Scientific report on a RGLIS short visit grant - Alexandre Stauffer

Purpose of visit

The goal was to spend one week in Paris at Université Paris Dauphine and École Normale Supérieure (ENS) in order to discuss with Hubert Lacoin (Universitè Paris Dauphine) and Vladas Sidoravicius (visiting professor at ENS Paris at the time of the visit) about a model for the growth of an aggregate or crystal called *multi-particle DLA*.

Work carried out during the visit

During the visit I worked not only with Hubert Lacoin and Vladas Sidoravicius, but also with Manuel Cabezas and Gady Kozma, who were visiting École Normale Supérieure at that time. Also, we worked both on the project of multi-particle DLA mentioned above and on a dependent percolation model that I had previously discussed with Gady Kozma when I was visiting the Weizmann Institute of Science, in Israel in 2011.

Main results obtained

Regarding multi-particle DLA, our discussions led to the introduction of a simpler version of the model in $1+1$ dimensions. In this model, the aggregate grows in $\mathbb{Z}^2$ and initially consists of the x axis, while particles are only allowed to move to the left or to the right and are added to the aggregate when they are adjacent to it. We considered two regimes: low density and high density of particles.

We first describe our results for the low density. We performed simulations of this model, which shows similar characteristics with the model in two dimensions. Based on the simulations, we observed, for example, that as the aggregate grows upwards, some pieces of the aggregate stretch out like fingers, and each such finger seems to stop growing eventually. In our discussions, we characterized the main features of this process that cause the aggregate to grow in this way. We believe this can be used to show that the fingers grew by the aggregate do not form an infinite component when the density of particles is small enough.

For the high density regime, we believe that we can now prove that even though the aggregate grows by stretching out fingers, some of these fingers eventually merge and cause the aggregate to have an infinite component almost surely. This, together with our observations for the low density regime, would establish an interesting phase transition phenomenon for this model.

Regarding the model of dependent percolation, we discussed a question initially proposed by Itai Benjamini about the percolation of a perturbed square circle packing. We showed that replacing circles by "curved lozenges" yields no percolation. We also discussed other shapes and variations which seem to capture the non-percolative conjectured by Itai Benjamini in the model with circles.

Future collaboration with host institution

We are scheduling a follow-up visit to Paris in January 2014 to continue our collaboration on these topics.

Projected Publications

We expect that the work carried out during this visit will lead to two papers regarding the multi-particle DLA model.