RIGIDITY AND FLEXIBILITY IN SYMPLECTIC TOPOLOGY AND DYNAMICS

Final report on the workshop held at the Lorentz Center, 21-25 July 2014
Application reference nr. 5381

1. Summary

The aim of this workshop was to address a number of questions and challenges arising from new developments in symplectic topology, such as: recently discovered symplectic flexibility phenomena, deep novel connections between symplectic topology and quantum mechanics and applications of symplectic and contact topological techniques to the existence of periodic orbits of Hamiltonian systems.

2. Description of the scientific content

P. Albers: *Orderability and Rabinowitz Floer homology*
I will explain a link between the notion of orderability in contact geometry and Rabinowitz Floer homology. The focus will be on recent results concerning connections to the Weinstein conjecture and the size of positive loops. This is joint work with Will Merry and Urs Fuchs.

B. Bramham: *Some links between finite energy foliations and Floer complexes*

L. Buhovski: *C^0 symplectic geometry of smooth submanifolds*
I will talk about a recent study of rigidity and flexibility of smooth submanifolds under the action of symplectic homeomorphisms, formulating some new results and questions. This study is a natural continuation of previous works of Emmanuel Opshtein, and of Vincent Humilière, Rémi Leclercq, and Sobhan Seyfaddini. My talk will be based on a joint work with Emmanuel Opshtein.

B. Chantraine: *Floer theory for Lagrangian cobordisms and topology of Lagrangian endocobordisms*
We will construct a Floer complex associated to a pair of exact Lagrangian cobordisms between Legendrian submanifolds of a contactisation when the negative ends of these cobordisms admits augmentations. This will allows us to describe an exact sequence relating the linearised Legendrian contact homology of the ends of a cobordism L to the topology of L which generalise a result of T. Ekholm. As an application we will find some strong restrictions on the topology of Lagrangian endocobordisms of certain Legendrian homotopy spheres. This is a joint work with P. Ghiggini, R. Golovko and G. Rizell.

Y. Eliashberg: *Existence and classification of overtwisted contact structures on high dimensional manifolds - Part II*

J. Evans: *Floer cohomology of the Chiang Lagrangian*
Joint work with Yanki Lekili. If we think of $\mathbb{CP}^3$ as the space of triples of points on the sphere then the Chiang Lagrangian is the subspace of triples with centre of mass at the origin. We will see that it has non-vanishing Floer cohomology if and only if the coefficient ring has characteristic 5. This calculation involves some general theory, true for all homogeneous Lagrangian submanifolds, and some very specific geometry in $\mathbb{CP}^3$ involving the twisted cubic.

B. Gürel: *Contact Conley conjecture and magnetic flows*

H. Hofer: *Construction of Moduli Spaces in Symplectic Geometry*

E. Kerman: *On the persistence of closed Reeb orbits*

O. van Koert: *Dynamics of the restricted three-body problem*
Inspired by holomorphic curves, we discuss how to find global surfaces of section to analyze the dynamics of the restricted three-body problem. We show how various dynamical features such as periodic orbits, invariant tori and heteroclinic orbits can be detected with this tool.

S. Lisi: *Spectral capacities and Lagrangian submanifolds*

E. Murphy: *Existence and classification of overtwisted contact structures on high dimensional manifolds - Part I*
The Lutz-Martinet theorem states that any 2-plane field on a 3-manifold is homotopic to a contact structure. This construction lead to Eliashberg's definition of overtwisted contact manifolds, and in this context the existence theorem of Lutz-Martinet can be extended to a uniqueness result: any two overtwisted contact structures which are homotopic as plane fields are in fact isotopic. We discuss a recent extension of these results to contact manifolds of all dimensions. We will focus on showing that any almost contact structure is homotopic to a contact structure, and seeing how this leads to a new definition of overtwistedness in high dimensions. As time allows we will discuss a proof that a homotopy class of almost contact structures is realized by a unique isotopy class of overtwisted contact structure. This project is joint work with Borman and Eliashberg.

K. Niederkrüger: *Filling by holomorphic disks in higher dimensions*
(joint work with Paolo Ghiggini and Chris Wendl) In this talk, I explain how it is sometimes possible to find families of Lobs (Legendrian open books) inside a contact manifold, and how these can be used to work with holomorphic disks. Some potential applications will be described.

S. Onaran: *Invariants of Legendrian Knots*
In this talk, we will focus on a class of knots in contact 3-manifolds called Legendrian knots. First, we will review known invariants for Legendrian knots. Then, we will define new invariants for Legendrian knots. We will discuss applications of these invariants and list several problems related to the invariants. This is joint work with K. Baker.

L. Polterovich: *Autonomous flows and Hofer's geometry*

I. Smith: *Categorical flux, after Seidel*
This will be an expository talk on Seidel's theory of families of objects in Fukaya categories, and the associated categorical abstraction of flux-type invariants in symplectic topology.

M. Usher: *Flexibility for Hofer's metric with respect to submanifolds*
We discuss geometric aspects of one parameter subgroups of Hamiltonian diffeomorphisms. Work in progress with Egor Shelukhin.

3. Assessment of the results and impact of the event

When we decided to submit a proposal to the Lorentz Center for a workshop around the theme of rigidity and flexibility in symplectic topology and dynamics we had a feeling that this would become an important topic: little did we know that, during the short time between our application and the actual workshop, it would grow and develop into one of the hottest topic in symplectic topology. The Lorentz Center workshop represented thus a highly necessary and appreciated opportunity for several researchers to get together and discuss their latest results and ideas.

We think it is fair to say that the work of Eliashberg and Murphy on existence and classification of contact structures will have a lasting impact on research in high dimensional contact manifolds.

Among the most exciting moments of the workshop were the very active discussions between the groups of Borman-Eliashberg-Murphy on one side and Casals-Presas on the other, concerning the notion of overtwistedness in higher dimensions.

Animated discussions were also produced by the presentation of Klaus Niederkrüger on filling of Legendrian open books by holomorphic disks. A preprint by Ghiggini-Niederkrüger-Wendl on this topic recently appeared (arXiv 1408.1051) and was written in parts during the workshop.

Several speakers addressed the question of existence of periodic orbits in Hamiltonian dynamics: Albers, for instance, explained how the notion of orderability in contact geometry has implications for the validity of the Weinstein Conjecture, while Gürel talked about how to approach the question of existence of infinitely many periodic Reeb orbits (Contact Conley Conjecture) and the results it produces for magnetic flows on surfaces.

Rigidity and flexibility in symplectic topology and dynamics: workshop programme

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Monday 21 July 2014

09:00 – 10:00 Welcome, office assignment and coffee in the Lorentz Center
10:00 – 10:15 Introduction by the manager of the Lorentz Center
10:15 – 11:00 **L. Polterovich:** *Autonomous flows and Hofer's geometry*
11:15 – 12:00 **B. Bramham:** *Some links between finite energy foliations and Floer complexes*

12:00 – 14:00 Lunch break

14:00 – 14:45 **P. Albers:** *Orderability and Rabinowitz-Floer homology*
14:45 – 15:15 Coffee and tea break
15:15 – 17:15 **Short presentations**

17:30 – Wine and cheese party

Tuesday 22 July 2014

09:00 – 09:45 **E. Kerman:** *On the persistence of closed Reeb orbits*
09:45 – 10:15 Coffee and tea break
10:15 – 11:00 **B. Gürel:** *Contact Conley conjecture and magnetic flows*
11:15 – 12:00 **S. Lisi:** *Spectral capacities and Lagrangian submanifolds*

12:00 – 14:00 Lunch break

14:00 – 14:45 **M. Usher:** *Flexibility for Hofer's metric with respect to submanifolds*
14:45 – 15:15 Coffee and tea break
15:15 – 17:15 **Short presentations**

17:15 – Free time / Discussion

Wednesday 23 July 2014

09:00 – 09:45 **I. Smith:** *Categorical flux, after Seidel*
09:45 – 10:15 Coffee and tea break
10:15 – 11:00 **B. Chantraine:** *Floer thoery for Lagrangian cobordisms and topology of Lagrangian endocobordisms*
11:15 – 12:00 **J. Evans:** *Floer cohomology of the Chiang Lagrangian*

12:00 – 14:00 Lunch break

14:00 – 16:00 **Work in groups**

16:30 – 17:00 Transfer by bus to harbor
17:00 – 21:00 Workshop dinner on boat through Kaag' lakes
21:00 – Transfer by bus to Leiden Centraal, lorentz Center, or Hotel Van der Valk

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Thursday 24 July 2014

09:00 – 09:45	**E. Murphy:** *Existence and classification of overtwisted contact structures on high dimensional manifolds – Part I*
10:00 – 10:45	**Y. Eliashberg:** E*xistence and classification of overtwisted contact structures on high dimensional manifolds – Part II*
10:45 – 11:15	Coffee and tea break
11:15 – 12:00	**K. Niederkrüger:** *Filling by holomorphic disks in higher dimensions*
12:00 – 14:00	Lunch break
14:00–14:45	**S. Onaran:** *Invariants of Legendrian knots*
14:45 – 15:15	Coffee and tea break
15:30 –	Free time / Discussions

Friday 25 July 2014

09:00 – 09:45	**O. van Koert:** *Dynamics of the restricted three-body problem*
09:45 – 10:15	Coffee and tea break
10:15 – 11:00	**L. Buhovski:** *C^0-symplectic geometry of smooth submanifolds*
11:15 – 12:00	**H. Hofer:** *Construction of moduli spaces in symplectic geometry*
12:00– 14:00	Lunch break
14:00 –	Coffee / Free time / Discussion

Rigidity and flexibility in symplectic topology and dynamics: list of participants

Peter Albers	(Münster, Germany)
Marta Batoréo	(Rio de Janeiro, Brazil)
Matthew Strom Borman	(San Francisco, United States)
Barney Bramham	(Bochum, Germany)
Lev Buhovski	(Tel Aviv, Israel)
Roger Casals	(Madrid, Spain)
Baptiste Chantraine	(Nantes, France)
Alexandru Cioba	(Hatfield, United Kingdom)
Sylvain Courte	(Lyon, France)
Álvaro Del pino	(Madrid, Spain)
Yakov Eliashberg	(Stanford CA, United States)
Jacqueline Espina	(London, United Kingdom)
Jonathan Evans	(London, United Kingdom)
Oliver Fabert	(Hamburg, Germany)
Yaniv Ganor	(Tel Aviv, Israel)
Hansjörg Geiges	(Köln, Germany)
Viktor Ginzburg	(Santa Cruz, United States)
Yusuf Goren	(Santa Cruz, CA, United States)
Basak Gurel	(Orlando, FL, United States)
Doris Hein	(Freiburg, Germany)
Helmut Hofer	(Princeton, United States)
Ely Kerman	(Urbana, United States)
Michael Khanevsky	(Chicago, United States)
Asaf Kislev	(Tel Aviv, Israel)
Samuel Lisi	(Nantes, France)
Maksim Maydanskiy	(Paris, France)
Marco Mazzucchelli	(Lyon, France)
Emmy Murphy	(Cambridge, MA, United States)
Klaus Niederkrüger	(Toulouse Cedex 9, France)
Juan Salvador Ojeda Santana	(Bochum, Germany)
Sinem Onaran	(Ankara, Turkey)
Federica Pasquotto	(Amsterdam, Netherlands)
Leonid Polterovich	(Tel Aviv, Israel)
Francisco Presas	(Madrid, Spain)
Daniel Rosen	(Tel Aviv, Israel)
Thomas Rot	(Amsterdam, Netherlands)
Nena Röttgen	(Freiburg, Germany)
Egor Shelukhin	(Jerusalem, Israel)
Richard Siefring	(Leipzig, Germany)
Kyler Siegel	(Palo Alto, United States)
Ivan Smith	(Cambridge, United Kingdom)
Dmitry Tonkonog	(Cambridge, United Kingdom)

Michael Usher	(Athens, United States)
Otto van Koert	(Seoul, Korea)
Chris Wendl	(London, United Kingdom)
Jagna Wisniewska	(Amterdam, Netherlands)
Kai Zehmisch	(Muenster, Germany)
Fabian Ziltener	(Utrecht, Netherlands)