SCIENTIFIC REPORT ABOUT THE SHORT VISIT SUPPORTED BY THE ESF PROGRAM "RGLIS"

Grantee:	**Balázs Ráth**
Hosts:	**Prof. Thierry Bodineau and**
	Prof. Vladas Sidoravicius
Title of research project:	**One-dimensional DLA / Branching interlacements**
Host institution:	**ENS, Paris**
Dates:	**26 May - 2 June 2013**

1 Purpose of the visit

The primary purpose of my visit to Paris was to make progress in our ongoing research with Prof. Sidoravicius on the one-dimensional diffusion-limited aggregation (DLA) model defined in [1]. We investigate the growth of a "sticky" aggregate which interacts with a cloud of particles that perform one-dimensional continuous-time simple symmetric random walks. We denote by $R(t)$ the position of the tip of the aggregate at time t. The walkers move freely until one of them tries to jump on the site $R(t)$ from the right: then $R(t)$ is increased by one and all of the particles at position $R(t)+1$ are removed from the system. The initial distribution of particles are i.i.d. Poisson with parameter μ. We are interested in the asymptotics of the growth rate of the length of the sticky aggregate $R(t)$ when $t \to \infty$, and in particular how the speed of growth depends on the parameter μ.

In [1] it is proved that $R(t)$ grows at most linearly for all $\mu \in \mathbb{R}_+$, and that if $\mu < 1$ then $R(t) \asymp \sqrt{t}$. It is conjectured that the model exhibits phase transition: if $\mu > 1$ then $R(t) \asymp t$. One of our current goals is to prove a linear lower bound for $R(t)$, at least for big values of μ.

The secondary goal of my visit was to discuss recent progress and future plans about a modification of the model of random interlacements [2] which involves branching random walks.

2 Description of the work carried out during the visit

With Prof. Sidoravicius, we discussed a new approach to the 1-dimensional DLA model. Instead of keeping track of the individual particles, one may look at them in an "annealed" way. This approach eliminates most of the randomness from the model, but it keeps all information about the evolution of the aggregate. In the annealed model we can perform a useful coupling of the process with two different initial conditions in a way which allows us to bring monotonicity into play. An associated PDE (the heat equation with a certain free boundary condition) also helps us in coming up with heuristic predictions and ideas for observables.

Our current focus is on the supercritical phase of the one-dimensional DLA model, i.e., the case when the initial density of particles μ is greater than 1. The aim is to combine the above sketched "annealed" approach with ideas from [1] involving the positive recurrence of certain observables in order to deduce the linear growth of the size of the aggregate. During my visit to Paris, we elaborated on these ideas as I will further discuss in Section 3 of this report.

The original purpose of my visit also included discussions about certain modifications of the model of random interlacements, but this goal was suppressed by the 1-dimensional DLA project, which consumed most of my time spent in Paris.

3 Description of the main results (1-dim DLA model)

The annealed approach which only keeps track of the evolution of the intensity of the particles rather than the particles themselves helps us in the construction of a coupling which allows us to make the following statement rigorous: if we start with more particles, then the aggregate grows faster. This result also implies the existence of a limiting stationary profile of particles (viewed from the tip of the aggregate). The goal is to prove that if the initial density μ is greater than one (or at least when μ is very big), then this limiting stationary profile has non-zero entries (unlike the $\mu < 1$ case, when the limiting profile is identically zero).

The next step we need to take in this direction is to find an observable (i.e., a Ljapunov function) of the system which is on one hand positively recurrent, on the other hand it has the property that it measures the mass of missing particles. Such an observable is easy to find for the associated PDE, but now it seems that in order to find a positively recurrent observable, one also needs to control the shape of the particle profile.

4 Projected publications (1-dim DLA model)

Based on the partial results already obtained during this visit we expect that our research will lead to a joint publication by the end of the year. The more delicate critical case $\mu = 1$ may be covered in another paper. We already have a heuristic argument which suggests that in this case $R(t) \asymp t^{2/3}$ holds, but the rigorous proof of the linear growth of $R(t)$ will be treated first.

5 Other comments

Besides the research and collaboration described above I also had the opportunity to discuss with multiple mathematicians (Laure Dumaz, Alexander Fribergh) in Paris during my visit. I also attended some lectures of the mini-course about "Threshold phenomena and random graphs" given at the spring school at Institut Henri Poincaré by Sourav Chatterjee.

References

[1] H. Kesten, V. Sidoravicius. A problem in one-dimensional diffusion-limited aggregation (DLA) and positive recurrence of Markov chains. *Ann. Probab.*, **36(5)**: 1838-1879, 2008.

[2] A.-S. Sznitman. Vacant set of random interlacements and percolation. *Ann. Math.* **171**: 2039-2087, 2010.