Summer School on Link Homology

IHP (Paris), 29 June - 3 July 2009

FINAL REPORT

1. Summary

The summer school took place from 29 June to 3 July 2009 at the Institut Henri Poincaré (Paris). The subject of the school were various interrelated aspects of link homology, which have recently been developed in quantum topology, symplectic geometry, and representation theory.

2. Organizers

The school was organizeed by :
- Christian BLANCHET (Université Paris Diderot)
- Emmanuel FERRAND (Université Pierre et Marie Curie)
- Nadine FOURNAISEAU (AI au CNRS, Institut de Mathématiques de Jussieu)
- Gregor MASBAUM (CNRS, Institut de Mathématiques de Jussieu)

3. Lecture Courses

There were four main lecture courses. Each lecture course consisted of four lectures of one hour. The lecture courses were given by
- Paolo GHIGGINI (CNRS, Univ. de Nantes)
- Peter OZSVATH (Columbia Univ., New York)
- Lev ROZANSKY (University of North Carolina, Chapel Hill)
- Catharina STROPPEL (Universität Bonn)

Titles and Abstracts of the Lecture Courses are attached below.

Note. Originally, we had planned a lecture course by Vincent COLIN (Univ. de Nantes). Unfortunately, Prof. COLIN had to cancel for family reasons. We are particularly grateful to Paolo GHIGGINI, who replaced him at an extremely short notice.

4. Additional Talks

Additional one hour talks were given
- Denis AUROUX (MIT)
- András JUHASZ (University of Cambridge, UK)
- Aaron LAUDA (Columbia University, New York)
- Jacob RASMUSSEN (University of Cambridge, UK)

– András STIPSICZ (Alfréd Rényi Institute, Budapest).

Titles and Abstracts of their lectures are also attached below.

5. Participants

There were 87 participants (including speakers). A little more than one third were graduate students, on third were Post-Docs, and the rest were confirmed mathematicians.

6. Discussion of the event

For the scientific content of the school, please see the abstracts of the various lectures.

The speakers of the summer school were among the best researchers currently working in the area. The interaction between the geometric side (OZSVATH, GHIGGINI) and the algebraic side (ROZANSKY, STROPPEL) was particularly instructive and opened new perspectives for many participants. The four main lecturers respected the spirit of the summer school in making their lectures accessible to graduate students. It is telling that all lectures (including the later ones within a lecture series) were very well attended. The same holds true for the five additional lectures. In a questionnaire distributed at the end of the summer school, most participants said the school matched their expectations and that it was profitable for their research. (All but four declared that they were working on a topic related to the subject of the school.)

In the same questionnaire, the participants also said that were satisfied with the exchanges they had among each other. The organizers provided a seminar room with a blackboard for disussions and some graduate students actually organized extra talks between themselves during the lunch break and at the end of the afternoon.

In conclusion, we believe that the school was very fruitful for most of the participants, both by stimulating their own ongoing research through the various lectures, and by giving them the possibility of interacting with the leaders in the field.

The following are attached :
– Titles and Abstracts
– Schedule

G. Masbaum (for the organizers)
4 October 2009
Contact : masbaum@math.jussieu.fr

Titles and Abstracts:

Lecture series:

The lectures of **Vincent COLIN** (Université de Nantes, IUF) : **Contact homology : computations and applications** have been cancelled.

Paolo GHIGGINI (CNRS, Université de Nantes) : **Introduction to Legendrian knots and contact homology**

Peter OZSVATH (Columbia University, New York) : **An Introduction to Heegaard-Floer homology**

Abstract: Heegaard Floer homology is an invariant defined in a number of low-dimensional situations (three-manifolds, knots, four-manifolds). I will give an introduction to this subject, with special emphasis placed on recent developments. I hope to focus on bordered Heegaard Floer homology, an invariant for three-manifolds with parameterized boundary, which is currently being developed in joint work with Robert Lipshitz and Dylan Thurston.

Lev ROZANSKY (University of North Carolina, Chapel Hill) : **Matrix factorizations and link homology**

Abstract: Quantum polynomial invariants of links, such as the Jones polynomial, remain a mysterious phenomenon, since their mathematically rigorous definition is purely combinatorial and their relation to classical topology is unclear. Khovanov's categorification program suggests that these polynomials are graded Euler characteristics of certain graded chain complexes related to links by special combinatorial constructions, and he constructed a chain complex related to the Jones polynomial.

In these lectures I will explain in details how to use simple tools of commutative algebra in order to construct complexes associated to 2-variable and SU(N) HOMFLY-PT polynomials. The categorification of the 2-variable HOMFLY-PT polynomial is based on Soergel's bimodules, while in the SU(N) case we use the so-called matrix factorizations.

I intend to provide the background material on links, braids, quantum polynomials and review basic facts about homological algebra.

Catharina STROPPEL (Universität Bonn) : **Category O, diagram algebras, and link homology**

Abstract: In this lecture series I will explain from scratch the construction of Khovanov homology (and its different disguises) using representation theory of Lie algebras. We will start by considering the Hecke algebra and its quotient the Temperley-Lieb algebra. I want to explain the notion of higher Schur-Weyl duality and its role in the categorification of link invariants. Finally I will apply this to deduce equivalences of categories between modules for the Lie superalgebra gl(m|n) and a generalised

Khovanov algebra.

Additional talks:

Denis AUROUX (MIT) : **Fukaya categories of symmetric products and bordered Heegaard-Floer homology**

Abstract: This talk will present an interpretation of the recent work of Lipshitz-Ozsvath-Thurston extending Heegaard-Floer homology to 3-manifolds with boundary in terms of the symplectic topology of symmetric products. More specifically, we will explain how to understand the algebra A(F) associated to a surface in terms of the (relative) Fukaya category of the symmetric product.

András JUHASZ (University of Cambridge, UK) : **Sutured Floer homology**

Abstract: Sutured manifolds provide powerful tools for studying knots and 3-manifolds. After briefly going through their definition and classical theory, I will define an invariant of sutured manifolds called sutured Floer homology. This is a common extension of the hat version of Heegaard Floer homology and knot Floer homology. It can be used to show that knot Floer homology detects the genus of a knot and also whether the knot is fibered. Furthermore, it helps in the classification of Seifert surfaces that a given knot bounds.

Aaron LAUDA (Columbia University, New York): **Categorifying quantum sl(2)**

Abstract: Crane and Frenkel conjectured that that the quantum enveloping algebra of sl(2) could be categorified at generic q using its canonical basis. In my talk I will describe a realization of this conjecture using a diagrammatic calculus.
If time permits I will also explain joint work with Mikhail Khovanov on how this construction can be generalized to quantum sl(n).

Jacob RASMUSSEN (University of Cambridge, UK) : **Dehn filling and the Thurston norm**

Abstract: Suppose Y is a 3-manifold whose boundary is a union of nontrivially linked tori. The Thurston norm on Y measures the geometric complexity (genus) of surfaces representing a given homology class. If Y' is obtained by Dehn filling one boundary component of Y, then the Thurston norm on Y gives an upper bound for the Thurston norm on Y'. I'll explain how sutured Floer homology can be used to show that this bound is an equality for all but finitely many filling slopes.

András STIPSICZ (Alfréd Rényi Institute, Budapest) : **Invariants of Legendrian knots in knot Floer homology**

Abstract: An invariant of Legendrian knots will be defined, taking values in the knot Floer homology of the underlying null-homologous knot. With the aid of this invariant we find transversely non-simple knots in many overtwisted contact structures, and show that the Eliashberg-Chekanov twist knots (in particular the 7_2 knot in Rolfsen's table) are not transversely simple.

IMJ Summer School 2009

Schedule (version June 30th)

Monday June 29	**Tuesday June 30**	**Wednesday July 1**	**Thursday July 2**	**Friday July 3**
Registration Coffee 9:00-10:30	GHIGGINI 2 9:00-10:00	OZSVATH 2 9:00-10:00	ROZANSKY 3 9:00-10:00	STIPSICZ 9:00-10:00
	Coffee break	Coffee break	Coffee break	Coffee break
ROZANSKY 1 10:30-11:30	STROPPEL 2 10:30-11:30	STROPPEL 3 10:30-11:30	OZSVATH 3 10:30-11:30	ROZANSKY 4 10:30-11:30
STROPPEL 1 11:45-12:45	ROZANSKY 2 11:45-12:45	GHIGGINI 3 11:45-12:45	STROPPEL 4 11:45-12:45	LAUDA 11:45-12:45
Lunch at ENS	Lunch at ENS	Lunch at ENS	Lunch at ENS	Lunch at ENS
GHIGGINI 1 15:00-16:00	OZSVATH 1 15:00-16:00	Free afternoon	GHIGGINI 4 15:00-16:00	OZSVATH 4 15:00-16:00
Coffee break	Coffee break		Coffee break	Coffee break
JUHASZ 16:30-17:30	RASMUSSEN 16:30-17:30		AUROUX 16:30-17:30	
			PARTY 18:00-20:00	