Report on summer school

Symplectic Field Theory V

Lagrangian Floer theory: computations and applications

1. Summary

The summer school "SFT V: Lagrangian Floer theory - computations and applications" took place at Hamburg University from August 22 to August 26, 2011. As usual for this series of summer schools, there were three introductory lecture courses on the weekend before the actual summer school. We had a total of 8 invited speakers during the week (one of which also gave a precourse), two more speakers for precourses, and a total of 71 participants.

The aim of the summer school was to acquaint beginning young researchers with the foundations of Lagrangian Floer homology, that have been the outcome of the work by Fukaya, Oh, Ohta and Ono done in the last 15 years. Since presenting the substantial technical machinery in detail currently appears to be beyond the scope of a single week event, we decided to emphasize recent applications by the main architects of the theory. The expectation was that by seeing the theory at work, Ph.D. students and young researchers would be able to get a sense of the basic ingredients and the great potential of the theory. Judging from feedback at coffee break conversations and comments at the end of the conference, this expectation has been fulfilled.

2. Scientific content

The central lectures of the main event, given by Kenji Fukaya, described some applications of Lagrangian Floer theory to questions in the symplectic topology of symplectic toric manifolds. Here he outlined the (partial) calculation of the potential, viewed as a function on H^1 of the torus fibers with values in the Novikov ring. Its critical points correspond to Maurer-Cartan elements for which the twisted Floer homology of the Lagrangian fiber does not vanish (which in particular implies that the fiber is not displaceable). This was illustrated with examples in the Fano case, were the full potential function can be computed using work of Cho and Oh. Fukaya also discussed bulk deformation, i.e. deformation of Floer homology using closed holomorphic curves, and the relation of the bulk deformed potential function to the big quantum cohomology. This leads to a bijective correspondence of the critical points with homomorphisms from the Jacobian ring of the potential function to the Novikov ring.

To properly introduce the audience to the techniques necessary for these applications, Fukaya's lectures were preceded by 3 lectures held by Yong-Geun Oh and Kaoru Ono, who systematically introduced Floer homology for Lagrangian submanifolds (starting from the monotone case) and discussed how filtered A_∞ structures arise from the behavior of compactified moduli spaces of holomorphic discs with boundary on the Lagrangian submanifold. In order not to completely loose sight of the analysis, Ono also gave one lecture on Kuranishi structures. Later in the week, Ono also described a continuum of Hamiltonianly non-isotopic Lagrangian tori in $S^2 \times S^2$. Oh also discussed torsion phenomena in Lagrangian Floer homology and the use of bi-disks in studying displaceability. Finally, Fukaya also gave one inspiring lecture discussing how one can sometimes control the behavior of moduli spaces of holomorphic disks under surgery of Lagrangian submanifolds.

The afternoon sessions were filled by several complementary talks, all with themes related to the main morning lectures. On Monday, Cheol-Hyun Cho discussed the generalization of Lagrangian Floer homology in the orbifold setting. On Tuesday, both talks (by Garrett Alston and Hiroshi Ohta) dealt with situations in the presence of an anti-symplectic involution. Whereas Ohta discussed the sometimes subtle orientation issues involved in counting holomorphic discs with boundary on the fixed point set of such an involution, Alston concentrated on the specific family of Lagrangian $\mathbb{R}P^3$'s in the quintic Calabi-Yau threefold $X = \{x_0^5 + \cdots + x_4^5 = 0\} \subset \mathbb{C}P^4$ obtained from the real part of X by acting with the automorphism group of the embedding

$X \subset \mathbb{C}P^4$. On Thursday, Mohammed Abouzaid explained some applications of Floer theoretic techniques to understanding Lagrangian submanifolds of cotangent bundles. Finally, on Friday, Sikimeti Ma'u explained generalizations of Floer theory to Lagrangian correspondences and illustrated the use of holomorphic quilts in giving alternative algebraic descriptions of the behavior of Floer homology under iterated Dehn twists.

On Monday and Thursday afternoons Abouzaid and Cho also gave two more basic lectures illustrating some examples. The talk by Abouzaid concentrated on Lagrangian tori in cotangent bundles of spheres, and the talk by Cho dealt with some aspects of homological algebra.

3. Assessment of results and impact

Judging from the feedback received at the end of the week, the summer school was very successful in introducing the younger researchers to this technically demanding topic, and so the primary goal has been achieved. Through numerous discussions between participants from different institutions the secondary goal of fostering more interaction between students at an early stage of their career has also been realized. In several talks, specific open questions were mentioned. To name one example, Abouzaid, after mostly discussing dimension 4 in his talk, formulated specific questions about Polterovich tori and other Lagrangian submanifolds in cotangent bundles of spheres. It was also clear from Fukaya's talk on Lagrangian surgery that in that area there are several potentially accessible open problems.

To summarize, I believe that the summer school was very successful, and I am greatful to the CAST network for its support.

For the organizers:

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