



Science Meeting – Scientific Report

Scientific report (one single document in WORD or PDF file) should be submitted online within two months of the event. It should not exceed seven A4 pages.

***Proposal Title:* Workshop on Noncommutativity and Physics: Quantization, topological models, and generalized geometries**

***Application Reference N°:* 4635**

1) Summary (up to one page)

Quantization, topological models and generalized geometries (Bayrischzell, May 24-27, 2013).

Started by Julius Wess some 30 years ago, the Bayrischzell workshops have a long tradition. They are devoted to foundational aspects of theoretical particle physics with an emphasis on new mathematical structures. The purpose of the workshop series is to foster new ideas and bring together established and junior researchers. During the past 10 years, the workshops have mainly been focused on noncommutativity and physics. More recently, the emphasis has moved towards the interplay between gravity and the quantum structure of spacetime, reflecting current progress and research interests. The present meeting was focused on quantization, generalized geometry, and topological models, and covered mathematical as well as physical perspectives of the subject, including e.g. 3-brackets, noncommutative extra dimensions, different applications of quantum spaces, and mathematical foundations.

It has been known for some time that non-commutative geometries arise quite naturally in open string theory, as deformations of Poisson structures that result from a 2-form background flux (B-field) on D-branes. The understanding of the underlying mathematical structures has given a great boost to the construction of models of the quantum structure of spacetime in physics. More recently it has been discovered that higher structures related to higher fluxes, play an equally important role in string/M-theory: Nambu-Poisson structures in the description of open membranes, Courant and AKSZ sigma models in the context of non-geometric backgrounds to name just a few. The (deformation) quantization of these structures is not well understood, but it is

expected to lead to non-associative as well as non-commutative structures. In some cases (that of twisted Poisson structures) the geometry is locally still associative but there is an obstruction (a gerbe) to a global associative product. In other cases, e.g., when the B-field is assumed to be dynamical, the Poisson constraint has to be relaxed and the geometry is expected to be non-associative also locally. The workshop brought together experts from the different fields to discuss the perspectives of such questions.

2) Description of the scientific content of and discussions at the event (up to four pages)

The workshop started out with a talk by Peter Bouwknegt on generalized geometry, 2 dimensional sigma models and T-duality. In this context, T-duality is an isomorphism of Courant algebroids. In the discussion, it was clarified that the nongeometric fluxes which appear in this setting in the T-dual of principal torus bundles, could also be found for noncommutative torus bundles. A relation to string theory via certain non-associativity terms was established. Peter Bouwknegt was then followed by Johannes Huebschmann who spoke about multi-derivation-Maurer-Cartan algebras and sh-Lie-Rinehart algebras. The latter are a generalization of quasi-Lie-Rinehart algebras, where one considers more than just one derivative operator. The relation to the Koszul resolution and Kontsevich's graph complex was a matter of discussion. It was clarified that the framework does not immediately allow for a noncommutative generalization due to the properties of derivatives of noncommutative algebras.

A (perturbative) quantization scheme of strings and membranes in the context of the Batalin-Vilkoviski complex was then outlined by Jochen Zahn; the fact that this scheme is anomaly free was discussed in detail. Following this talk, a different direction in string theory was illustrated by Urs Schreiber who gave an introduction to higher geometric prequantum theory. Chenchang Zhu introduced the audience to the abstract notion of n-groupoids; it will be interesting to understand how this might provide a good framework to understand e.g. symmetries of noncommutative spaces.

Klaas Landsman addressed the problem of spontaneous symmetry breaking in finite quantum systems. His explanation for its occurrence is that exponential sensitivity (with growing system size) to (asymmetric) perturbations of the (symmetric) dynamics causes symmetry breaking already in finite but very large quantum systems.

Harold Steinacker explained his ideas on how extra dimensions can be compactified in matrix models. His approach is based on a deformation of the embedding functions.

The Sunday session opened with Katrin Wendland's talk who gave an introduction to the elliptic genus of a Calabi-Yau manifold X and explained it as a modular function which interpolates between some of the known topological invariants of X . She discussed a number of open conjectures related to the so-called "Mathieu Moonshine Phenomenon" for the elliptic genus of $K3$ (the latter is based on the fact that a symmetry group of a $K3$ surface M is isomorphic to a (small) subgroup of the (rather large!) Mathieu group). The interpretation of these conjectures in the context of (half twisted) sigma models was discussed. Gregor Masbaum then gave a talk on modular representations in finite characteristic of mapping class groups of surfaces coming from the theory of Integral $SO(3)$ Topological Quantum Field Theory. It will be interesting to fully understand e.g. the geometric meaning of the relation of some dimensions occurring in this context and Verlinde's formula.

Ivo Sachs made a connection between homotopy algebras and quantum string field theory, thereby explaining the existence, background independence and uniqueness of closed, open and open-closed bosonic- and topological string field theory. He explained for instance, how the decomposition theorem for loop homotopy Lie algebras leads to a proof of the uniqueness of the quantum theory of closed strings. Branislav Jurco then complemented this talk by a survey on the use of operads and homotopy algebras in string theory.

Taking a turn towards geometry again on Sunday afternoon, Jørgen Andersen established the existence of Hitchin's connection for certain symplectic manifolds. He showed that it induces a unique formal connection which by parallel transport produces equivalences between the corresponding Berezin-Toeplitz deformation quantizations. In particular cases, he can use this to find a symmetry-invariant deformation quantization. Thomas Strobl then explained the connection between generalized geometries, topological models and quantization and illustrated his ideas using the Dirac sigma models as an example. He was followed by Francesco Bonecchi who gave a talk on how a Poisson manifold can be quantized via the quantization of its symplectic groupoid. The framework combines tools of geometric quantization with the results of Renault's theory of groupoid C^* -algebras. It allows for very singular polarizations.

The first talk on Monday was given by Richard Szabo who spoke about quantization techniques to describe the nonassociative geometry probed by closed strings in flat non-geometric R-flux backgrounds. Starting from a suitable Courant sigma-model on an open membrane, one is led to nonassociative star products which via the Seiberg Witten map can be mapped to associative ones. The approach also leads to a consistent quantization of Nambu-Poisson 3-brackets.

Nicolas Boulanger gave an introduction to an off-shell formulation of higher-spin gravity. Based on a generalized Hamiltonian sigma model, a (non-standard) action principle for Vasiliev's fully nonlinear equations of motion for bosonic higher spin gauge fields in four spacetime dimensions, produces the amplitudes related to the 3D free $O(N)$ model. The quantization is then achieved within the BRST-BV framework in the Alexandrov-Kontsevich-Schwarz-Zaboronsky approach.

Domenico Fiorenza continued Urs Schreiber's talk on higher geometric prequantum theory, and in particular spoke about loop-algebras of local observables from higher prequantum bundles. Stefan Waldmann gave the last talk of the conference, speaking about convergent star products. As an example he discussed the quantization of Peierls' bracket and free quantum field theory.

In addition to the scientific talks, we had a short 'gong show' on Saturday, allowing younger participants (A. Munch, F. Arici, J. Ysoky, and M. Matassa) to introduce themselves to the audience by giving a very short presentation (a few minutes each) of their doctoral theses' subjects.

3) Assessment of the results and impact of the event on the future directions of the field (up to two pages)

The workshop provided a unique opportunity for scientists working in different fields to exchange their ideas. The ideas presented at the workshop will be pursued further in the future and we expect that many of the mathematical ideas and constructions will find further applications in physics, in particular in string and M theory and in higher order approaches to noncommutative geometry. Following its tradition, the workshop provided a forum for fruitful exchange not only during the detailed discussions after the talks, but also in numerous informal discussions that took place during the course of the workshop. It has laid the foundations for further scientific exchange across the fields.

As the field is still new, many attempts will be needed to establish a well-founded theory of quantum spacetime and it is unclear which of the algebraic and geometric constructions sketched above will provide a good foundation, and how these concepts might still have to be further generalized. Beyond the new research ideas stemming directly from the talks sketched in the previous section, one might mention three exemplary lines of research that will be further pursued: Courant sigma-models, Nambu brackets and their quantization, the application of operads and homotopy algebras in string theory, and more generally, of higher order structures in quantum geometry.

4) Annexes 4a) and 4b): Programme of the meeting and full list of speakers and participants

Annex 4a: Programme of the meeting

Saturday, May 25

- 9.00-9.45 Peter Bouwknegt
Generalized Geometry, 2D sigma models and T-duality
- 9.50-10.35 Johannes Hübschmann
Multi derivation Maurer-Cartan algebras and sh-Lie-Rinehart algebras
- 11.00-11.45 Jochen Zahn
Quantization of submanifold embeddings
- 11.50-12.35 Urs Schreiber
Higher geometric prequantum theory I – General theory
- 16.30-17.15 Klaas Landsman
Spontaneous Symmetry Breaking in Quantum Systems:
Emergence or Reduction?
- 17.20-18.05 Chenchang Zhu
Lie n-groupoids and their action
- 18.15-19.00 Harold Steinacker
Compactified extra dimensions in matrix models

Sunday, May 26

- 9.00-9.45 Katrin Wendland
The elliptic genus of K3
- 9.50-10.35 Gregor Masbaum
Integral TQFT and applications to the mapping class group
- 11.00-11.45 Ivo Sachs
Homotopy Algebras and String Field Theory
- 11.50-12.35 Branislav Jurco
Operads, homotopy algebras and strings
- 16.30-17.15 Jørgen Andersen
The Hitchin connection, Toeplitz operators and deformation quantization
- 17.20-18.05 Thomas Strobl
Generalized geometries, Topological models and Quantization:
The example of Dirac sigma models
- 18.15-19.00 Francesco Bonechi
Multiplicative integrability of the modular function

Monday, May 27

- 9.00-9.45 Richard Szabo
Quantization of non-geometric flux backgrounds
- 9.50-10.35 Nicolas Boulanger
An off-shell formulation of higher-spin gravity
- 11.00-11.45 Domenico Fiorenza
Higher geometric prequantum theory II –
Loop-Algebras of local observables from higher prequantum bundles
- 11.50-12.35 Stefan Waldmann
Convergence of star products and the nuclear Weyl algebra

Annex 4b: Full list of speakers and participants

Speakers:

Jørgen Andersen	(U Aarhus, DK)
Francesco Bonechi	(INFN Sezione di Firenze, Florence, I)
Nicolas Boulanger	(UMH, Mons, BE)
Peter Bouwknegt	(National Univ., Canberra, AUS)
Domenico Fiorenza	(Univ. Roma 1, Rome, I)
Johannes Hübschmann	(Univ. Lille 1, Lille, F)
Brano Jurco	(Charles University, Prague, CZ)
Klaas Landsman	(U Nijmegen, NL)
Gregor Masbaum	(Jussieu, Paris, F)
Ivo Sachs	(LMU München, Munich, D)
Urs Schreiber	(U Utrecht, NL)
Harold Steinacker	(TU Wien, A)
Thomas Strobl	(U Lyon, F)
Richard Szabo	(Heriot-Watt U, Edinburgh, UK)
Stefan Waldmann	(U Würzburg, D)
Katrin Wendland	(U Freiburg, D)
Jochen Zahn	(TU Wien, A)
Chenchang Zhu	(U Göttingen, D)

Participants:

Francesca Arici	(SISSA, Trieste, I)
Joakim Arnlind	(U Linköping, SE)
Paolo Aschieri	(U del Piemonte Orientale, Alessandria, I)
Dorothea Bahns	(U Göttingen, D)
Daniel Blaschke	(Los Alamos, USA)
Maja Burič	(U Belgrade, RS)
Leonardo Castellani	(U del Piemonte Orientale, Alessandria, I)
Bianca Cerchiai	(U Milano, I)
Lucio Cirio	(U Münster, D)
Marija Dimitrijevic	(U Belgrade, RS)
Andreas Döring	(U Oxford, UK)
Gaetano Fiore	(U Naples, I)
Domenico Fiorenza	(U Roma 1, Rome, I)
Olivier Gabriel	(U Göttingen, D)
T R Govindarajan	(CMI, Chennai, IN)
Jose Gracia-Bondia	(U Zaragoza, ES)
Harald Grosse	(Vienna, A)
Florian Hanisch	(U Potsdam, D)
Omid Hurson	(Vienna, A)
John Madore	(U Paris Sud 11, Paris, F)
Pierre Martinetti	(U Naples, I)
Marco Matassa	(SISSA, Trieste, I)
Albert Much	(MIS, Leipzig, D)
Voja Radovanovic	(U Belgrade, RS)
Christian Sämann	(Heriot-Watt University, Edinburgh, UK)
Hisham Sati	(U Pittsburgh, USA)
Alexander Schenkel	(U Wuppertal, D)
Martin Schlichenmaier	(U Luxembourg, LU)

Peter Schupp	(Jacobs Univ. Bremen, D)
Zoran Skoda	(U Zagreb, HR)
Christoph Stephan	(U Potsdam, D)
Josip Trampetic	(U Zagreb, HR)
Patrizia Vitale	(U Naples, I)
Jan Vysoky	(TU Prague, CZ)
Jean-Christoph Wallet	(U Paris Sud 11, Paris, F)
Michael Wohlgemannt	(Vienna, A)
Hyun Seok Yang	(U Sogang, KR)
Jiangyang You	(U Zagreb, HR)
George Zoupanos	(U Athens, GR)