ESF Workshop - Scientific Report

Title	Mathematics and Physics of Moduli spaces
Dates	September 24–28, 2012
Place	Mathematics Center (MATCH), University of Heidelberg, Germany
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Organizing	Prof. Dr. M. Banagl, Prof. Dr. A. Hebecker, Prof. Dr. D. Roggenkamp
Committee	Dr. T. Weigand, Prof. Dr. A. Wienhard

1 Summary

The workshop "Mathematics and Physics of Moduli Spaces" took place from September 24 - 28, 2012 at the University of Heidelberg. The thematic focus of the workshop was higher Teichmüller spaces and their occurrence in various contexts in theoretical physics. Many recent developments in this subject have taken place rather separately in the mathematics and physics communities respectively, and remained largely unnoticed by researchers in the other field.

The workshop was an encounter of mathematicians and physicists interested in higher Teichmüller theory. It aimed at laying common ground and exploiting resulting opportunities for cross-overs of ideas. In order to achieve this goal, the program consisted of several series of two 90 minute talks (Fock, Gukov, Labourie, Moore and Teschner), 90 minute background talks (Fomin, Dimofte and Guichard), and 60 minute research talks (Alim, Boalch, Guilloux, Pantev and Wentworth). The late afternoons were often free for additional discussion among the participants. The lunch breaks and social activities (welcome reception, conference dinner, excursion, reception at the Institute for Theoretical Physics) played an important role in fostering discussion between mathematicians and physicists. In total 66 researchers participated in the workshop, over 20 of which were PhD students or young postdocs.

2 Scientific Content

The central theme of the workshop were moduli spaces of flat G-bundles on manifolds (in particular of dimension two and three), representation varieties and moduli spaces of G-Higgs bundles, and their appearance in theoretical physics. A special focus was on higher Teichmüller spaces, their cluster variety and Poisson structure, and on the study of BPS states in supersymmetric quantum field theories. (For the scientific program see below.)

Several talks (Gukov, Dimofte, Guilloux) dealt with moduli spaces of flat G-bundles on three-manifolds (in particular knot or link complements), their quantizations and para-

metrizations, and their role in understanding moduli spaces of flat G-bundles and G-Higgs bundles on surfaces. Gukov's talks focussed on relations of these moduli spaces and their deformations with knot homologies. In an additional talk he explained their emergence in the context of gauge theories. Dimofte and Guilloux presented independent but closely related work on Fock-Goncharov like parametrizations of the space of framed local systems on three-manifolds, which sparked several intense discussion between mathematicians and physicists about this topic afterwards.

The talks of Fock and Fomin focussed on cluster variety structures which arise in many, quite diverse mathematical areas, in particular in the study of moduli spaces of local systems, but also in the study of dimers and integrable system. The combinatorial structure of cluster varieties is encoded in quivers. Alim explained in his talk how quiver varieties appear in determining BPS spectra in N=2 gauge theories in four dimensions.

Labourie's and Guichard's talks dealt with the Hitchin components of representation varieties, different characterisations and infinite dimensional versions of them. Labourie constructed a Poisson structure on Hitchin components and related it to the Poisson structure on spaces of opers. Spaces of opers also played an important role in Teschner's talks, which was devoted to the quantization of Teichmüller space and its relation to Liouville theory, one of the best understood conformal field theories.

Moore explained several aspects of his work with Gaiotto and Neitzke on spectral networks, wall-crossing and BPS-states, which gives rise to many new implications on higher Teichmüller spaces (e.g. new cluster coordinates) and provides a new understanding of the space of holomorphic differentials. His talks sparked many very lively discussions among mathematicians and physicists.

Boalch, Wentworth and Pantev talked about wild character varieties, gluing formulas for determinants of Laplacians and symplectic structures on more general moduli spaces.

3   Assessment of Results

The workshop was very successful in bringing together mathematicians and physicist from communities which, despite some common interests, usually hardly interact. It fostered intense scientific discussions between mathematicians and physicists, which in part triggered new collaborations. Many participants attested that it was one of the few occasion where during a workshop a deep and fruitful exchange between mathematicians and physicists took place. The workshop laid a common ground between researchers of very different communities, and we hope that there will be other opportunities to continue the fruitful interactions and collaborations fostered at this workshop.

Many young researchers participated in the workshop and learned about these very interesting topics at the interface between mathematics and theoretical physics. We hope that the experience they had at this workshop will encourage them to start or continue research in this area.

Together with Springer we are planning to produce a proceedings volume with several introductory and survey articles to make the scientific content of the workshop available to the larger mathematics and physics communities.

Annex A: Final Programme

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
$9^{00} - 10^{30}$	GUKOV	GUKOV	FOMIN	LABOURIE	LABOURIE
$10^{30} - 11^{00}$	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
$11^{00} - 12^{30}$	FOCK	FOCK	TESCHNER	TESCHNER	GUICHARD
$12^{30} - 14^{00}$	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
$14^{00} - 15^{00}$	BOALCH	ALIM	Excursion $14^{00} - 18^{30}$	MOORE	MOORE
$15^{00} - 15^{30}$	*Coffee*	*Coffee*		*Coffee*	*Coffee*
$15^{30} - 16^{30}$	DIMOFTE	GUILLOUX		WENTWORTH	PANTEV
$16^{30} - 17^{00}$		GUKOV		———	———
	Welcome reception, 17^{00}	Dinner 19^{00}		Reception at Institute for Theoretical Physics, 17^{30}	

Speakers and Titles:

Murad Alim: BPS spectra of N=2 Theories in 4d
Philip Boalch: Fission and wild character varieties
Tudor Dimofte: The Virtues of 3-Manifolds in Teichmller Theory
Vladimir Fock: Integrable systems, dimers and affine Lie groups
Sergey Fomin: Cluster structures in rings of SL_3 invariants
Olivier Guichard: Different characterisations of representations in Hitchin components
Antonin Guilloux: Representations of 3-manifolds groups in PGL(3,C)
Sergei Gukov: Knot Homologies from Quantization of Moduli Spaces
Francois Labourie: Asymptotics of Gauge Theory for SL(n,R)
Greg Moore: Spectral Networks and Snakes
Tony Pantev: Shifted symplectic structures and quantization
Jörg Teschner: Quantization of moduli spaces of flat SL(2)-connections
Richard Wentworth: Gluing formulas for determinants of Laplacians