1. Title:

Interactions of Low-Dimensional Topology and Geometry with Mathematical Physics

2. Acronym:

ITGP

3. Abstract:

The goal of this network is to facilitate, stimulate, and further promote the many interactions of low-dimensional topology and geometry with various fields including (in no particular order) gauge theory, quantum topology, symplectic topology and geometry, Teichmüller theory, hyperbolic geometry, string theory and quantum field theory.

The network is intended to be organized on a European wide scale, reflecting the global nature of the ongoing research in these areas. The planned activities of workshops and conferences, schools and programmes of research visits will reach across international and disciplinary lines to stimulate current and future progress.

At the same time, this network will bring together leading experts in the above-mentioned areas with a new generation of researchers, providing them with the interdisciplinary perspective and training which will plant seeds for the breakthroughs of the future.

4. Keywords:

Low-dimensional Geometry and Topology, Gauge theory, Quantum Topology, Symplectic Geometry and Topology, Hyperbolic Geometry, Quantum Field Theory and String Theory.

5. Status of the relevant research and scientific context:

Low dimensional topology and geometry comprise one of the most currently active fields in mathematics as exemplified by a number of Fields Medals in the recent past, starting with Thurston and Yau in 82, Donaldson and Freedman in '86, Witten and Jones in '90, McMullen in '94, Kontsevich in '98, and Okunkov and Perelman in ‘06. In large part, the recent vitality of these fields is derived from interactions with theoretical physics that have seen dramatic developments over the last 3 decades.

In the 70's the interaction between classical Yang-Mills gauge theories and integrable field theories with geometry and topology was pioneered by, among others, Atiyah and Faddeev.

In the early 80's Donaldson's celebrated theorems showed that the h-cobordism theorem, in fact, fails for 4-manifolds, and this failure provides many unexpected consequences, most notably the existence of uncountably many exotic 4-dimensional Euclidian spaces, when combined with Freedman's proof of the topological Poincaré conjecture in dimension 4. Since then the study of manifolds of dimension less than or equal to 4 (e.g., 3- and 4-dimensional topology, knots in 3-manifolds, mapping class groups of surfaces) have formed a new branch of geometry and topology, called *low dimensional geometry and topology*.

Further interest in low dimensional topology was provided by Jones’ construction of a new and power full knot invariant arising from his basic constructions for sub-factors, which are also deeply linked to mathematical physics.

The manifest interaction between low dimensional topology and *Quantum* field theory was provided by Witten in the late 80's, when he established that the Jones polynomial is a Wilson loop expectation value in quantum Chern-Simons theory in 3 dimensions. His observation, that this theory is completely solvable via conformal field theory, led to the rigorous construction of this theory via quantum groups by Reshetikhin and Turaev shortly there after.

Since then, the effort to understand classical and quantum field theories and string theories, in all their ramifications, has motivated some of the most extraordinary advances in mathematics of the last three decades. This progress has had a profound impact in all the areas of mathematical research included in the present proposal, some of which have been connected in totally unexpected ways. The most spectacular developments include:

● Kontsevich's universal Vasilliev invariant of links.

● The discovery of Mirror symmetry by Vafa, Greene, Candelas, and others, and its relation to T-duality by Strominger, Yau, and Zaslow.

● Le, Murakami and Ohtsuki's universal invariant of homology sphere which vastly generalized the Casson invariant.

● Kontsevich's proof of the mirror conjecture.

● Quantum cohomology relating back to Gromov's study of pseudo holomorphic curves in symplectic geometry.

● Kontsevich's proof of Witten's conjecture on the cohomology ring structure of the tautological classes.

● Seiberg-Witten equations and their simplification of Donaldson invariants.

● Taubes' relation between solutions of the Seiberg-Witten equations and pseudo holomorphic curves.

● Donaldson's results on almost holomorphic sections of high powers of prequantum line bundles over symplectic manifolds.

● Seiberg-Witten Floer homology and its relation to instanton Floer homology. The proof of property P by Kronheimer and Morowka, combining Seiberg-Witten Floer homology with the results of Gabai, and Eliashberg and Thurston.

● Ozsvath-Szabo Floer homology.

● Kashaev's volume conjecture and its relation to the Jones Polynomial by Murakami and Murakami.

● Deformation quantization of all Poisson manifolds by Kontsevich.

● Construction of a canonical deformation quantization of any compact Kähler manifold via explicit asymptotic analysis of Berezin-Toeplitz operators.

● Symplectic Floer homology of Eliashberg and Hofer.

● The "AJ"-conjecture of Gukov, further elaborated by Garoufalidis and Le.

● Khovanov's construction of a new link homology, which provides a categorification of polynomial link invariants.

● Kapustin and Witten's Electric-Magnetic duality interpretation of the geometric Langlands program.

Two spectacular developments in three-manifold theory, building on the geometrization program initiated by Thurston some three decades ago with far-reaching consequences that should surely be mentioned, but which appear for now to be unrelated to developments in physics, are:

● Perelman's proof of the Poincaré Conjecture and Thurston's Geometrization Program via PDEs, using and greatly extending Hamilton's work on Ricci-flow

● The Minsky-Masur-Brock-Canary proof of Thurston's Ending Lamination Conjecture together with the Agol/Calegari-Gabai proof of Marden's Tame Ends Conjecture, each of which is proved using techniques of combinatorial topology and hyperbolic spaces.

6. Scientific Objectives and Envisioned Achievements:

The role of the proposed network will be to stimulate and facilitate new interactions between the different subfields of low dimensional topology and geometry with various subfields of mathematical physics related to quantum field theory and string theory. The main general objectives are to stimulate exchanges of ideas between the sub-fields as these develop, fuse and split up into new formations.

We have organized the subject matter under a number of headings. These headings are best though of as perspectives on the whole subject, focusing on various aspects of the entirety. Some of these perspectives are already strongly related in a number of ways, whereas other relations are still developing. A prime objective of the network is to promote further relations and connections between all the perspectives, in order to benefit from the unity and strong interaction of all the perspectives on the subject matter. The aim is to obtain a deeper insight into the entirety of the field - something which will be of benefit to all existing and future perspectives

A number of open problems which are of central importance have been highlighted below. They will receive particular attention by the teams involved in the network.

Gauge theory

Since the work of Donaldson in the 1980's, gauge theory has evolved as an indispensable tool in the study of smooth 4-manifolds. After the introduction of the Seiberg-Witten invariants in 1994, the theory was both simplified and extended, and the relation to Gromov-Witten invariants (for symplectic 4-manifolds) was established by Taubes. The equivalence of the Donaldson and Seiberg-Witten invariants, conjectured by Witten in 1994, now seems close to being proved by Feehan--Leness. Kronheimer--Mrowka have used part of the work of Feehan-Leness combined with that of Taubes to complete the proof of Property P for knots. Recently, the Seiberg-Witten invariants together with the rational blow-down technique of Fintushel--Stern have been applied to prove the existence of exotic smooth structures on $CP^2\#k(-CP^2)$ for small values of k according to work of Akhmedov-Park after initial results of J. Park.

The introduction of "finite-dimensional approximation" by Furuta signaled a new direction in gauge theory. He first used it to prove a weakened version of the 11/8 conjecture, and later, Bauer-Furuta defined the *refined* Seiberg-Witten invariant, which is stronger than the classical one.

In the theory of knots and 3-manifolds, the various kinds of Heegard Floer homologies introduced by Ozsvath-Szabo have become a highly active area of research. It is expected that the Heegard Floer homologies for 3-manifolds are essentially isomorphic to the monopole Floer homologies constructed by Kronheimer-Mrowka. Different schemes and ideas towards proving this conjecture have been presented by Y.-J. Lee and Hutchings-Sullivan. There is furthermore major work by Salamon-Wehrheim towards the classical Atiyah-Floer conjecture, which motivated Ozsvath-Szabo's construction.

We emphasis the following important open problems in this area:

- Settle the 11/8-conjecture for 4-manifolds.

- Find the smallest k such that $CP^2\#k(-CP^2)$ admits an exotic smooth structure.

- Establish ismomorphisms between the Heegard and monopole Floer homologies for 3-manifolds.

- (Kronheimer--Mrowka) Is there a connection between Khovanov homology for knots in S^3 and some version of the Collin-Steer instanton knot Floer homology?

- (Donaldson) Construct equivariant instanton Floer homology groups with arbitrary coefficients and good functorial properties with respect to cobordisms.

- What relation is there between instanton and monopole Floer homology (with rational coefficients) for integral homology 3-spheres? In particular, do the corresponding h-invariants of Frøyshov agree?

- Do the generalized monopole equations of Taubes and Pidstrygach (which involve the choice of a hyperkähler manifold with special properties) yield new invariants of 3- or 4-manifolds?

Symplectic and Contact geometry

The theory of symplectic and contact manifolds has come to play an essential role in modern topology though the full scheme and its ramifications are yet to be understood. These theories arise naturally in the setting of smooth manifolds via Hamiltonian mechanics and geometric optics, for example.

In dimensions three and four, these relationships have been solidified over the past 15 years and produced fundamental contributions as detailed in the above section on gauge theory. The latest development is a purely combinatorial interpretation of the Heegaard Floer invariants of knots by Manolescu Ozsvath Sarkar.

With this perspective, the following problem is natural to address:

- Provide purely combinatorial models for Heegaard Floer invariants of general 3- and 4-manifolds.

These Heegaard Floer invariants are expected to be equivalent to Donaldson and Seiberg-Witten invariants. Hence the solution of this problem will also lead to a combinatorial description of these invariants.

Much of the recent progress in these areas involves techniques of counting holomorphic curves, which were pioneered in the 80's by Gromov (Gromov-Witten invariants) and Floer (Floer homology) and have developed ever since. An important recent addition is symplectic field theory (SFT) introduced in the late 90's by Eliashberg, Givental, and Hofer. These theories all associate an algebraic object to geometric data, where the algebraic object is defined in terms of moduli spaces of holomorphic curves. Some homology quotient of the object is invariant under deformation of the geometric data, and these quotients thus appear as obstructions for geometric deformations.

Although the holomorphic curve techniques have indeed been successfully applied to find obstructions, one might ask whether they can be used in some more constructive manner. There are many instances for symplectic 4-manifolds when this is so, where positivity of intersection allows for the application of so-called filling techniques. Recent developments involving more explicit representations of moduli spaces of holomorphic curves in terms of

Morse-theoretic objects indicate that in important special cases holomorphic disks may be used as Whitney disks to design deformations. This is a promising line of research aimed at determining the implications of vanishing holomorphic curve obstructions. We expect progress to be made in the coming years on the following problems.

- Is every exact Lagrangian submanifold in a cotangent bundle Hamiltonian isotopic to the 0-section?

- If a contact manifold (Legendrian submanifold) admits an exact filling (is null cobordant in the exact category), then its contact homology admits a linearization. Is every linearization of contact homology geometrically induced in this way?

The holomorphic curve theories mentioned above come in two types: absolute and relative. In the absolute case, the geometric data is a contact or a symplectic manifold whereas in the relative case it is a contact or symplectic manifold with a Legendrian or Lagrangian submanifold. A major stumbling block has been to include all holomorphic curves in the relative case. Recently, these problems were overcome on the conceptual level, and a solution involving the string topology of the submanifold is emerging. Further work must be done in order to put this conceptual picture on a rigorous footing, and we again mention a concrete problem:

● The relative versions involving string topology are not immediately effectively computable. Formulate more effectively computable theories and relate the full string topology theories to already existing simpler versions.

Another example of a successful application of holomorphic curve techniques to smooth topology is Ng's powerful knot invariants which are defined as the simplest version of relative SFT of the conormal lift of a knot. One might hope to use similar constructions to get non-trivial information about 4-dimensional smooth topology, and a starting point would be the following:

● Give a combinatorial description of the contact homology of the conformal lift of surface in 4-space.

● Does the SFT invariants of conformal lifts of surfaces inside a smooth 4-manifold carry information of the smooth structure of the manifold?

Quantum Topology

Shortly after Witten's work on quantum Chern-Simons theory, Reshetikhin and Turaev used the theory of quantum groups to give mathematical constructions of what is now known as the quantum SU(N) RT-TQFT's, which includes the SU(N) quantum invariants and the quantum SU(N) representations of the mapping class groups. These are the key examples of TQFT and at the center of activity in quantum topology. A few years later Blanchet, Habbeger, Masbaum and Vogel gave a purely skein-theoretic construction of SU(2) theory and later generalized this to SU(N). Following these constructions the field has branched into a number of research areas, the most predominant of which are listed and detailed below. Furthermore, quantum topology is closely related to the study of skein theory, various quantizations of moduli spaces, and classical topological/geometric invariants of 3-manifolds. The finite type invariants, also discussed below, are of course motivated by perturbative analysis of the Feynmann path integral formula for the quantum Chern-Simons invariants by Witten.

<u>Finite type invariants of links:</u> The theory of *finite type invariants* of links in the 3-sphere is a well-developed theory based on the contributions of Vasilliev, Birman-Lin, Bar Natan, Kontsevich, Vogel, and others. All quantum invariants of links (which includes the Jones and Homfly polynomials) have asymptotic expansions in terms of Vasilliev invariants. Moreover there is the universal invariant due to Kontsevich. An open problem is:

● Do Vasilliev invariants distinguish the unknot?

The AJ-conjecture, the volume conjecture and the AE-conjecture (see below) imply an affirmative answer to this problem. The main outstanding question is whether Vasilliev invariants classify knots and links. Recently there has been some very interesting work by Fielder, who has constructed invariants of knots in the 3-sphere which are very similar to Vassiliev invariants and which completely determine some simple knots. Another problem is to:

● Describe the class of knots classified by Fiedler's constructions.

<u>Finite type invariants of 3-manifolds:</u> Finite type invariants of 3-manifolds have primarily been developed to study integral homology 3-spheres, and the lowest degree non-trivial finite type invariant is equal to the Casson-Walker-Lescop invariant. There is a universal finite type invariant constructed from the Kontsevich integral for links by Le, Murakami and Othsuki, the LMO-invariant. D. Bar-Natan, S. Garoufalidis, L. Rozansky and D. Thurston have used a diagrammatic analogue of the Gaussian integration to give a new construction of the LMO invariant for rational homology 3-spheres. By the work of Ohtsuki and Habiro, the LMO-invariant determines the SU(2)-quantum invariant.

It is conjectured that the LMO invariant distinguishes integral homology 3-spheres. As a more tractable sub-problem we propose:

● Prove that the LMO invariant distinguishes the 3-sphere from all other homology 3-sphere.

It is known that the LMO invariant does not separate lens spaces, and a natural problem is to construct refinements of the LMO invariant. There exists other constructions of a universal perturbative invariant based on integration over configuration spaces due to S. Axelrod and I. Singer, M. Kontsevich, R. Bott and A. Cattaneo, and G. Kuperberg and D. Thurston. These are closer to the original physical inspiration, and it is an interesting problem to establish their connection to the LMO invariant and to quantum invariants, possibly via the AE-Conjecture (see below).

<u>Link polynomials, the A-polynomial and the AJ-conjecture:</u> Gukov proposed based on a quantum field theory argument, that there should be a relation between the colored Jones polynomials and the A-polynomial from hyperbolic geometry through classical asymptotics. Subsequently Garoufalidis and Le proved that the colored Jones polynomials are q-holonomic and they further observed that the ideal of operators that annihilate the colored Jones polynomials of a knot is principal and is generated by the so-called *noncommutative A-polynomial* of the knot. On the other hand, one has the A-polynomial, which cuts out the deformation variety of a knot, which is the variety of characters of representations of the fundamental group of the knot into PSL(2,C).

● *The AJ-Conjecture:* Prove that the noncommutative A-polynomial at q=1 equals to the A-polynomial of a knot, and hence is determined by the colored Jones polynomials.

In the late 90'ties Murakami and Murakami converted a conjecture of Kashaev about the asymptotics of his invariants to a conjecture about the asymptotics of certain evaluations of the colored Jones polynomials of a knot.

● *Volume Conjecture:* Prove that these asymptotics determine the Gromov-Thurston norm of the compliment of the knot.

Quantum hyperbolic field theory: Having as model the TQFT-axioms the so called "quantum hyperbolic field theory" QHFT has been constructed by Baseilhac and Benedetti. This provides quantum invariants of pairs, 3-manifolds equipped with a flat PSL(2,C) principal bundle. The fundamental examples of such pairs are given by topologically tame hyperbolic 3-manifolds. This theory is a generalization of Kashaev's invariants of knots in the 3-sphere, and we have the following problem:

● Understand the relation between these invariants and quantum Teichmüller space.

A generalization of the volume conjecture has been formulated for these QHFT invariants:

● Test, refine and prove the generalized volume conjecture.

Asymptotic expansions of quantum invariants: Right from the outset, Witten suggested that quantum invariants should have asymptotic expansions in the level of the theory. During the past decade, a precise conjecture regarding these asymptotics has been formulated which is known as the AE-conjecture. Lately Andersen has proposed a program using the theory of coherent states and Toeplitz operators to approach the AE-conjecture in the general case. This is a natural extension of his proof of Turaev's conjecture regarding the Asymptotic faithfulness of the quantum SU(N)-representations of the mapping class groups and his proof that the mapping class groups do not have Kashdan's property T.

● *AE-conjecture (Witten):* Establish the precise formulation of the asymptotics of the quantum Chern-Simons invariants.

Another interesting problem is to understand how the AE-conjecture is related to the asymptotic expansions of Ohtsuki and Habiro.

Integral TQFT: Gilmer and Masbaum have studied the natural integral lattices inside the TQFT vector spaces, on which the mapping class groups acts, reflecting integrality properties of the TQFT-invariants of manifolds with boundary. An explicit basis for these lattices has been given, facilitating explicit calculations and Masbaum has shown that integral TQFT's lead to perturbative expansion of the quantum representations. One problem is to:

● Understand these integral structures in terms of Habiro expansions.

Another fascinating question is to determine how this is related to the AE-conjecture.

Khovanov homology: Around 1998, Khovanov invented a new way of viewing the Jones polynomial for links. By using a topological quantum field theory he constructed a homology theory for knots and links whose quantum Euler characteristic is the Jones polynomial. The importance of his construction lies not only in the fact that it provides a stronger invariant than the Jones polynomial, but that it is functorial with respect to link cobordisms. Lately Khovanov and Rozansky have invented a generalization: a homology theory whose quantum Euler characteristic is a specialization of the HOMFLY polynomial. A connection has been conjectured between Khovanov-Rozansky homology and the spectrum of BPS states for open topological strings. This is a relatively new area ripe for further development.

● Understand the implications for four dimensional quantum invariants arising from the algebra and geometry emerging from Khovanov-Rozansky homology.

The quantum polynomials of a given Lie algebra can be combined to define the Witten-Reshetikhin/Turaev invariants of 3-manifolds and, more generally, the corresponding topological quantum field theories. It would also be important to categorify the full TQFT's, a problem which at present is completely open. Part of this program would be to lift the TQFT representations of mapping class groups to some category associated to a surface.

● Find a direct relation with Witten's original approach to quantum invariants from Chern-Simons theory.

Link homologies have already been used to give new, purely combinatorial proofs of theorems in four-dimensional topology which formerly had depended on the heavy machinery of gauge theory, for instance in the work of Rasmussen, which includes the proof of Milnor's conjecture on the slice genus of torus knots, and the existence of exotic differential structures on 4-space. One would expect more such proofs to appear as the understanding of the theories grows.

Recent work of Seidel/Smith and Manolescu and of Cautis and Kamnitzer give symplectic and geometric settings, respectively, for Khovanov homology, thus providing good opportunities for exploring the relation of quantum topology to these other fields.

A different construction due to Oszvath-Szabo expresses the Alexander polynomial as an Euler characteristic.

● Find the precise relations between Khovanov homology, Heegard-Floer homology, and gauge theory.

Geometry and Quantization of Moduli spaces

Geometric structures on moduli spaces of Higgs bundles: Higgs bundles over Riemann surfaces were introduced by Hitchin (1987) almost 20 years ago in the study of the self-duality Yang-Mills equations. A Higgs bundle is a pair consisting of a holomorphic bundle and a holomorphic one-form with values in the adjoint bundle. The moduli space of Higgs bundles with complex reductive structure group has a tremendously rich geometric structure. It is a completely algebraic integrable system and it is a hyperkähler manifold. It also has a topological interpretation: It can be identified with the moduli space of representations of the fundamental group of the surface in the complex reductive group. Further, Hitchin constructed an integrable system on these moduli spaces, which has had a significant impact on the study of these moduli spaces from many points of view. It has been applied to several other moduli problems, e.g. in Hitchin's proof of the projective flatness of his connection over Teichmüller space in the bundle arising from the quantization of the moduli space of semi stable bundles and more recently, in a very prominent way, in Witten and Kapustin's Electric-Magnetic duality interpretation of the geometric Langlands program. In fact a recent conjecture of Hausel and Thaddeus states that the generic torus fibers of the Hitchin map for an algebraic group G and its Langlands dual LG are in an appropriate sense relative mirror partners (in the sense of Strominger-Yau-Zaslow). This is related to the conjectured existence of a general Fourier--Mukai transform underlying the geometric Langlands duality:

● The Fourier-Mukai transform takes a solution to the Higgs bundle equations on a Riemann surface to a bundle with a unitary connection compatible with the flat hyperkähler structure on the cotangent bundle of the Jacobian. Characterize this connection.

Bonsdorff's work showed that one holomorphic structure extends to the projective bundle compactification of the cotangent bundle. Doing this for fixed points of the circle action on the moduli space would give another approach to determining the hyperbolic metric on a Riemann surface.

The moduli of Higgs pairs and the moduli of stable bundles can also be studied as subschemes of the infinite Grassmannian of the ring of Laurent series, and natural actions of the Virasoro groups on them will be deduced: We study the Toda hierarchy algebraically as well as to analyze of the relation of the multicomponent KP hierarchies that generalize it. The study of the hierarchies arising from the Hurwitz schemes is related with the Beilinson-Drinfeld formulation of the geometric Langlands program over the complex numbers. We propose to continue the study of the moduli of Higgs pairs and of the Hitchin morphism by means of the soliton theory. In particular:

● Characterize such moduli space as subschemes of an infinite Grassmannian.

Hitchin has introduced higher analogues of Teichmüller spaces inside the representation varieties of higher rank groups. Labourie has shown that the mapping class group acts properly on these spaces.

● Find geometric structures on surfaces which are parametrized by the Teichmüller component: the distingushed contractible component of the moduli space of representations of a surface group into a split real form of a simple Lie group.

For PSL(3,R), Goldman and Choi showed that this space parametrizes convex RP^2 structures. Labourie has also shown that for PSL(n,R) there is an (in general, non-smooth) curve in $RP^{(n-1)}$ which describes the situation, but one would like a smooth structure. For example, in the case of PSL(3,R), Labourie's curve is the non-smooth boundary of a convex set, but a flat projective structure is a smooth concept. Fock and Goncharov have results about these spaces using tropical geometry further discussed below.

Quantization of moduli spaces: Within the last decade quantizations of the moduli spaces have been constructed using both algebraic, geometric and topological techniques.

The geometric quantization of moduli spaces of flat G-connections on a 2-dimensional manifold with G a compact Lie group is well understood. The classical results of Narasimhan and Seshadri state that Teichmüller space is a natural parameter space of Kähler structures on the moduli spaces. By associating the space of holomorphic section of a power of the Chern-Simon's prequantum line bundle over Moduli space to every point over Teichmüller space, a finite rank vector bundle arises. This bundle caries a proejctively flat connection, which was constructed by Axelrod, Della Pietra and Witten from the quantum Chern-Simons point of view and by Hitchin from a purely differential geometric view point.

The Berezin-Toeplitz operators provide natural quantum operators in the context of compact Kähler manifolds. These are asymptotically equivalent to the traditional quantum operators of geometric quantization, and the relationship can be explicitly described. Furthermore, results of Bordemann-Meinrenken-Schlichenmaier, Schlichenmaier, and Karabegov-Schlichenmaier show that the Berezin-Toeplitz quantization scheme has excellent semi-classical behavior (it is a strict quantization in the sense of Rieffel) and that there exists a canonical unique star product associated to it, providing the Berezin-Toeplitz deformation quantization.

The relation between the Berezin-Topelitz operators and Hitchin's projectively flat connection has been understood by the work of Andersen. This has led to a proof of Turaev's asymptotic faithfulness conjecture. By further considering the coherent state constructions and applying the results on Toeplitz structures by Boutet de Monvel and Guillemin and Sjøstrand, Andersen has established that the mapping class groups do not have Kazhdan's property T.

Some parts of this program have so far only been developed in the non-singular setting, while many of the moduli spaces are singular. We thus propose the problem to:

● Extend the above program fully to singular moduli spaces.

Quantum group techniques have been used by Alexseev, Fock, Kashaev, and others to construct *quantum moduli space* for various complex groups and non-compact real groups (including as a special case quantum Teichmüller space). Andersen, Mattes, and Reshetikhin constructed a deformation quantization of the moduli spaces of flat SL(N,C)-connections using Vassiliev invariants of knots in surface cylinders. Frohman, Shikora and others constructed a deformations of the SL(2,C)-character variety using skein theory. Recently, Andersen has used the Berezin-Toeplitz deformation quantization in combination with the Hitchin connection to construct a mapping class group invariant deformation quantization of the SU(N)-moduli spaces.

● Understand the relation between all these different quantizations of moduli spaces.

Understanding relations to quantizations of the Higgs bundle moduli space will no doubt also be of fundamental importance. In particular we propose the problem to:

● Geometrically quantize Teichmüller space using the complex structure inherited from its embedding in the PSL(2,C) Higgs bundle moduli space, which is just $C^{(3g-3)}$. The resulting symplectic structure then gives a complete S^1-invariant Kähler metric.

Construction of TQFT's via quantization of moduli space: Witten proposed in his initial paper on the subject that the TQFT's could be constructed by applying geometric quantization to the moduli spaces of flat SU(N)-connections. There was intensive activity from the point of view of algebraic geometry in early 90's, but a full construction was not presented. From the conformal field theory, major advances were made by Segal, Beilinson, Feigin and Masur, Tsuchiya, Ueno and Yamada, Teleman, and others. Laszlo subsequently established that the projectively flat bundles constructed from conformal field theory agree with the ones constructed via quantization of moduli spaces above. Recently, Andersen and Ueno have completed the program and provided an explicit isomorphism between the geometric construction and the quantum SU(N) RT-TQFT in the BHMV-model. Another problem is to:

● Provide a complete gauge theory construction of the RT-TQFT.

A satisfactory solution to this problem should include a truly 3-dimensional geometric construction of the boundary states of the theory.

Three dimensional hyperbolic geometry

The most dramatic development in three dimensional hyperbolic geometry recently is the proof of the Poincaré conjecture and the completion of Thurston's geometrization program by Perelman. It builds and greatly extends on Hamiltons work on the Ricci flow program.

The positive solutions of the Tameness Conjecture (TC) by Agol/Calegari-Gabai the Ending Laminations Conjecture (ELC) by Minsky-Masur-Brock-Canary open the way to an effective understanding of hyperbolic 3-manifolds. Given a complete hyperbolic 3-manifold M with finitely generated fundamental group, the TC guarantees that the ends of M are homeomorphic to a product S X R, where S is a surface (called a boundary surface). The ELC on the other hand ensures that the set of complete hyperbolic structures on M is parameterized by the so-called end-invariants, namely, by elements of the closures of the Teichmüller spaces of the boundary surfaces.

The core of the proof of the TC is the study of CAT(-1)-surfaces immersed in the ends. For the ELC, the main point is to build a bi-Lipschitz model for M depending only on the end-invariants. The new techniques introduced have been further developed so to provide more information about hyperbolic manifolds. For instance, the volume, the length spectrum, and other geometric quantities can be efficiently estimated using the bi-Lipschitz model.

The study of the space of end-invariants is currently one of the most active research fields. The phenomena occurring are slightly different if the end-invariants are taken in the interior of the Teichmüller space or in its boundary. In the latter case the complete characterization of those laminations that occur as end-invariants has been recently achieved, and the attention is now devoted to understanding the local structure of the space of parameters (e.g. discontinuity and bumping phenomena). When the end-invariants lie in the interior of the Teichmüller space, the local structure is smooth, and the most intriguing problem is to understand the relations between the invariants of different ends. For instance, if one associates to an end-invariant its inner correspondent, one gets the so-called skinning map. The fact that this map is a contraction was crucial in Thurston's proof of geometrization for Haken manifolds, but the fact that it is not a constant was established only very recently, and many questions about its behavior remain open and deserve investigation:

● Hyperbolic structures in higher dimensions. Deformation of three-dimensional manifolds embedded in higher dimensional spaces.

● Local structure of the end-invariants.

Recently, Gabai, Meyerhoff, and Milley have proven that the Weeks manifold is indeed the unique smallest closed hyperbolic manifold. Their proof builds on results by Agol that describe how the volume of a closed 3-manifold changes under drilling out a simple closed geodesic. This result was subsequently strengthened by Agol, Storm, and Thurston using Perelman's work on the Ricci flow and the geometrization of closed 3-manifolds. Agol, Culler, and Shalen have shown that any compact manifold with volume less than the Weeks manifold can be obtained by Dehn filling a cusped manifold of volume less than 2.848. Such cusped manifolds can be classified using "Mom technology" and finally explicit enumeration shows that there is no manifold smaller than the Weeks manifold. Some interesting problems are to:

● Enumerate closed hyperbolic manifolds with small complexity and comparison between volume and complexity.

● Use the MOM technology for a thorough understanding of exceptional slopes on hyperbolic manifolds.

● Examine the behavior of complexity under finite cover.

● Progress on the Hurwitz existence problem for surface branched covers with prescribed branch data, and prove the conjecture of realizability for prime degrees.

Agol, Storm and Thurston also apply their result to provide evidence for the positivity of the universal TQFT pairing for 3-manifolds as conjectured by Freedman. Very recently, Freedman has proven that the universal 3-manifold pairing is indeed positive using virtually all the new machinery provided by Perelman, Agol, Storm, Thurston, and others. We would like to:

● Understand the further relations and implications of this for TQFT.

The moduli space of flat G-connections on 3- and 2-dimensional manifolds can be interpreted as evolution operators and phase spaces of the Chern-Simons field theory, respectively. The Einstein gravity in 3d can be also interpreted as a version of Chern-Simons field theory. In Euclidean signature of the space-time and negative cosmological constant, the role of the space of 2d flat connections is played by the space of quasifuchsian groups, though the spaces of Kleinian groups play role of flat 3d connections. Both kinds of spaces are certain open subsets of the moduli of flat SL(2, C)-connections. The theory of quasifuchsian and Kleinian groups is a rapidly developing subject due to Perelman's geometrization theorem, the Brok-Canary-Minsky ending lamination theorem, and several others. It is also clear that due to Hitchin's Higgs field construction, the moduli spaces of SL(2,C)-connections can be interpreted as certain subspaces of the moduli of flat connections with affine group G. On the other hand Kashaev's volume conjecture suggests a very close relation between quantum invariants for compact groups and their hyperbolic analogues. One of the aims and challenges of the network is to:

● Unify the approaches to this subject coming from these different branches of mathematics in order to develop a quantization scheme for 3d gravity.

Combinatorics of moduli spaces and quantum Teichmüller theory

The moduli space of flat G-connections on a smooth punctured surface S is an algebraic manifold admitting several descriptions using methods relying on different structures on the surface. One of the approaches was initiated by Thurston, Strebel, Mumford, Harer and Penner and originally applied to the Teichmüller space. In this approach, the topology of the surface is encoded into combinatorics of its triangulations with vertices in the punctures, or, equivalently, of the dual trivalent graph that the surface is retractable to. In this direction the following results were obtained:

● Strebel, Mumford, Harer and Penner have described a cell decomposition of the moduli spaces of curves by cells numerated by fat graphs thus reducing the study of topology of the moduli space to combinatorics.

● Kontsevich has used this reduction to compute the intersection indices of Mumford cycles on moduli and presented a generating function for these numbers as a matrix integral giving a tau-function of an integrable system, a statement conjectured by Witten.

● Madsen-Weiss-Tillman has proven Mumford's Conjecture on the stable cohomology of M(F).

● Penner introduced explicit coordinates on a version of Teichmüller space and described the action of the mapping class group on it.

● Penner coordinates and the action of the mapping class group on them were generalized in many ways, namely to different flavours of Teichmüller spaces, to the space of measured geodesic laminations, to moduli of flat connections with simple Lie groups over any ground field, and to the higher analogues of Teichmüller spaces introduced by Hitchin as certain connected components of discrete surface fundamental group representations to split real forms of simple Lie groups.

● The combinatorial approach to the spaces of flat connections turned out to have many common features with the so-called cluster theory introduced by Fomin--Zelevinsky in their study of combinatorics related to Lie groups and Lusztig's canonical basis.

● Combinatorial coordinates on Teichmüller spaces allowed Kashaev--Chekhov--Fock to find a noncommutative (quantum) deformation of this space equivariant with the mapping class group action. It was proven by Teschner that the representation space for these quantum algebras are isomorphic to the space of conformal blocks of the Liouville conformal field theory.

We suggest the following list of problems to focus on:

● Explicitly understand the unstable cohomology of the mapping class group and its natural Deligne-Mumford compactification with consequent implications for string theory, string topology, and 2-dimensional topological quantum field theory.

● There has already been progress in applying these techniques to the mapping class group action on the pronilpotent and profinite completions of the fundamental group of the surface with consequent implications of the former towards the understanding of finite-type invariants of 3-manifolds and links in 3-manifolds, and perhaps of more general quantum invariants as well.

● By the work of Donaldson, symplectic 4-manifolds stabilize to Lefshetz fibrations, which over the 2-sphere correspond to special relations in the mapping class group that should also be illuminated by these combinatorial techniques.

● Though Teichmüller space is very well understood from the combinatorial viewpoint, nothing comparable is done for the moduli space of quasifuchsian groups which is a natural complexification of the Teichmüller space and plays a role of a phase space in the three-dimensional gravity. We plan to study this space using some relation of it to KdV-like integrable systems.

● Detailed study of the mapping class group representation arising from quantum Teichmüller space, the question of its reducibility, relations to the geometric quantisation, corresponding 3d invariants with possible application to the volume conjecture.

● Generalization of Penner's cell decomposition to arbitrary cluster varieties, in particular to higher Teichmüller spaces and to simple Lie groups.

● The Thurston boundary for Teichmüller space can be described using cluster approach as a tropical limit of it. This approach generalizes to arbitrary cluster varieties. For ordinary Teichmülller space, the boundary can be interpreted geometrically as the set of projectivized measured geodesic laminations. In particular, every such lamination provides a function on the Teichmüller space itself, defining a canonical basis in the space of functions on it. For higher Teichmüller space, such an interpretation is unknown, and the canonical basis is not yet constructed. We hope to attack this problem using representation theory in the form of the Knudsen--Tao construction for Littlewood-Richardson coefficients as well as the combinatorics of Lusztig's canonical basis.

Mirror symmetry and derived categories

Mirror symmetry is a correspondence between symplectic and complex geometry developed to understand formulas discovered by physicists for the numbers of rational curves on certain 3-dimensional Calabi-Yau manifolds. These formulas involve Hodge theory on mirror dual Calabi-Yau manifolds, while the problem of counting rational curves in Calabi-Yau manifolds has now been formalized in the general context of {Gromov-Witten invariants}.

Kontsevich proposed the idea of {homological mirror symmetry}: there should be an equivalence of categories behind mirror duality, where one category is the derived category of coherent sheaves on a Calabi-Yau manifold X and the other is the Fukaya category of the mirror manifold X'.

One passes from the abelian category Coh(X) of coherent sheaves on X to its derived category $D^b(X)$ by formally inverting all quasi-isomorphic complexes of coherent sheaves. From a topological point of view, complexes of coherent sheaves are D-branes of the B-model (twist of a N=2 super conformal field theory), morphisms between the objects of $D^b(X)$ are identified with the states of the topological string, and composition of morphisms is computed by correlators of the B-model. The mirror of $D^b(X)$ is the category of A-branes on X'. The fundamental problem which would establish homological mirror symmetry is thus:

● Prove that the category of A-branes is equivalent to the Fukaya category.

This has only been proved so far for elliptic curves. As a consequence of the general problem, it would follow that two Calabi-Yau manifolds that have the same mirror must have equivalent derived categories. In this direction, the most spectacular result is due to Bridgeland: If X is a projective threefold with terminal singularities and $Y_1 \rightarrow X$, $Y_2 \rightarrow X$ are crepant resolutions, then there is an equivalence of derived categories of coherent sheaves $D(Y_1) \rightarrow D(Y_2)$.

In particular, two birational Calabi-Yau threefolds do have equivalent derived categories and therefore have the same Hodge numbers, but not all Calabi-Yau threefolds that are derived equivalent are actually birational. In fact, Andrei Caldararu has shown that there are non-birational counterexamples to the Torelli-problem for Calabi-Yau threefolds.

In 6 dimensions there are moduli spaces with good special properties when the underlying manifold is a Calabi-Yau 3-fold (the objects of interest in mirror symmetry and string theory). In particular, moduli spaces are often naturally complex, and admit the structure of a ``gradient scheme''. They are usually still singular, but their special structure means a ``virtual cycle'' can be defined, and so an interesting invariant may be defined, regularizing the Euler characteristic of the moduli space. This applies to moduli of curves, surfaces, bundles and coherent sheaves on CY 3-folds, defining the relevant Gromov-Witten and Donaldson-Thomas invariants.

These invariants are hard to calculate in even a very small number of cases, but recent work of Maulik-Nekrasov-Okounkov-Pandharipande conjectures a relationship between GW and DT invariants which has been checked (by localization) in many noncompact cases with symmetry.

Dualities and effective theories: From the point of view of mathematical physics, one fundamental breaktrough has been the realisation that certain QFT models, which are formulated very differently, can be related in some regime or even sometimes be understood to be equivalent in a precise sense. One example is the relationship between nonabelian gauge theories in four (spacetime) dimensions and sigma-models in two dimensions: the sigma-model describes the low-energy dynamics of the vortex string solutions of the 4d theory. Upon quantisation, the 2d theory on the string worldsheet captures certain correlation functions and vacuum information about the strongly coupled 4d theory. For example, in QFTs with N=2 supersymmetry, the 2d theory can be used to rederive the Seiberg-Witten curve and the spectrum of the 4d theory. One of the many issues still awaiting investigation is:

● Study the quantitative features of this correspondence for QFTs with N=1 supersymmetry.

The gauged linear sigma-model provides a method to construct Calabi-Yau geometries from gauge theories. By studying the quantum dynamics of the gauge theory, one may extract important geometric information about the moduli space and geometry of the underlying manifold. Recently, Hori and Tong studied the quantum dynamics of 2d nonabelian gauge theories, and the associated Calabi-Yau manifolds which are complete intersections in Grassmannians. This provided a proof of several new features, including a new topology changing transition, reminiscent of the the flop, but where the two Calabi-Yau's are not birationally equivalent. Open questions to be addressed at this point include:

● Calculate Gromov-Witten invariants for these Calabi-Yau's and obtain a more precise understanding of mirror symmetry in this context, which is somewhat more complicated than for intersections in toric varieties.

Conformal field theory: Conformal field theories (CFTs) occupy a special place among QFTs because they possess an infinite dimensional symmetry algebra. In the basic examples, this is just the Virasoro algebra, but other infinite algebras, like affine Lie algebras, are possible. CFT is therefore intimately tied on the one hand to the representation theory of these algebras and on the other hand to the theory of Riemann surfaces, which specify the worldsheet for which a CFT amplitude is computed. Physically relevant applications of CFT are to the description of critical behaviour in statistical mechanics, percolation and stochastic Loewner evolution, and the worldsheet description of string theory. While CFTs are not generic, they play an important role as limits of QFTs under `renormalisation group flows'; in this sense, the moduli spaces of CFTs can themselves be interpreted as a boundary of a much larger space of quantum field theories. A fruitful point of view has been to understand CFT as the boundary degrees of freedom of a three-dimensional theory which is topological in the bulk. The prototypical example is the relation between two-dimensional Wess-Zumino-Witten models and three-dimensional Chern-Simons gauge theory. Mathematically this can be phrased as a highly nontrival equivalence of two functors: one constructed analytically in terms of moduli spaces of solutions to certain differential equations, and the other combinatorially in terms of representation theory. While very powerful, this approach is at the moment only applicable to so-called rational CFTs, which lie at discrete points in the moduli space of all CFTs, and they necessarily belong to components which from a string theory point of view correspond to compact directions in the target space. This sets out a number of important tasks where progress is feasible, which include to:

● Understand properties of CFTs belonging to components of the moduli space that contain rational points. Of particular interest are the N=2 super-CFTs arising as sigma-models for compact Calabi-Yau manifolds, given their strong ties to algebraic geometry.

● Obtain a better control over non-compact CFTs and their moduli, if possible by extending the topological methods to this class of CFTs.

● Give a useful and adequate mathematical definition of the moduli space of CFTs (e.g. an appropriate 2-category), which should work beyond the class of models for which the concept of moduli space is meaningful at present.

Non-compact CFTs are particularly relevant to investigate decay processes of unstable configurations in string theory (`tachyon condensation') and to study non-static string backgrounds which are important in cosmological applications.

Topological solitons: Another central theme in quantum field theory has been the understanding of the role played by topological solitons such as monopoles and instantons. These objects affect certain mechanisms encompassing supersymmetry and integrability, which have provided a fertile ground for ideas in symplectic and complex geometry. They were also one of the fundamental ingredients leading to the proposal of electric-magnetic duality, which has been recently revived and extended by Kapustin and Witten in the context of the geometric Langlands programme.

Moduli space methods have become an essential tool in the analysis of topological solitons, producing profound insight into the classical, quantum and statistical mechanics of solitons in relativistic field theories. Many of these theories coincide, in the static regime, with important models of condensed matter physics (e.g. sigma-model instantons coincide with ferromagnetic bubbles). An emerging theme is the quest to adapt geometric moduli space methods to deal with these condensed matter contexts. Most obviously, this approach involves deriving moduli space approximants to soliton dynamics in realistic models; however, there is considerable potential for these methods to address more subtle problems, for instance to:

● Develop moduli space techniques to calculate geometric phases associated with topological defect transport (e.g. in fractional quantum Hall systems).

Very recently, new insights have been emerging in this direction such as the relationship between geometric phases and supersymmetry. Further progress is expected to have fundamental impact on both theoretical physics and applications as geometric phases are a crucial component in experimentally feasible approaches to topological quantum computation.

Three-dimensional gravity: While QFTs have provided a basic framework for phenomenological models of fundamental interactions, incorporating gravity has proved elusive. However, in three spacetime dimensions Einstein's theory of gravity simplifies dramatically. Physically, this manifests itself through the absence of gravitational waves. Mathematically, it allows for the formulation of the theory as a Chern-Simons gauge theory. The gauge group G depends on the signature of spacetime and on the cosmological constant, but is typically non-compact. The phase space of the theory is the moduli space of flat G-connections. In this formulation, quantizing gravity amounts to the quantisation of this moduli space, thus relating one of the deepest problems in theoretical physics with one of the most studied objects in modern differential geometry. Correspondingly, research in 3d gravity is interdisciplinary. A number of research groups employ a Hamiltonian approach and address fundamental physics questions concerning the role of geometry at the Planck scale in the language of quantum groups and noncommutative geometry. A different approach via so-called spin foam models leads to interesting links with knot theory and 3-manifold invariants. Problems that can be addressed within the time-scale of this proposal are for example:

● Provide a rigorous quantisation of Lorentzian 3d gravity in the Hamiltonian framework and its relationship to the spin foam approach.

● Classical limit: how does spacetime geometry emerge from a diffeomorphism-invariant quantum theory of 3d gravity?

7. European Context: The goal of this proposal is to consolidate many excellent individual and institutional efforts in low dimensional topology, geometry, and mathematical aspects of quantum field theory and string theory into a European-wide network. Such consolidation is long awaited and will make participating groups more competitive worldwide and will facilitate exchange of information, research, and educational experience. This field experienced tremendous development in the last two or three decades with many discoveries, and it has been crowned with several Fields medals; however, it still only has a relatively modest representation in the European networks. A small part of activities of this network overlaps with the focus area of the ESF network “MISGAM”, with funding expiring in the summer 2009. This network is chaired by Professor Boris Dubrovin, who also serves on the steering committee of this network to ensure the transition and the transfer of the expertise. The proposed network will coordinate activities with the ESF network “QG” which is devoted to quantum gravity. The chair of this network, Professor John Barrett, has expressed his strong support and enthusiasm to collaborate with this network. In particular there is a plan to organizing a joint workshop on quantum Chern-Simons theory and quantum gravity. Professor Jürg Frölich is on the steering committee for QG and he is a member of the steering committee of this network as well. This will guarantee the coordination of the two networks. A grant of the ESF will facilitate the communication between researchers across Europe and different areas of mathematics and physics, and a substantial part of it will be focused on involving young people in the research.

8. European added value: The European research landscape with benefit from the proposed network in at least two ways. Firstly, the network will facilitate research in one of the most actively developing branches of mathematics by bringing researchers in quantum field theory and string theory together with geometers and topologists. This will make scientists in these areas more competitive worldwide. Secondly, the network will also provide new possibilities for training graduate students and post-docs in a new environment where they will be exposed to a variety of scientific traditions. Finally, we will also set up and maintain an electronic database accessible through the World Wide Web, which will contain preprints and information related to the network.

9. Duration and budget estimates

We propose the following program for the duration of five years. In order to develop a strong European group and to establish fruitful collaborations among the members of the ITGP-network, we plan individual visits, small research meetings, and workshops with or without summer school. The progress made in first years of the program will be evaluated, and a larger conference (50 people) will be organized in the third year. This will also help to shape details for the research in the remaining two years.

The following figures have been estimated on a basis of a cost of about: One-week visits: 1000 euros per week; One month visits: 2100 euros per month; Workshop/conference: 700 euros per participant. We will seek to co finance all events with local sources. We consider this possible, given the enthusiastic attitude from all the involved members of this network.

● A **Main conference** during the third year of the project with 50 participants, most from the network, including the word renown experts particiants, plus some of the international collaborators. Our experience has shown that 40 000 euros is the bare minimum required to successfully organize such a gathering and that such an amount does not even allow full support of the participants. This meeting would take place e.g., at CTQM (who would co-finance half of the cost), in order to:

a) evaluate the progress made in the first two/three years of the program

b) plan research directions for the remaining years

c) expose the participants to the latest research in and around the field

This would involve an annual budget of 20 000 euros / 5 = 4 000 euros

● During the five years of the project we plan to organize a total of:

a) eight small workshops (involving about 15 participants)

b) two larger workshops (involving about 25 participants)

c) Two worldwide, well advertised, two-week **summer school** with expository lectures aimed at young researchers (30 advanced PhD students and post-docs). They would be given by members of the project and outside scientists. The school would go back-to-back with one of the workshops Topics of summer schools and workshops will include

Combinatorics of Moduli Spaces, Generalized Teichmüller spaces and their quantization

Khovanov homology and related topics from topology and representation theory

Geometry of Moduli Spaces and the Geometric Langlands Program

Quantization of moduli spaces and topological quantum field theory

Finite type invariants, Asymptotic analysis of quantum invariants and Hyperbolic geometry

Gauge theory, Floer Homologies and quantum field theory

Quantum hyperbolic field theory and quantum gravity

Conformal field theories on manifolds with boundaries

These workshops will all be given by the world experts involved in this network.

The estimated costs are as follows: Small workshop: 15 000 euros; Larger workshop: 25 000 euros. The school would involve some 35 people, and we estimate a bare minimum of 50 000 euros per school to mainly cover expenses to PhD-students and young post docs from the network. These events will be arranged with local co-funding.

This would involve an annual budget from ITGP of 100 000 euros/ 5 = 20 000 euros

- The project must also support our core activity, which is **research**; i.e., this includes about 20 small research meetings involving 3-4 people gathering in a geographically convenient location for one or two weeks and collaborating on a common project. We wish to allocate an equivalent of 30 weeks to this activity.

In the first year, this part of the budget will cover the expenses of the first meeting of the steering committee, which we estimate at 7 000 euros. The figure per year from ITGP is:

20 000 euros

- Individual short scientific visits: 30 000 euros

- We also consider it as a matter of priority to enable young researchers (**pre-docs and post-docs**) to spend several months in one of the centers mentioned in the proposal. Our allocation is about the equivalent of 16 months per year, as there is very little alternative funding for that kind of activity:

35 000 euros

- We shall also require clerical support for setting up an electronic database and maintenance of the web pages; external administrative costs (part-time assistant) and honorarium for the Project coordinator; and publication expenses. This would cover the expenses for publishing the Proceedings of the major conferences, the publication of the brochures, and, possibly, the Publication of thematical monographs on specific parts of the research program.

10 000 euros

TOTAL ANNUAL BUDGET per year: 119 000 euros

Note to budget: ● All numbers includes the ESF administrative fee. ● The size of the budget should be seen in relations to the number of involved researchers needed to guarantee the European integration of the field.

Annex to Proposal

Short Curriculum Vitae of Program Proponent Jørgen Ellegaard Andersen

Address: Center for the Topology and Quantization of Moduli Spaces, Department of Mathematical Sciences, University of Aarhus, Ny Munkegade, DK 8000 Aarhus C, Denmark. Phone: +45 89423432. Fax: +45 86131769. Email: andersen@imf.au.dk

Present position: Professor of Mathematics, University of Aarhus. Director of the "Center for the Topology and Quantization of Moduli spaces" (CTQM). Researcher in charge of a Danish Research Foundation Grant attaching Prof. Nicolai Reshetikhin (UCB) as Visiting Niels Bohr Professor to CTQM (2006-2011).

Education: 1988-1992 University of Oxford D. Phil. in Mathematics. 1984-1988 Odense University, BA in Mathematics

Previous Positions: 1992-1994 C.B. Morrey Jr. Assistant Professor at University of California, Berkeley. 1994-1996 Assistant Professor at University of Aarhus. 1995 General membership at Mathematical Sciences Research Institute, Berkeley. 1996-1997 Research Associate Professor at University of Aarhus. 1997-2007 Associate Professor at Univeristy of Aarhus. 2000 Visiting Associate Professor at University of California, Berkeley. 2001 General membership at MSRI. 2001-2002 Visiting Clay Professor at University of California, Berkeley. Funded by the Clay Mathematical Institute.

Awards and Honors: 1988-1992 Scholarship from The Danish Academy of Science. 1989-1992 Candidatus Scholarship from The Danish Research Council. 1992-1994 Fulbright scholar. 1996-1997 Scholarship from Danmark-Amerika fondet. 2000-2001 The Danish Bank scholarship price.

Research groups, Centers and Networks: 1998-2005 Principal investigator at MaPhySto research center in Aarhus. 2000-2004 Aarhus coordinator of the EU-Network EDGE, European Differential Geometry Endeavour. 2006- Director of the "Center for the Topology and Quantization of Moduli spaces" (CTQM). 2006-2011 Researcher in charge of a Danish Research Foundation Grant attaching Prof. Nicolai Reshetikhin (UCB) as Visiting Niels Bohr Professor to CTQM.

Editorships: 2005- Academic editor of Jour. of Knot Theory and its Ramifications._2007- Member of the Editorial board of Geometriae Dedicata.

Selected Key Publications:

1. J.E. Andersen, J. Mattes & N. Reshetikhin, "The Poisson Structure on the Moduli Space of Flat Connections and Chord Diagrams". Topology 35, (1996), 1069--1083.

2. J.E. Andersen, J. Mattes & N. Reshetikhin, "Quantization of the Algebra of Chord Diagrams". Math. Proc. Camb. Phil. Soc. 124 (1998), 451-467.

3. J.E. Andersen & G. Masbaum, "Involutions on moduli spaces and refinements of the Verlinde formula". Mathematice Analen 314 (1999), 291-326.

4. J. E. Andersen, "Asymptotic faithfulness of the quantum SU(n) representations of the mapping class groups". Annals of Mathematics, 163 (2006), 347 -- 368.

5. J.E. Andersen & K. Ueno, "Geometric construction of modular functors from conformal field theory", Journal of Knot theory and its Ramifications. 16 (2007), 127 -- 202.

Steering Committee:

Jørgen Ellegaard Andersen (Denmark), Jean-Bernard Zuber (France), Ursula Hamenstaedt (Germany), Gunter Cornellisen (Holland), Andras Stipsicz (Hungary), Boris Dubrovin (Italia), Peter Gothen (Portugal), Garcia-Prada (Spain), Jurgen Fuchs (Sweden), Jürg Fröhlich (Switzerland), Nigel Hitchin (UK).

Names and affiliations of researchers/research groups that want to participate in the Program's activities:

Note: In addition to those listed, research groups contain numerous post docs and PhD-students.

(All are departments of mathematics unless otherwise stated):

Bulgaria: IRNE, Sofia: Valentina Petkova.

Belgium: Univ. Libre de Bruxelles: Felix Schlenk, Frederic Bourgeois, Simone Gutt;

Czech Repulic: Inst. Math., Czech Acad. Science, Prague: Miroslav Englis; Masaryk Univ.: Jan Slovak, Rikard von Unge; Charles Univ., Prague: Vladimir Soucek;

Denmark: CTQM and GEOMAPS, Aarhus Univ.: CTQM Director Jørgen Ellegaard Andersen, Vladimir Fock, Bob Penner, Nicolai Reshetihin; Copenhagen Univ.: Berfinnur Durhuus, Hans P. Jakobsen, R. Nest, Henrik Schlichtkrull; Niels Bohr Institutet: Jan Ambjørn, Paul Di Vecchia, Charlotte Kristjansen, Holger B. Nielsen, N. Obers; Univ. of Southern Denmark: Andrew Swann, Martin Svensson and Francesco Sannino (Phys).

Estonia: Univ. Tartu: Viktor Abramov; Tallinn Univ. of Technology: Eugen Paal.

France: IHES: Maxim Kontsevich; Univ. Strasbourg: Athanase Papadopoulos, Christian Kassel; Univ. Paris VII: Gregor Masbaum, Christian Blanchet, Pierre Vogel; LPTHE, Univ. Paris VI: Jean-Bernard Zuber; ENS, Ulm: Philip Boalch; Univ. Grenoble: Christine Lescop, Stephane Baseilhac; Univ. Toulouse: Thomas Fiedler, Michel Boileau; ENS Lyon: Emmanuel Giroux; Univ. Montpellier: Alain Bruguieres; Univ. Marseille: Daniel Matignon; Univ. Lille I: Johannes Huebschmann; Univ. Orsay: Francois Labourie; Univ. Nantes: Vincent Colin; Univ. Metz: Tilmann Wurzbacher.

Germany: MPIM, Bonn: Yuri Manin, Matilde Marcolli; Univ. Hamburg: Christoph Schweigert; Univ. Koeln: Hansjoerg Geiges; Univ. Bonn: Ursula Hamenstaedt; Univ. Leipzig: Matthias Schwarz; Jacobs Univ., Bremen: Ivan Penkov; Univ. Bielefeld: Kim Froyshov; Univ. Munchen: Dieter Kotschick; Univ. Ausbourg: Katrin Wendland; Inst. Theoretical Phys, Freie Univ., Berlin: Robert Schrader; Ludwig-Maximilians Univ.: Kai Cieliebak;

Greece: Univ. Athens: Charitos Charalampos, Papadoperakis Ioannis.

Hungary: Renyi Inst.: Andras Stipsicz, Andras Nemethi; Eotvos Univ.: Balazs Csikos; Budapest Tech. Univ.: Gabor Etesi.

Ireland: Trinity College, Dublin: Samson Shatashvili, Sergey Cherkis.

Italia: Univ. Pisa: Riccardo Benedetti, Carlo Petronio, Paolo Lisca; SISSA, Trieste: Boris Dubrovin; ICTP, Trieste: George Thompson;

Luxemburg: Univ. Luxemburg: Martin Schlichenmaier;

Nederland: Geometry and Quantum Theory (GQT) Utrecht (UU): Erik van den Ban, Gunther Cornelissen, Eduard Looijenga, Ieke Moerdijk, Dirk Siersma (director of GQT) Nijmegen (RU): Erik Koelink, Gert Heckman, Klaas Landsman Amsterdam (UvA): Robbert Dijkgraaf, Gerard van der Geer, Eric Opdam

Norway: Univ. Tromsø: Valentin Lychagin.

Poland: Univ. Warsaw: Piotr Stachura; Univ. Lublin: Jozef Zajac.

Portugal: IST, Lisbon: Miguel Abreu, Rui Loja Fernandes, Jose Mourao, Roger Picken; Univ. Porto: Peter Gothen; Univ. Algarve: Marco Mackaay;

Spain: Univ. Complutense: Ignacio Sols Lucia; CSIC, Madrid: Oscar Garcia-Prada, Vicente Munoz, Luis Alvarez-Consul; Univ. Autonoma: Joachim Kock; Univ. Salamanca: Beatriz Graña, E. Gomez, A.C. Lopez J.M. Muñoz Porras, F. Plaza , Daniel Hernandez Ruiperez, F. Sancho, C. Tejero.

Sweden: MP2, Chalmers: Director Alexander Stolin; Uppsala Univ.: Tobias Ekholm, Antti Niemi, Ryszard Rubinsztein; Dept. Theoretical Phys., Uppsala Univ: Maxim Zabzine; Stockholm Univ.: Ari Laptev, Dmitry Leites, Sergei Merkulov; Dept. Theoretical Phys., Karlstads Univ.: Jurgen Fuchs; Dept. Theoretical Phys., KTH: Jouko Mickelsson.

Switzerland: Univ. Basel: Norbert A'Campo; Univ. Geneve: Anton Alekseev, Rinat Kashaev, Andras Szenes; Univ. Neuchatel: Matthias Blau; Zurich Univ.: Anna Beliakova; ETH, Zurich: Giovanni Felder, Alberto Cattaneo; Dept. Theoretical Phys., ETH, Zurich: Juerg Froehlich; CERN, Geneve: Marcos Marino Beiras.

Turkey: METU, Ankara: Mustafa Korkmaz; Koc Univ.: Burak Ozbagci, Tolga Etgu, Baris Coskunuzer, Tekin Dereli.

UK: Oxford Univ.: Tamas Hausel, Nigel Hitchin, Frances Kirwan, Graeme Segal, Lionel J Mason, Ulrike Tillmann, Dominic Joyce; DAMTP, Cambridge: Maciej Dunajski, Nicholas Manton, David Tong; DPMMS, Cambridge: Ivan Smith; Imperial College, London: Simon K. Donaldson, Richard Thomas; King's College, London: Ingo Runkel, William Harvey; Heriot-Watt, Edinburgh: Bernd Schroers; Edinburgh Univ.: Michael Atiyah, Jose Figueroa-O'Farrill, Michael Singer; Univ. Aberdeen: Vassily Gorbunov, Alexey Bondal; Univ. Durham: Richard Ward, Paul Sutcliffe; Univ. Leeds: Roger Bielawski, John Wood, Martin Speight; Univ. Manchester: Marta Mazzocco; Univ. Liverpool: Peter Newstead;

INTERNATIONAL COLLABORATORS:

Australia: ANU, Canberra: Peter Bouwknegt; Univ. Adelaide: Michael Eastwood, Michael Murray, Nicholas Buchdahl; Univ. Melbourne: Paul Norbury, Hyam Rubinstein;

Brazil: IMPA, Rio: Henrique Bursztyn; Univ. Campinas: Marcos Jardim.

Canada: Univ. British Columbia: Jim Bryan, Hendryk Pfeiffer; Univ. Toronto: Dror Bar-Natan, Marco Gualtieri, Lisa Jeffrey, Yael Karshon, Joel Kamnitzer; McGill Univ.: Jacques Hurtubise; Univ. Montreal: Francois Lalonde.

Israel: Technion, Haifa: Michael Polyak, Gelaki Shlomo.

Japan: Kyoto Univ.: Kenji Ueno; Osaka Univ.: Hitoshi Murakami; Tokyo Univ.: Nariya Kawazumi, Shigeyuki Morita; RIMS, Kyoto: Kazuo Habiro, Tomotada Ohtsuki; TITech, Tokyo: Sadayoshi Kojima.

Russia: Inst. Theoretical Physics, Chernogolovka: Aleksander Belavin, Boris Feigin; Steklov Inst. Math., St. Petersburg: Ludwig Faddeev.

USA: Harvard Univ.: S.T. Yau; Univ. Cal. Berkeley: Vaughan Jones, Peter Teichner, Nicolai Reshetikhin; Princeton Univ.: Dave Gabai, Rahul Pandharipande; Univ. Chicago: Alex Eskin; MIT, Cambride: Denis Auroux; Yale: Yair Minsky; George Washington Univ.: Jozef H Przytycki; CalTech, Pasadena: Sergei Gukov; Columbia Univ.: Mikhail Khovanov, Dylan Thurston, Peter S. Ozsvath; Indiana Univ.: Vladimir Turaev; Georgia Tech.: Stavros Garoufalidis; Microsoft Research: Michael Freeman.