidity of coisotropic submanifolds*

Michael Khanevsky: *Lagrangian displacement energy and Hofer's norm of commutators*

Chekanov proved that displacement energy of a Lagrangian L is bounded from below by minimal area of holomorphic spheres and disks in (M, L) . We describe a similar bound for the energy needed to separate a pair of Lagrangians in M . An application of this estimate shows that positive genus surfaces admit Hamiltonian commutators with large Hofer's norm.

Asaf Kislev: *Compactly supported Hamiltonian loops with a non-zero Calabi invariant*

We give examples of compactly supported Hamiltonian loops with a non zero Calabi invariant on certain open symplectic manifolds.

Will J. Merry: *Orderability and the Weinstein Conjecture*

In 2000 Eliashberg–Polterovich introduced the natural notion of orderability of contact manifolds; that is, the existence of a natural partial order on the group of contactomorphisms. I will explain how one can study orderability questions using the machinery of Rabinowitz Floer homology. We establish a link between orderable and hypertight contact manifolds, and show that the Weinstein Conjecture holds (i.e. there exists a closed Reeb orbit) whenever there exists a positive (not necessarily contractible) loop of contactomorphisms. This is a joint work with Peter Albers and Urs Fuchs.

Sheila Sandon: *Translated points and contact rigidity*

A point p in a contact manifold is said to be a translated point of a contactomorphism ϕ (with respect to a given contact form) if p and $\phi(p)$ are in the same Reeb orbit and ϕ preserves the contact form at p . As I first discussed in 2011, translated points seem to satisfy an analog of the Arnold conjecture on fixed points of Hamiltonian symplectomorphisms. Moreover, at least in the case of the contact manifold $\mathbb{R}^{2n} \times S^1$ that I studied in my thesis (2009), they turn out to be the geometric objects that underlie some manifestations of contact rigidity such as the non-squeezing phenomenon (first discovered by Eliashberg, Kim and Polterovich) and the existence of bi-invariant metrics on the contactomorphism group. Although these contact rigidity phenomena can be seen as a natural contact analog of the corresponding results in symplectic topology, they also have some specific features that makes them still quite mysterious. A first specific feature of the contact case is the relation of the non-squeezing phenomenon with the notion of orderability, which was introduced by Eliashberg and Polterovich in 2006 and which has motivated the development of this whole area of research. A second specific feature is the fact that in the contact case the above mentioned rigidity phenomena have a discrete character (for example bi-invariant metrics are integer valued) which, in my eyes, is due to the fact that translated points, in contrast to fixed points of Hamiltonian symplectomorphisms, also have a flexible side (for example they are not invariant by conjugation). In my minicourse I will discuss rigidity and flexibility of translated points. I will support my discussion by describing in details the construction via generating functions of spectral invariants for contactomorphisms of $\mathbb{R}^{2n} \times S^1$ (following my thesis), of the non-linear Maslov index in $\mathbb{R}P^{2n-1}$ (following Givental) and of their applications to contact rigidity phenomena. I will also show (following a joint work with Vincent Colin) how

to obtain, by exploiting both rigidity and flexibility of translated points, a natural definition of a bi-invariant metric on the universal cover of the contactomorphism group, whose non-degeneracy is equivalent to orderability of the given contact manifold.

Felix Schlenk: *Growth of geodesics between two points in manifolds of non-finite type*

This is a talk on Reeb flows on spherizations, but to bring out the main point I will focus on the Riemannian situation. Consider a closed Riemannian manifold M , and two non-conjugate points p, q in M . The growth function $CF(L; p, q)$ counts the number of geodesics from p to q of length at most L . The study of this function is a traditional topic, with major results by Morse, Serre, Gromov, and Paternain-Petean. The answers are quite complete for manifolds of finite type, which means that the universal cover of M has finitely generated homology groups, or, equivalently, that the universal cover of M has the homotopy type of a finite CW complex. For manifolds of non-finite type, it was conjectured by Paternain and Petean that $CF(L; p, q)$ grows exponentially. While geometric and topological approaches don't give much for this problem, one can use the Hopf algebra structure of the based loop space to prove "half of" this conjecture: If M is not of finite type, then $CF(L; p, q) \geq e^{C\sqrt{L}}$ for a constant $C > 0$. This is work joint with Urs Frauenfelder.

Sobhan Seyfaddini: *The displaced disks problem via symplectic topology*

We will show that a C^0 -small area preserving homeomorphism of S^2 cannot displace a disk of large area. This resolves the displaced disks problem posed by F. Béguin, S. Crovisier, and F. Le Roux.

Jake Solomon: *Moduli of marked disks, open KdV and Virasoro*

Witten conjectured a relationship between intersection theory on the moduli space of marked stable curves and the KdV integrable hierarchy. Alternatively, the KdV hierarchy can be rephrased in terms of a representation of half the Virasoro algebra. Wittens conjectures were proved by Kontsevich. I will present analogous conjectures for moduli of Riemann surfaces with boundary. These conjectures have been proved in genus 0. This is joint work with R. Pandharipande and R. Tessler.

Alfonso Sorrentino: *Mather's β function and Hamiltonian dynamics.*

In the study of the dynamics of twist maps and Tonelli Hamiltonian systems, a central role is played by the so-called Mather's minimal average action (or β -function). Roughly speaking, this is a convex superlinear function on the first homology group of the base manifold, which represents the minimal action of invariant probability measures within a prescribed homology class (i.e., rotation vector). In this talk we shall discuss the formidable problem of understanding the relation between the regularity properties of this function and the dynamics of the system, and describe some results in the case of Tonelli Hamiltonians on surfaces and Birkhoff billiard maps.