Final Report ESF Short Visit Grant 4465

Purpose of the visit

As described in the grant proposal, the purpose of this short visit to the Centre of Quantum Geometry of Moduli Spaces in Aarhus was to discuss and develop a joint project with Prof. Jørgen Ellegaard Andersen, Johan Martens, and Shehryar Sikander from Aarhus and Prof. Peter Gothen from Oporto, concerning geometric quantization of the Hitchin fibration on the moduli space of semistable Higgs bundles of rank two and trivial determinant.

Description of the work carried out during the visit

The work carried out during my stay in Aarhus concerned on quantization of the Hitchin system, and the relation of this problem to the abelianization programme for non-abelian theta functions, concentrating essentially on the case of bundles of rank 2 with trivial determinant.

The non-singular part of the Hitchin fibration on the moduli space of semistable Higgs bundles consists, in this case, of the Prym varieties over a certain family of ramified double covers of the base curve. During our work, team members updated each other on the state of the art of various topics that enter the problem, including

- the topology of the regular part of the base of the Hitchin fibration and its relation to the topology of the fibration of Prym varieties it carries;
- aspects of the algebraic geometry of the natural projections from the Prym varieties to the moduli space of vector bundles;
- the natural projective connection on the relevant spaces of theta functions on the Prym varieties;
- some of its properties, mainly concerning its monodromy both for fixed base curve and moving base curve;
- comparison with known properties of the monodromy of Hitchin's connection on the spaces of non-abelian theta functions for moving base curve.

These discussions permitted in particular the definition of a further line of investigation.

Description of the main results obtained

As noted already by Beauville[1] in the 1980ies, the space of non-abelian theta functions of level 1 is "abelianized" by theta functions of level 2 on the Jacobian

[1]A.Beauville, "Fibrés de rang 2 sur une courbe, fibré déterminant et fonctions thêta". Bull. Soc. Math. France 116 (1988), no. 4, 431–448 (1989).

variety of the base curve by applying results Mumford[2] had obtained while studying Prym varieties of ramified double covers and certain linear systems on them (with other applications in mind).

These works do not mention or use the Hitchin fibration (not yet discovered at the time of Mumford's paper and also not directly related to the problem investigated there), which in this context corresponds essentially to a variation of the ramified double cover of the base curve over a certain parameter space. Therefore our first task was to understand this relation. Closer investigation revealed that Mumford's argument linking the linear systems on a double cover (fixed, in his case) and the base curve, corresponds to identifying the non-abelian theta functions with the space of abelian theta functions over the family of Prym varieties that is parallel with respect to the natural connection on this space[3].

The arguments used are, of course, specific to the case of level 1; in particular it became clear through our discussions that a better understanding of the quantization of Hitchin's fibration would follow from a generalization of Mumford's construction to higher level, which is therefore one of our aims in forthcoming work.

On the other hand it also became clear that a better understanding of the pullback of non-abelian to abelian theta functions and its variation along the family of Pryms is desireable: this is necessary since otherwise discrepancies between the monodromies of the Hitchin connection and the abelian connection arise. It appears that these can not be resolved by the essentially representation-theoretic constructions of the abelian case alone, and understanding it is the second main aim for our collaboration.

Future collaboration with host institution

The members of our team from Aarhus plan to visit the Center of Mathematics at University of Porto between November and Dezember of the current year to pursue the investigations; the precise dates have not yet been fixed.

Projected publications/articles resulting or to result from your grant

These have not yet been outlined.

Other comments

During my stay in Aarhus I participated in QGM's annual Nielsen Retreat from October 10^{th} to 13^{th}. I also gave a seminar talk based on not directly related

[2]D.Mumford, "Prym varieties. I". Contributions to analysis (a collection of papers dedicated to Lipman Bers), pp. 325–350. Academic Press, New York, 1974.

[3]This connection goes back to previous work by Mumford in D.Mumford, "On the equations defining abelian varieties. I.". Invent. Math. 1 1966 287–354.

work[4] at QGM on October 17$^{\text{th}}$, entitled "Toric Kähler metrics at infinity and compact tropical amoebas".

[4]T.Baier, C.Florentino, J.M.Mourão, J.P.Nunes, "Toric Kähler metrics seen from infinity, quantization and compact tropical amoebas", to appear in J. Differential Geom.