Workshop on Interactions between Algebra and Dynamics in Symplectic Topology

Final Report

1 Summary

The workshop on Interactions between Algebra and Dynamics in Symplectic Topology was successfully held in the Technion - Israel Institute of Technology (Haifa, Israel) on June 17-21, 2012. About 20 leading researchers in symplectic topology gave one-hour invited talks on their recent work and exchanged ideas in conversations between the lectures.

2 Description of the scientific content of the event

The following are brief descriptions of the talks given at the workshop (in alphabetical order by the speaker's last name).

Miguel Abreu (IST-Lisbon, Portugal) gave a talk on "Displacing Lagrangian toric fibers by extended probes". He described his recent joint work with Matthew Strom Borman and Dusa McDuff, where they introduce a new way of displacing Lagrangian fibers in toric symplectic manifolds, a generalization of McDuff's original method of probes. Extended probes are formed by deflecting one probe by another auxiliary probe and can be used to displace all toric fibers in Hirzebruch surfaces except those already known to be nondisplaceable. Miguel also discussed what extended probes can tell us about displaceability of toric fibers in certain weighted projective spaces, sectors and their resolutions, where there still remain many simple cases with continuous and even open sets of fibers whose displaceability status is unknown.

Peter Albers (University of Münster, Germany) gave a talk on "Contact non-squeezing and Rabinowitz Floer homology", describing his recent joint work with Will Merry. He explained how Rabinowitz Floer homology can be used to prove a new result about contact non-squeezing and for a construction of a new metric on the universal cover of the identity component of the group of contactomorphisms of a contact manifold.

Paul Biran (ETH-Zurich, Switzerland) and Octav Cornea (University of Montreal, Canada) talked about their joint work on Floer theory for Lagrangian cobordisms and its relation to Fukaya category.

Mihai Damian (University of Strasbourg, France) gave a talk on the topology of monotone Lagrangian submanifolds. He recalled the method of Lagrangian surgery developed by Leonid Polterovich in the 90's that yields many examples of Lagrangian submanifolds in $\mathbb{C}^n$ and showed that many of these manifolds do not admit monotone Lagrangian embeddings.

Urs Frauenfelder (Seoul National University, Korea) talked about "Dihedral homology and the moon". He explained how twisted O(2)-action on the free loop space of a sphere combining the flip of a loop with a reflection at the equator appears in the restricted problem of three bodies.

Kenji Fukaya (Kyoto University, Japan) gave a talk about Floer homology of arbitrary genus. He presented a construction defining an invariant of a Lagrangian submanifold using the moduli space of pseudo-holomorphic curve with arbitrary genus. This is defined as an element of certain generalization of Maurer-Cartan equation that is called BV-Master equation. It gives a series of elements of symmetric bar complex of the cyclic barcomplex of the de Rham complex.

Viktor Ginzburg (University of California at Santa Cruz, USA) talked about his joint project with Başak Gürel discussing the effect of hyperbolic fixed points on the dynamics of Hamiltonian diffeomorphisms.

Başak Gürel (Vanderbilt University, USA) talked about "Periodic orbits of Hamiltonian systems hyperbolic and linear at infinity" discussing the question of existence of infinitely

many periodic orbits for a certain class of Hamiltonian systems on the Euclidean space. She proved that a compactly supported perturbation of a hyperbolic quadratic Hamiltonian has infinitely many periodic orbits, provided that it possesses more than one homologically essential one-periodic orbit.

Richard Hind (Notre Dame University, USA) gave a talk on "Ellipsoid embeddings and packing stability" about his joint work with Olguta Buse and Emmanuel Opshtein. He discussed symplectic embeddings of ellipsoids and the packing problem for symplectic manifolds establishing packing stability for many manifolds, that is, a sufficiently large number of disjoint identical balls of total volume less than the volume of the manifold can always be embedded symplectically.

Yael Karshon (University of Toronto, Canada) talked about her joint work with Susan Tolman on "Hamiltonian torus actions with two dimensional quotients". They classified the Hamiltonian torus actions for which all the non-empty reduced spaces are two dimensional and the manifold is connected and compact, or, more generally, the momentum map is proper as a map to a convex set.

Jarek Kedra (University of Aberdeen, UK) gave a talk on "Distortion in groups of homeomorphisms" based on his joint work with Światosław Gal. They investigated the distortion in groups of homeomorphisms of closed manifolds with positive first Betti number and provided practical criteria for checking that a homeomorphism of such a manifold is undistorted. This gives restrictions on actions of, for example, certain lattices in semisimple Lie groups.

François Lalonde (Université de Montréal, Canada) talked about the injectivity radius in Hofer's geometry. In his talk, based on a joint work with Yasha Savelyev, he discussed the conjecture stating that the ball of radius ρ in the Hofer metric is relatively contractible inside the full group of Hamiltonian diffeomorphisms when ρ is small enough.

Alexandru Oancea (University of Strasbourg, France) gave a talk on "Algebraic construction of S^1-equivariant Floer homology" based on a joint work with F. Bourgeois. He explained two equivalent definitions of S^1-equivariant Floer homology. One is more geometric and goes along the original lines of Viterbo using the coarse Borel construction, the other one is more algebraic and goes along lines of Seidel, using the fact that the Borel construction is naturally a fibered space. The second definition allows for a shorter functorial proof of the isomorphism between S^1-equivariant symplectic homology and linearized contact homology using rational coefficients. Applications on the contact homology side include a rigorous definition that solves some transversality issues, a subcritical surgery long exact sequence, and the computation of linearized contact homology for unit cosphere bundles.

Yong-Geun Oh (University of Wisconsin at Madison, USA) talked on "Hamilton-Jacobi equation and continuous Hamiltonian dynamics". He explained how a natural continuous solution to Hamilton-Jacobi equation (HJE) can be constructed by Floer theory in the framework of continuous Hamiltonian dynamics and related its initial value problem and boundary value problem to various constructions arising from Floer theory.

Kaoru Ono (RIMS, Kyoto University, Japan) gave a talk on "Non-displaceable Lagrangian tori" based on a joint work with K.Fukaya, Y.-G.Oh and H.Ohta. He discussed a relation between two criteria for displaceability for Lagrangian submanifolds. One is the non-vanishing of Lagrangian Floer cohomology and the other is (super)-heavyness in the sense of Entov-Polterovich.

Dietmar Salamon (ETH Zürich, Switzerland) talked about "Hyperkähler Floer theory, the Fueter equation, and divergence free frames". As he explained, a divergence free frame on a closed three manifold is called regular if every solution of the linear Fueter equation is constant and is called singular otherwise. Singular divergence free frames form an analogue of the Maslov cycle. Regular divergence free frames satisfy an analogue of the Arnold conjecture for flat hyperkähler target manifolds. The Seiberg–Witten equations can be viewed as gauged versions of the Fueter equation, and so can the Donaldson–Thomas equations on certain seven-dimensional product manifolds.

Felix Schlenk (Université de Neuchâtel, Switzerland) gave a talk on "Filling 4-tori with a symplectic ball" based on his recent work with Dusa McDuff and Janko Latschev. They have shown that any 4-torus with a linear symplectic form can be fully filled by one symplectic ball. Felix presented an explicit and elementary construction of such a full embedding for

the standard torus $\mathbb{R}^4/\mathbb{Z}^4$. For most other symplectic 4-tori our construction of a full filling is not elementary, but uses tools from algebraic geometry.

Dmitry Tamarkin (Northwestern University, USA) discussed his work in progress on using a microlocal approach in order to associate a dg category to a compact symplectic manifold.

Michael Usher (University of Georgia, USA) talked about "Submanifolds and the Hofer norm". Hofer's norm on the Hamiltonian diffeomorphism group of a symplectic manifold induces a natural pseudometric on the orbit of any submanifold under the action of the group. It is trivial to see that the pseudometric vanishes identically when the submanifold is a point, whereas Chekanov showed that for a compact Lagrangian submanifold of a tame symplectic manifold the pseudometric is nondegenerate. In his talk Michael discussed the situation for more general submanifolds, showing on the one hand that the pseudometric continues to be nondegenerate for many classes of coisotropic submanifolds, and on the other that it vanishes identically for generically-embedded submanifolds having codimension at least two.

Jean-Yves Welschinger (CNRS/Université Lyon 1, France) gave a talk on "Open Gromov-Witten invariants in dimensions four and six". He explained how, given a closed orientable Lagrangian submanifold of a closed symplectic manifold of dimension four or six, it is sometimes possible to extract Gromov-Witten invariants from the count of pseudo-holomorphic discs with boundary on this Lagrangian submanifold.

3 Assessment of the results and impact of the event on the future direction of the field

The workshop has served as a platform for leading experts from all around the world to present their cutting edge results in symplectic topology and exchange ideas. We hope that it will lead to new mathematical results and fruitful cooperation between the researchers.

4 Final program of the meeting

Sunday, June 17, 2012

09:30-10:00 Registration
10:00-11:00 Yael Karshon (U. of Toronto) *Hamiltonian torus actions with two-dimensional quotients*
11:00-11:30 Coffee break
11:30-12:30 Michael Usher (U. of Georgia) *Submanifolds and the Hofer norm*
12:30-14:00 Lunch break
14:00-15:00 Octav Cornea (U. of Montreal) *Lagrangian Topology and Homotopy Functors*
15:00-15:30 Coffee break
15:30-16:30 Paul Biran (ETH - Zurich) *Lagrangian cobordisms and Fukaya categories*
16:45-18:15 Reception

Monday, June 18, 2012

10:00-11:00 Basak Gurel (Vanderbilt U.) *Periodic orbits of Hamiltonian systems hyperbolic and linear at infinity*
11:00-11:30 Coffee break
11:30-12:30 Viktor Ginzburg (U. of California at Santa Cruz) *Hyperbolic fixed points in Hamiltonian dynamics*
12:30-14:00 Lunch break
14:00-15:00 Jean-Yves Welschinger (U. Lyon 1) *Open Gromov-Witten invariants in dimensions four and six*
15:00-15:30 Coffee break
15:30-16:30 Urs Frauenfelder (Seoul National U.) *Dihedral homology and the moon*

16:30-16:45 Coffee break
16:45-17:45 Yong-Geun Oh (U. of Wisconsin at Madison) *Hamilton-Jacobi equation and continuous Hamiltonian dynamics*

Tuesday, June 19, 2012

09:15-10:15 Felix Schlenk (U. of Neuchatel) *Filling 4-tori with a symplectic ball*
10:15-10:30 Coffee break
10:30-11:30 Francois Lalonde (U. of Montreal) *On the injectivity radius in Hofer's geometry*
11:30-12:15 Lunch
Afternoon & evening: Excursion to Beit She'arim and Tel Megiddo National Parks followed by dinner

Wednesday, June 20, 2012

10:00-11:00 Peter Albers (U. of Munster) *Contact non-squeezing and Rabinowitz Floer homology*
11:00-11:30 Coffee break
11:30-12:30 Alexandru Oancea (U. of Strasbourg) *Algebraic construction of S^1-equivariant Floer homology*
12:30-14:00 Lunch break
14:00-15:00 Dietmar Salamon (ETH-Zurich) *Hyperkahler Floer theory, the Fueter equation, and divergence free frames*
15:00-15:30 Coffee break
15:30-16:30 Kenji Fukaya (Kyoto U.) *Floer homology of arbitrary genus*
16:30-16:45 Coffee break
16:45-17:45 Kaoru Ono (Kyoto U.) *Non-displaceable Lagrangian tori*

Thursday, June 21, 2012

10:00-11:00 Mihai Damian (U. of Strasbourg) *On the topology of monotone Lagrangian submanifolds*
11:00-11:30 Coffee break
11:30-12:30 Richard Hind (Notre Dame U.) *Ellipsoid embeddings and packing stability*
12:30-14:00 Lunch break
14:00-15:00 Miguel Abreu (IST-Lisbon) *Displacing Lagrangian toric fibers by extended probes*
15:00-15:30 Coffee break
15:30-16:30 Jarek Kedra (U. of Aberdeen) *Distortion in groups of homeomorphisms*
16:30-16:45 Coffee break
16:45-17:45 Dmitry Tamarkin (Northwestern U.) *Microlocal category*