Report for RGLIS Short Visit Grant 5429

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Details of the visit

1. Project title:

   A general pruning process & convergence of $\mathbb{R}$-trees with locally infinite measures

2. Host: Guillaume Voisin, Université Paris-Sud 11

3. Date: Feb. 11 – Feb. 26, 2013

Purpose of the visit

The purpose of the visit was to fix some technical issues in a joint project with Anita Winter, so that the results can be published shortly afterwards.

The project is to provide a unified description, generalisation, and tool for the analysis of different existing pruning processes of (random) discrete and continuous $\mathbb{R}$-trees [AP98, ADH12, AS02, AD12, ADV10], thereby in particular obtaining convergence in Skorohod path space of certain discrete pruning processes (e.g. of Galton-Watson trees) to corresponding pruning processes on continuous $\mathbb{R}$-trees (e.g. the Brownian CRT or Lévy-trees).

Such pruning processes are naturally measure $\mathbb{R}$-tree valued, and the Gromov-weak topology [GPW09] is a natural and fruitful topology on the space of measured $\mathbb{R}$-trees. The idea was to extend the state space to also include a pruning measure on the $\mathbb{R}$-tree, thereby obtaining one general process on a space of bi-measure $\mathbb{R}$-trees instead of several processes for different pruning measures. A technical issue here is that typical pruning measures, like the length measure on the Brownian CRT, are not locally-finite. Therefore, we had to develop a new topology for $\mathbb{R}$-trees with two measures, a finite sampling measure and the pruning measure which does not have to be locally finite, but finite on finite subtrees. The idea of the topology, which we call leaf-sampling weak vague (LWV) topology, is to sample finite subtrees and equip these with the restrictions of the pruning measure. For LWV-convergence, the resulting distribution of measure $\mathbb{R}$-trees is then required to converge Gromov-weakly in distribution.

Results

We were able to resolve the main technical issues after a slight change in the topology, namely we strengthened the above mentioned Gromov-weak convergence in the definition of the LWV-convergence to an n-pointed Gromov-weak convergence, which is a straight-forward extension to n-pointed spaces. In particular, we obtain that our process is Feller-continuous on the (non-locally compact) space of bi-measure $\mathbb{R}$-trees with LWV-topology.

Publications and Future Projects

The results of the project will be published in the near future, after a final face-lifting. The support of the ESF will be acknowledged.

We plan to continue the collaboration and analyse further examples within our new framework. We expect that the LWV-topology is not only useful for pruning processes but could also have other applications.

References

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[ADH12] Romain Abraham, Jean-François Delmas, and He Hui. Pruning Galton-Watson trees and tree-valued Markov process. *Ann. Inst. H. Poincaré Probab. Statist.*, 48(3), 2012.

[ADV10] Romain Abraham, Jean-François Delmas, and Guillaume Voisin. Pruning a Lévy continuum random tree. *Elect. Journal of Probab.*, 15(46), 2010.

[AP98] David Aldous and Jim Pitman. Tree-valued Markov chains derived from Galton-Watson processes. *Ann. Inst. H. Poincaré Probab. Statist.*, 34(5):637–686, 1998.

[AS02] Romain Abraham and Laurent Serlet. Poisson snake and fragmentation. *Elect. Journal of Probab.*, 7(17):1–15, 2002.

[GPW09] Andreas Greven, Peter Pfaffelhuber, and Anita Winter. Convergence in distribution of random metric measure spaces (Λ-coalescent measure trees). *Prob. Theo. Rel. Fields*, 145(1-2):285–322, 2009.