Report on our work on Lagrangian product tori during my stay at Neuchâtel

A submanifold L of a symplectic manifold (M, ω) is called *Lagrangian* if the symplectic form ω vanishes along L. Lagrangian submanifolds are, in some sense, "the interesting" submanifolds of symplectic manifolds. Indeed, according to Weinstein, "Everything is Lagrangian", by which he meant that most problems and concepts in symplectic topology can be reformulated in terms of Lagrangian submanifolds. Particularly interesting Lagrangian submanifolds are tori: They always exist, and they are particularly relevant since they arise in integrable and almost integrable systems.

Lagrangian product tori in standard symplectic space $\mathbb{R}^{2n}$ are tori of the form $S^1(a_1) \times \cdots \times S^1(a_n)$ where $S^1(a)$ denotes the circle in the plane that incloses are a. A product torus in a symplectic manifold is the image of a product torus under a Darboux chart. Product tori in $\mathbb{R}^{2n}$ have been classified up to Hamiltonian isotopy in [1].

In this work we extend this classification to tame symplectically aspherical symplectic manifolds. Here, the tameness condition is a technical condition that makes J holomorphic curves well-behaved, and "symplectically aspherical" means that the symplectic form and the Maslov index vanish on 2-spheres. Our classification in spaces such as cotangent bundles is complete, and in tame symplectically aspherical manifolds it is almost complete. In symplectic manifolds that are not symplectically aspherical, the classification is different. In fact, we give a construction that shows that symplectically essential 2-spheres can be used to obtain Hamiltonian isotopies between product tori that are not Hamiltonian isotopic in $\mathbb{R}^{2n}$. During my stay of two weeks at the Université de Neuchâtel, we have carried out this construction (Chapter 6 of the appended preprint).

References

[1] Yu. V. Chekanov. Lagrangian tori in a symplectic vector space and global symplectomorphisms. *Math. Z.* **223** (1996) 547–559.