Teichmüller theory : quantization and relations with physics
April 15-19 2013

Erwin Schrödinger Institute, Vienna

Organized by: Louis Funar, Athanase Papadopoulos and Bob Penner

1 Summary

The conference took place at the Erwin Shrödinger International Institute for Mathematical Physics in Vienna, from April 5 to 19, 2013. It was part of a 3-month activity at the institute, whose subject is Teichmüller theory and its interactions with physics. The subject of the conference was "Teichmüller theory : quantization and relations with physics". There were 14 talks that covered the recent advances in the theory. The talks were given by the main specialists in the field. In addition to the talks there was a series of advanced lectures by Rinat Kashaev on quantization of Teichmüller space and relations with physics. The conference was attended by more than 45 persons, and among them about 20 people were funded in part from the ESF program ITGP. The participants came from several countries from Europe, Asia and America.

2 Description of the scientific content of and discussion at the event

The conference covered the major recent advances in Teichmüller theory and its interactions with physics, including quantization of Teichmüller theory, clusters, quantum symmetry, Feynman diagrams, TQFT, and Liouville theories. The follwoing is a list of the major subjects presented and discussed at the conference:

1. Ideal triangulations of the Milnor fibers of plane curve singularities and the variation of the corresponding complex structure.

2. The study of the shape of the subset of moduli space consisting of surfaces whose systoles fill by using combinatorial methods and an appropriate graph.

3. Classical and semiclassical TQFTs.

4. Infinite symmetric group, their relation with the universal Teichmüller theory, topological field theories, and Feynmann diagrams.

5. Representations of mapping class groups, from TQFT and from the Goldman-Turaev algebra point of views. The infinitesimal version of the Dehn-Nielsen theorem

6. Tropical geometry.

7. Quantum continued fractions.

8. Quantization of Teichmüller space via mirror symmetry.

9. Generalizations of higher Teichmüller theory (a.k.a. the study of framed flat SL(K) connections on punctured surfaces) to 3- and higher-dimensional manifolds with boundary.

10. Ptolemy-Thompson groups. The Funar-Sergiescu computation of the central extension of the Ptolemy-Thompson group. Relation with the Kashaev quantization of the universal Teichmüller space and with the relative abelianization of the braided Ptolemy-Thompon group of Funar and Kapoudjian.

11. Thurston's gluing equations in PGL(n,C) and applications to quantum topology, including the duality between the shape coordinates and the Ptolemy coordinates of Garoufalidis-Thurston-Zickert. The shape coordinates and Ptolemy coordinates as 3-dimensional analogues of the X- and A-coordinates on higher Teichmüller spaces due to Fock and Goncharov.

12. algorithms for countingt intersections of normal curves and a matrix presentation of mapping class groups.

13. Quantum symmetry and homological representations of braid groups

The series of lectures by Kashaev were on quantum Teichmüller theory. The specific content was the following: [1] Groupoid of ideal triangulations of punctured surfaces; [2] Penner's coordinates in the decorated Teichmüller

space; [3] Ratio coordinates; [4] Quantization and Faddeev's quantum dilogarithm; [6] Length spectrum of simple closed curves; [7] 3-dimensional symmetries and dihedral angles; [8] Partition functions on shaped triangulations; [9] Quasi-classical behavior and hyperbolic volume.

Each lecture was followed by a discussion, and each afternoon several discussions wee held at the ESI about the days' lectures and about the whole subject.

3 Assessment of the results and impact of the event on the future direction of the field

This encounter between mathematicians working in different aspects on Teichmüller theory (geometric, algebro-geometric and analytic) was very important in terms of exchanging ideas an establishing new contacts between these mathematicians. The speakers at the conference were first-rate mathematicians, and they presented in a comprehensive way important new ocntributions to the subject. Many young mathematicians were present, and they all learned a lot by participating to that conference.

The atmosphere during the conference was very good in terms of willingness to learn and to exchange new ideas. . Several new collaborations were born during this conference. The conference took place at the beginning of the three-month special period on Teichmüller theory at ESI Vienna and several people, at the end of the conference, decided to come back to ESI a few weeks later during the prigram, in order to contunue collaborations which started during the conference. In the attendance there were also several PhD students, from Europe, Asia and America. Although some of the advances lectures were difficult for them to follow, the students have certainly gained some experience And new ideas and they learned what are the most active problems in the area. It eas also the occasion for them to encounter several mathematicians from all over the world and to discuss with them about their work. the contacts they made will be very helpful for them when they will be looking for a job.

4   Final program

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All lectures take place in the ESI Boltzmann Lecture Hall

- **Monday, April 15, 2013**

  08:45 − 10 Registration

  10:00 − 11:00 Yurii Neretin
  Infinite symmetric group, checker topological field theories, and Feynmann diagrams

  11:00 − 11:20 Coffee break

  11:20 − 12:20

  Gregor Masbaum
  TQFT and modular representations of mapping class groups

  12:30 − 14:00 lunch break

- **Tuesday, April 16, 2013**

  09:00 − 10:00 Bob Penner
  Quantum continued fractions

  10:00 − 10:20 Coffee break

  10:20 − 11:20 Sergei Gukov
  Quantization of Teichmuller space via mirror symmetry

  11:30 − 12:30 Ivan Dynnikov
  An algorithm to count intersections of normal curves and a matrix presentation of mapping class groups

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12:30 − 14:00 lunch break

14:00 − 15:00 Rinat Kashaev
Lecture I

- **Wednesday, April 17, 2013**

  09:00 − 10:00

  Tudor Dimofte
  K-decompositions and framed flat connections on 3-manifolds

  10:00 − 10:20 Coffee break

  10:20 − 11:20 Nariya Kawazumi
  An infinitesimal version of the Dehn-Nielsen theorem

  11:30 − 12:30 Hyun Kyu Kim
  The dilogarithmic central extension of the Ptolemy-Thompson group via the Kashaev quantization of the universal Teichmuller space

  12:20 − 14:00 lunch break

  14:00 − 15:00 Rinat Kashaev
  Lecture II

- **Thursday, April 18, 2013**

  09:00 − 10:00 Nicolai Reshetikhin
  Semiclassical topological quantum field theories

  10:00 − 10:20 Coffee break

  10:20 − 11:20

  Joergen Andersen
  Mapping class group invariant unitarity of Hitchin connection

  11:30 − 12:30 Christian Zickert
  Thurston's gluing equations for PGL(n,C)

  12:30 − 14:00 lunch break

  14:00 − 15:00 Rinat Kashaev
  Lecture III

- **Friday, April 19, 2013**

    09:00 − 10:00

    Julien Roger
    Quantum Teichmueller theory at a root of unity

    10:00 − 10:20

    Coffee Break

    10:20 − 11:20 Francesco Costantino
    On an analytic family of representations of mapping class groups

    11:30 − 12:30 Toshitake Kohno
    Quantum symmetry in homological representations of braid groups

    12:30 − 14:00 lunch break

    14:00 − 15:00 Rinat Kashaev
    Lecture IV

5 List of registered participants

Toshiyuki Akita, Joergen Ellegaard Andersen, Stephane Baseilhac, Francesco Costantino, Tudor Dimofte, Ivan A. Dynnikov, Michihiko Fujii, Louis Funar, Sourav Ghosh, Sergei Gukov, Rinat Kashaev, Nariya Kawazumi, Hyun Kyu Kim, Teruaki Kitano, Alexander Klyachko, Hiroki Kodama, Toshitake Kohno, Julien Marche, Gregor Masbaum, Gabriele Mondello, Yurii Neretin, Athanase Papadopoulos, Robert Penner, Wolfgang Pitsch, Romain Ponchon, Nikolai Reshetikhin, Julien Roger, Adam Sikora, Jeremy Toulisse, Barbara Tumpach, Binbin Xu, Tian Yang, Christian Zickert.