GESTA 2011: NEW TRENDS IN SYMPLECTIC AND CONTACT GEOMETRY

Scientific summary and abstract

Place: Castro Urdiales, Santander, Spain
Venue: CIEM (Centro Internacional de Encuentros Matemáticos)
Dates: June 27- Juy 1, 2011
Organizing Committee:
Jaume Amorós (Universitat Politècnica de Catalunya)
Fernando Etayo (Universidad de Cantabria)
Eva Miranda (Universitat Politècnica de Catalunya)
Francisco Presas (Consejo Superior de Investigaciones Científicas)

Scientific Committee:
Vincent Colin (Université de Nantes)
Maria Luisa Fernández (Universidad del País Vasco)
Viktor Ginzburg (University of California at Santa Cruz)
David Martínez-Torres (Instituto Superior Técnico, Lisboa)
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Scientific Summary:

GESTA is a research team in Spain which is currently very visible in the international research scene. The seed of GESTA was raised in a mathematical meeting in Symplectic Geometry that some of us were attending in Lisbon in June 1999. The craddle of the group came up from a very natural context: impressed by the work of Donaldson and by the recent developments in Symplectic Geometry in the last years, the group formed by then by only some of us decided to start some regular meetings to understand these techniques.

This year the group has celebrated its eleventh meeting. It has been held in CIEM, a research center at Castro Urdiales (Spain). It has lasted from June 27th to July 1st 2011.

The final programme of the conference has been

Schedule	Monday	Tuesday	Wednesday	Thursday	Friday
9:30-10:30	Swann	Kerman	Müller	Ghiggini	Sandon
Coffee					
11:00-12:00	Gil	Swann	Swann	Oancea	Tralle
12:15-13:15	Gurel	Gil	Oancea	Usher	Eynard
Lunch			Free Afternoon		
15:00:16:00h	Oancea	Sepe		Vertesi	Gil
16:00-16:30h	Coffee	Poster Presentations (16:15-17:30) Wine break and Poster session (17:30-18:30)		Coffee	Free afternoon
16:30-17:30	Nemirovski			Bazzoni	
				OFFICIAL DINNER AT 21:00 h	

The social activities of the conference was a Wine break on Tuesday with some appetizers and an Official dinner on Thursday.

We had a total of 47 participants. There were 12 1-hour talks and 3 minicourses of 3 hours. Moreover, we have a poster presentation session with a total of 4 short communications.

Description of the scientific content of and discussion at the event

We had a total of 13 speakers who contributed with 1-hour talks and three minicourses. Moreover, we had 4 contributed talks held in a poster session.

It follows the list of speakers with the abstract of their talks:

3-hour minicourses:

Cavalcanti, Gil Ramos (Utrecht)
Constructions of generalized complex structures

In these lectures I will introduce/review the concept of generalized complex structures and focus on ways of constructing generalized complex manifolds. I hope I will be able to cover some of the following constructions: T-duality, generalized complex surgery and reduction

Oancea, Alexandru (Strasbourg)
Relationship between symplectic homology and linearized contact homology
The goal of the lecture course is to present the way in which linearized contact homology (LCH) and symplectic homology (SH) are isomorphic theories, within the setup of contact manifolds endowed with an exact symplectic filling. The two main statements that I shall prove are the following : (i) LCH is isomorphic to the S1-equivariant version of SH; (ii) there is a Gysin long exact sequence relating the equivariant and the non-equivariant theory. The lecture course is based on joint work with Frédéric Bourgeois.

Swann, Andrew (University of Aarhus)
Moment maps and the geometry of three-forms
The lectures will discuss geometries with a closed three-form. These are less uniform than symplectic geometries. Nevertheless, there is a useful notion of multi-moment map for symmetries of such geometries (developed in joint work with Thomas Bruun Madsen). The lectures will discuss the basic geometries and construction of multi-moment maps, describe a distinguished class of symmetry groups and apply multi-moment maps to relate seven-manifolds with holonomy in G_2 and T^2-symmetry to coherent triples of symplectic structures on four-manifolds.

Lectures:

Bazzoni, Giovanni (ICMAT, Madrid)
Classification of minimal algebras over any field up to dimension 6
In this talk we sketch the classification of minimal algebras generated in degree 1, defined over any field k of characteristic different from 2, up to dimension 6. This recovers the classification of nilpotent Lie algebras over k up to dimension 6. If the field k has characteristic zero, we obtain the classification of nilmanifolds of dimension less than or equal to 6, up to k-homotopy type. We will see that the algebraic setting of the proof provides interesting features on the (algebraic) topological part. This is joint work with Vicente Muñoz.

Eynard, Hélène (Paris)
Homotopy of codimension one foliations on 3-manifolds.
In 1969, J. Wood showed that any plane field on a closed 3-manifold can be deformed into the tangent plane field to a foliation. We will present a one-parameter version of this result: if two C^∞ foliations have homotopic tangent plane fields, they can be connected by a path of C^1 foliations.

Ghiggini, Paolo (CNRS, Nantes)

From Heegaard Floer to embedded contact homology via open book decompositions

Heegaard Floer homology and embedded contact homology are invariants for smooth three-manifolds defined via symplectic geometry. After breafly recalling the definition of both, I will sketch the construction of an isomorphism between the two.

Gurel, Basak (Vanderbilt University)
On the periodic orbits of Hamiltonian systems hyperbolic quadratic at infinity

A general, but not universal, feature of Hamiltonian systems is that such systems tend to have numerous periodic orbits. For example, the number of periodic orbits is infinite, i.e., the Conley conjecture holds, for any Hamiltonian diffeomorphism of a closed, symplectically aspherical manifold. On the other hand, it is easy to see that the conjecture fails for a general symplectic manifold such as the two dimensional sphere or the Euclidean space. One variant of the Conley conjecture asserts that the number of periodic orbits of a Hamiltonian diffeomorphism is infinite, provided that the map has more fixed points than absolutely necessary. For instance, Franks and Handel proved that this is the case for the two dimensional sphere. We consider this question for a certain class of Hamiltonian systems on the Euclidean space. Namely, we prove that a compactly supported perturbation of a hyperbolic quadratic form has infinitely many periodic orbits, provided that it has a one-periodic orbit which is homologically essential in a certain sense. In this talk, we will discuss this work-in-progress.

Kerman, Ely (Urbana, Illinois)
Resonance and the periodic points of area-preserving diffeomorphisms of the sphere

A remarkable theorem by John Franks asserts that every area-preserving diffeomorphism of the sphere has either two or infinitely many periodic points. In this talk I will describe a Floer theoretic proof of this result. More precisely, Franks' theorem is recovered under some additional nondegeneracy assumptions, and it is strengthened through the detection of new restrictions on the periodic points which can occur in area-preserving diffeomorphisms with only two such points. This is joint work with B. Collier, B. Reiniger, B. Turmunkh, and A. Zimmer.

Mueller, Stefan (KIAS, Korea)
Topological character of smooth invariants and topologically conjugate smooth dynamical systems

Suppose one is given an invariant of smooth vector fields that takes the same value on X and $\phi_* X$ for any C^1-diffeomorphism ϕ. Is this invariant also invariant under topological conjugation, and is it possible to extend the invariant to isotopies of homeomorphisms (at least in the presence of some additional geometric structure that is preserved)? V. I. Arnold originally posed this question for the helicity of a divergence-free vector field on a (closed) three-manifold. A natural question in this context is whether it is possible for two smooth vector fields to be topologically conjugate but not C^1-smoothly so. For Hamiltonian and (strictly) contact vector fields, we give an affirmative answer to

both questions (for many such invariants). The proofs are based on C^0-symplectic and contact topology, and this talk will survey these new tools.

Sandon, Sheila (CNRS, Nantes)
On existence of translated points for contactomorphisms
A point p in a contact manifold is called a translated point for a contactomorphism \phi with respect to a fixed contact form if p and \phi(p) belong to the same Reeb orbit and if the contact form is preserved at p. In my talk I will discuss the problem of existence of translated points, and its relation with the Arnold conjecture, the chord conjecture and the problem of leafwise coisotropic intersections. If I will have the time I will also explain how to use generating functions techniques to study this problem for contactomorphisms of the euclidean space, the sphere and the projective space.

Sepe, Daniele (University of Leicester)
Lagrangian bundles and affinely flat geometry
Lagrangian fibrations arise naturally in various areas of symplectic geometry, including Hamiltonian mechanics and mirror symmetry. Their topology and geometry encode information about the dynamics of completely integrable Hamiltonian systems, which are characterised by the existence of sufficiently many distinct constants of motion. These fibrations are entwined with affinely flat geometry, which studies manifolds equipped with an atlas whose changes of coordinates are constant affine transformations of R^n. In this talk, I shall illustrate how deep this relation goes by concentrating on the case of Lagrangian bundles, i.e. when all fibres are diffeomorphic. As a consequence of the Liouville-Mineur-Arnol`d theorem, the fibres and base space of a Lagrangian bundle are affinely flat manifolds. The affinely flat geometry of the former allows to classify these bundles topologically by constructing universal Lagrangian bundles. However, this classification is not sufficient to solve the problem of constructing an appropriate symplectic form on the total space of bundles which topologically "look" Lagrangian, as there is a cohomological obstruction class constructed by Dazord and Delzant. Using the topology of universal Lagrangian bundles, I shall describe this cohomology class in terms of an important affine invariant of the base space, called the radiance obstruction, introduced by Goldman, Hirsch and Smillie in the '80s. Time permitting, I will illustrate the theory with some examples.

Tralle, Aleksy (U. of Warmia and Mazury in Olsztyn)
On the group of hamiltonian symplectomorphisms and a conjecture of Reznikov
In the talk I will present some recent results on the topology of the group of hamiltonian symplectomorphisms of some compact homogeneous spaces K/H. In particular, I will address to a conjecture of Reznikov that the natural homomorphism from the cohomology of the classifying space of this group to the cohomology of the classifying space BK is a surjection. We prove that the conjecture is true generically for coadjoint orbits of K and find conterexamples. This is a joint work with S. Gal and J. Kedra.

Vertesi, Vera (MIT)
A "true" transverse invariant in Heegaard Floer homology

Using the language of Heegaard Floer knot homology recently two invariants were defined for Legendrian knots. One in the standard contact 3-sphere defined by Ozsvath, Szabo and Thurston in the combinatorial settings of knot Floer homology, one by Lisca, Ozsvath, Stipsicz and Szabo in knot Floer homology for a general contact 3--manifold. Both of them naturally generalizes to transverse knots. In this talk I will give a characterization of the transverse invariant, similar to the one given by Ozsvath and Szabo for the contact invariant. Namely for transverse braids both transverse invariants are given as the bottommost elements with respect to the filtration of knot Floer homology given by the axis. The above characterization allows us to prove that the two invariants are the same in the standard contact 3--sphere. This is a joint work with J. Baldwin and D.S. Vela-Vick.

Poster session:

Grama, Lino (Universidade Compinas, Brasil)
Equigeodesics on generalized flag manifolds
In this work we study homogeneous curves in generalized flag manifolds with the additional property that such curves are geodesics with respect to each invariant metric on the flag manifold. These curves are called equigeodesics. We give an algebraic characterization for such curves and we exhibit families of equigeodesics in several flag manifolds of classical and exceptional Lie groups. This work is joint with Caio Negreiros and Nir Cohen.

Sabatini, Silvia (EPFL, Switerland)
A necessary condition for S^1-Hamiltonian actions with isolated fixed points (joint work with L. Godinho, IST)
For every compact symplectic manifold M with a Hamiltonian circle action and isolated fixed points, a simple algebraic identity involving the first Chern class is derived. When the manifold satisfies an extra ``positivity condition", this enables us to construct an algorithm to obtain linear relations among the isotropy weights at the fixed points. If $\dim(M)\leqslant 8$ and the number of fixed points is minimal, this algorithm quickly determines all the possible families of isotropy weights, simplifying the proofs due to Ahara and Tolman, and generalizing their results to $\dim(M)=8$. From the fixed set data, we are able to recover the equivariant cohomology ring and Chern classes of the manifold.

Solha, Romero (UPC, Barcelona)
Foliated cohomology and geometric quantisation of integrable systems
This poster shows an attempt to extend some results by Sniatycki, Guillemin and Sternberg in geometric quantisation considering regular fibrations as real polarisations to the singular setting. The generic real polarisations concerned here are given by integrable systems with nondegenerate (in the Morse-Bott sense) singularities. And the definition of geometric quantisation used is the one suggested by Kostant; via higher cohomology groups. The case of nondegenerate singularities was obtained in dimension 2 by Hamilton and Miranda and the completely elliptic case was considered by Hamilton in any dimension. The idea is to combine previous results of Miranda and Presas on a

Künneth formula to reduce to the 2-dimensional case. One of the key tools used here is an extension of the results of Rawnsley on the Kostant complex. This is based on a joint work in progress with Eva Miranda.

Zambon, Marco (UAM, Madrid)
Moment map up to homotopy
We consider a manifold endowed with a closed 3-form invariant under a Lie group action. Assuming that the vector fields generating the action are hamiltonian (in the sense of Baez-Rogers), we show that the action admits a "moment map up to homotopy", which is closely related to Masden's and Swann's multimoment maps. We provide some examples. Finally, we put the above into perspective by briefly discussing general "moment maps up to homotopy" as well as "actions up to homotopy" on Poisson manifolds.

Assessment of the results and impact of the event on the future direction of the field

The Workshop entailed a close interaction between Symplectic Topology and Generalized Geometries researchers. This has been fruitful for both sides. The conference has been full of informal discussions and many open problems have been proposed and partially solved.

We tried to enhance communication among participants by providing long coffee breaks and leaving Wednesday's afternoon free. We also decided to prepare a special poster session with short communications by the poster presenters. This gave them the opportunity to present shortly their results. After the presentation of the poster session there was a Wine Break with appetizers in front of the posters. This nice spot and break also contributed to the active exchange of ideas. This also fostered the communication between poster presenters and the rest of the audience.

GESTA has been faithful to its initial spirit of bringing together scientists from different areas and disciplines, related by the common interest in Symplectic Geometry. Once again, the talks in the Workshop have shown a whole array of different ideas and techniques.

The experience of organizing a conference at CIEM has been very positive. The local structure has been really helpful and though most of the Organizers were not local, this has never been a problem. The research institute provides an ideal framework for a Workshop as GESTA. We plan to come back in the future.

Concerning budget, the main economical contribution to this conference came from CAST. We also had some contributions from other sources: namely the Spanish ministry of science and several nets in Geometry and Mechanics. Some of the funds have been difficult to handle from an administrative viewpoint. This is not the case of the ESF contribution which was really easy to

handle: the freedom given to the Organizers is wide and we have been able to cover most of the expenses that we had envisaged.

The journal *Geometriae Dedicata* will devote a special issue to the results presented in this Workshop . It will be the second time in which a Proceedings Issue for a GESTA conference is published. The editors will be Oscar García-Prada, Viktor Ginzburg, William Goldman, Eva Miranda and Vicente Muñoz.

The global assessment of this conference is very positive. It will continue the line of the previous GESTA conferences. We can proudly say that the GESTA Workshop has become a meeting point for researchers in Symplectic Geometry all over the world. Next year the GESTA conference will be fusioned with the sixth edition of the *Workshop on Symplectic Geometry, Contact Geometry and Interactions*. This workshop will also be the meeting point of the Steering Committee of the CAST network. It will be held at ICMAT in Madrid on the 2-4 of February 2012.

Meeting photo: The participants of the GESTA conference during the Poster session and Wine Break.