LINT
Logic for Interaction

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Fundamental question of mathematical logic:

Can mathematics, and thereby many other sciences which are based on mathematics, be treated effectively, that is, based on simple logical rules that even a computer can follow?

What has been the main source of ideas for mathematics and thereby for logic in the past?

What will probably be the main source of ideas for mathematics and thereby for logic in the future?

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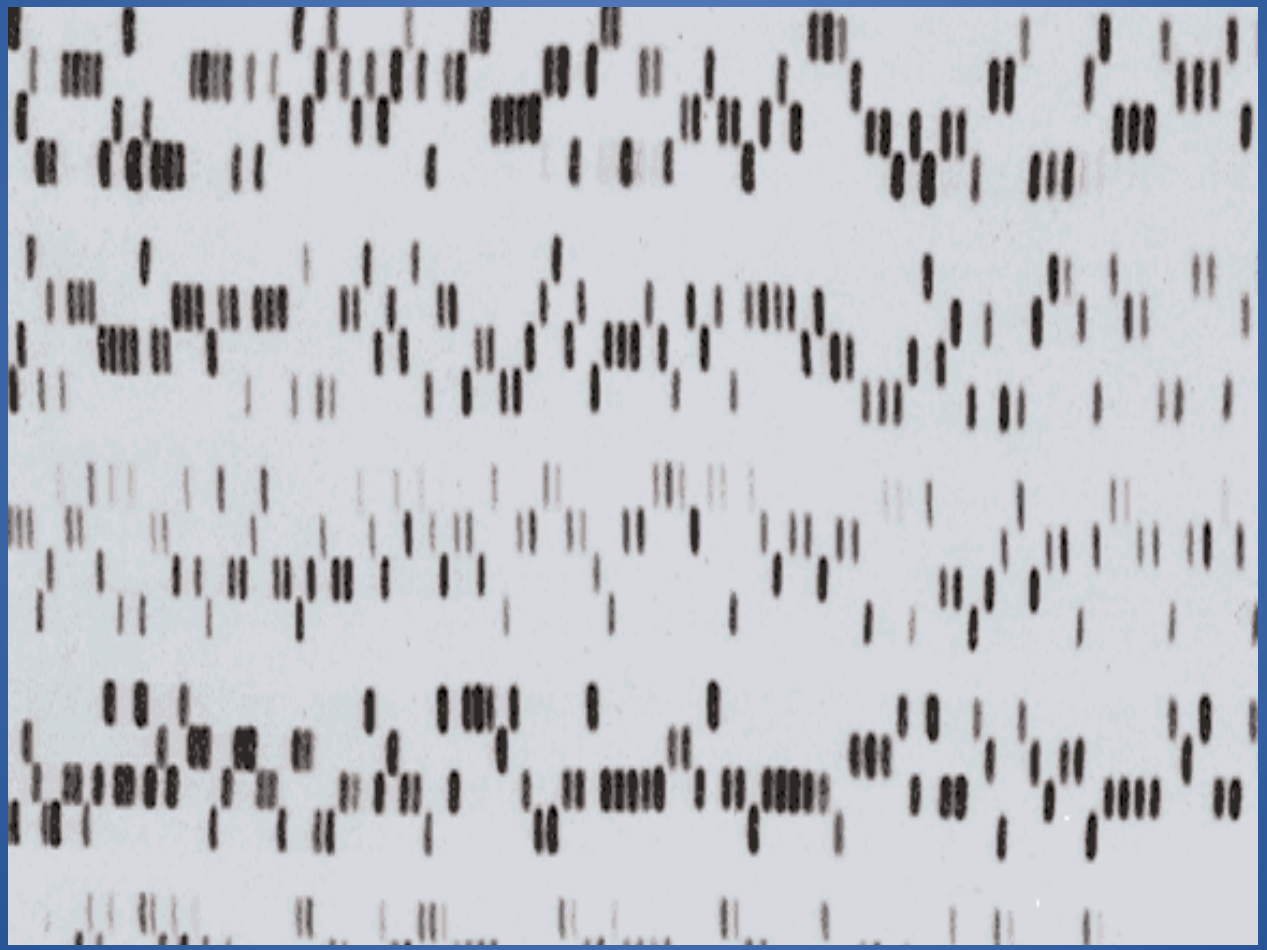


What will probably be the main source of ideas for mathematics and thereby for logic in the future?

Customers who bought this item also bought

Theory of Moves by Steven J. Brams

Biblical Games: Game Theory and the Hebrew Bible by Steven J. Brams

Negotiation Games by Steven J. Brams

The Strategy of Conflict by Thomas C. Schelling

Fair Division: From Cake-Cutting to Dispute Resolution by Steven J. Brams

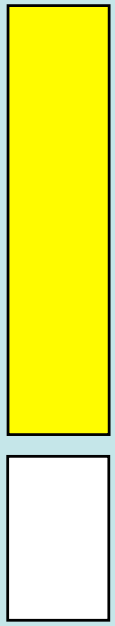
Logic and games - background

- Dialogical logic (Lorenzen)
- Game theoretic semantics (Henkin)
- Semantic tableaux (Beth)
- EF game (Ehrenfeucht, Fraïssé)

Change we can

From	To
individual	**group**
perfect information	**inperfect information**
classical logic	**classical+constructive+linear**
agent makes choices	choices make the agent
features are fixed predicates	features are fixed variables

Is the shirt color dependent on the pants color?



meaningless question

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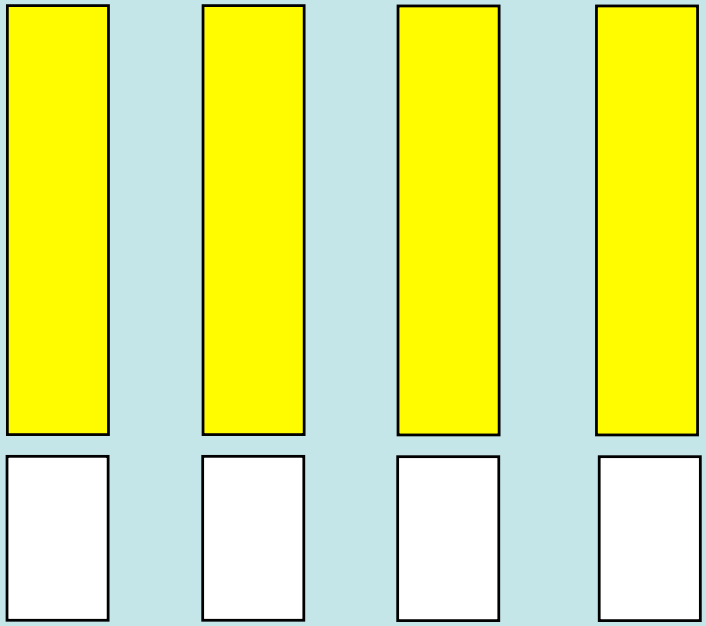
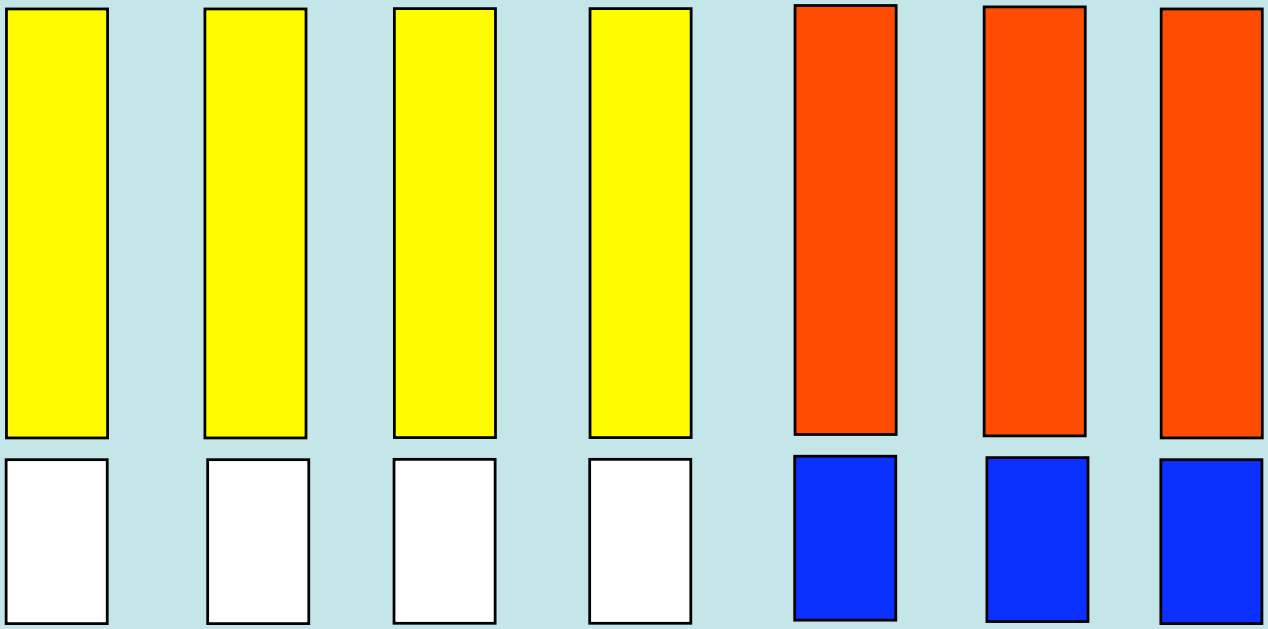


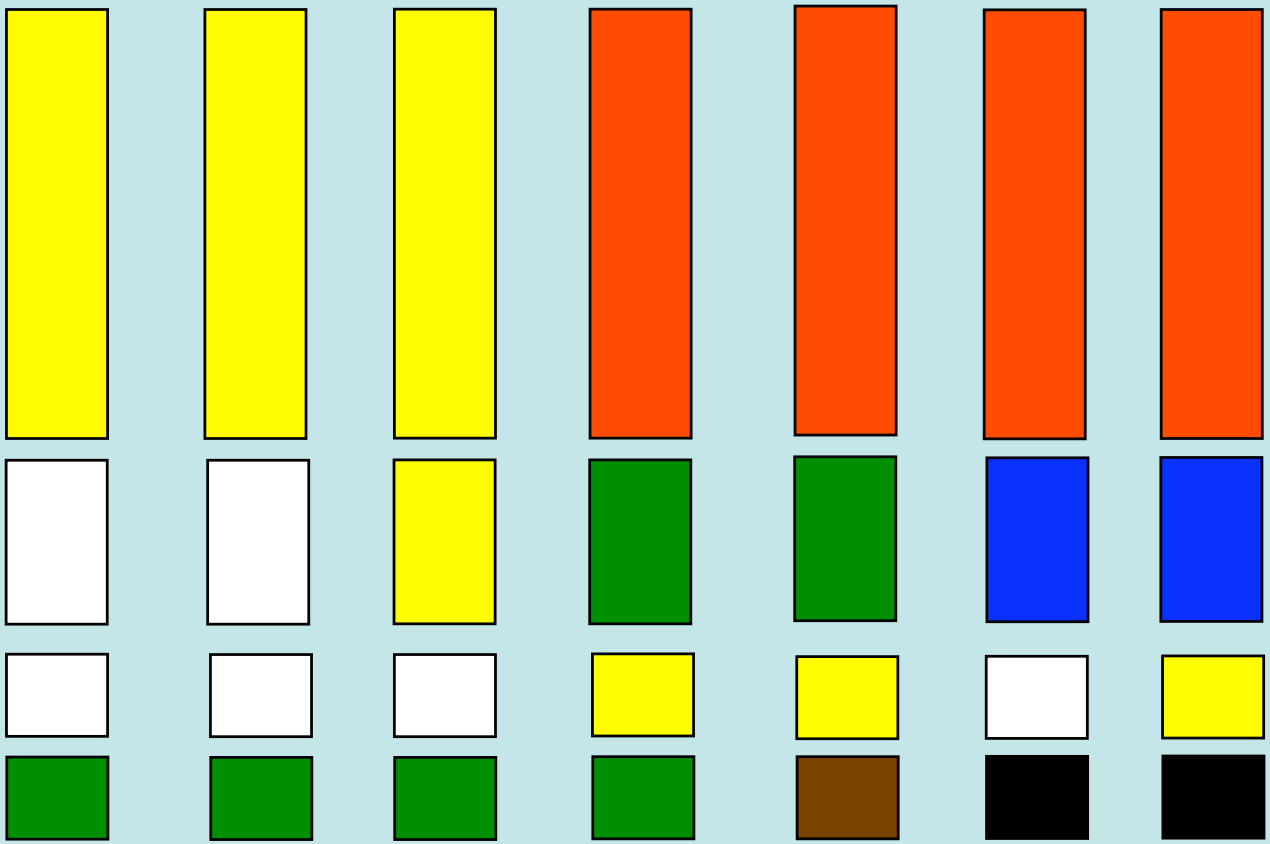
meaningless question

Is the shirt color dependent on the pants color?



Yes! White pants → yellow shirt, blue pants → red shirt

Is the shirt color dependent on the socks and shoes color?



Yes!

record	A1	A2	A3	A4	A5	A6
100000	8	6	7	3	0	6
100002	7	5	6	3	0	6
100003	4	8	7	3	0	6
100004	6	5	4	3	0	6
100005	6	12	65	3	0	6
100006	5	56	9	3	0	6
100007	6	23	0	4	0	8
…	…	…	…	…	…	…
408261	77	2	11	1	0	2

Can you write this in first order or modal logic:

Whatever decisions the governments make in the next 10 years, it seems likely that by the year 2050 the sea levels will rise, but whether the rise is over 25 cm depends on whether the industrialized nations start reducing their greenhouse gas emissions now. (Modal dependence logic)

Can you write this in a decidable fragment of first order:

A passport official at an airport only wants to know whether you have a valid visa or not. If you do, she lets you in; if not, she sends you back on the next flight. *(Social software, Rohit Parikh)*

Can you write this in a decidable fragment of first order logic:

If the social welfare function respects unanimity and independence of irrelevant alternatives, it is a dictatorship. (Social choice theory, Arrow's Theorem)

In a game a play is built up from the choices of the players.

By looking at many plays we can learn about the players.

The game BIG

- First I picks a natural number x_0.
- Then II picks a natural number x_1.
- Then II picks a natural number x_2, without now using knowledge of what x_0 is.
- II wins if $x_2 > x_0$.

Is II obeying the rules?

I: x_0	II: x_1	II: x_2
1	1	2
3	3	4
3	3	4
100	100	101
53111468	53111468	53111469

I: x_0	II: x_1	II: x_2
1	0	2
3	0	4
3	0	4
100	0	101
53111468	0	53111469

Summing up: dependence arises in

- Causality
- Data
  - Biological
  - Financial
  - Sociological
  - Cosmological
- Social software
- Social choice
- Natural language
- Games

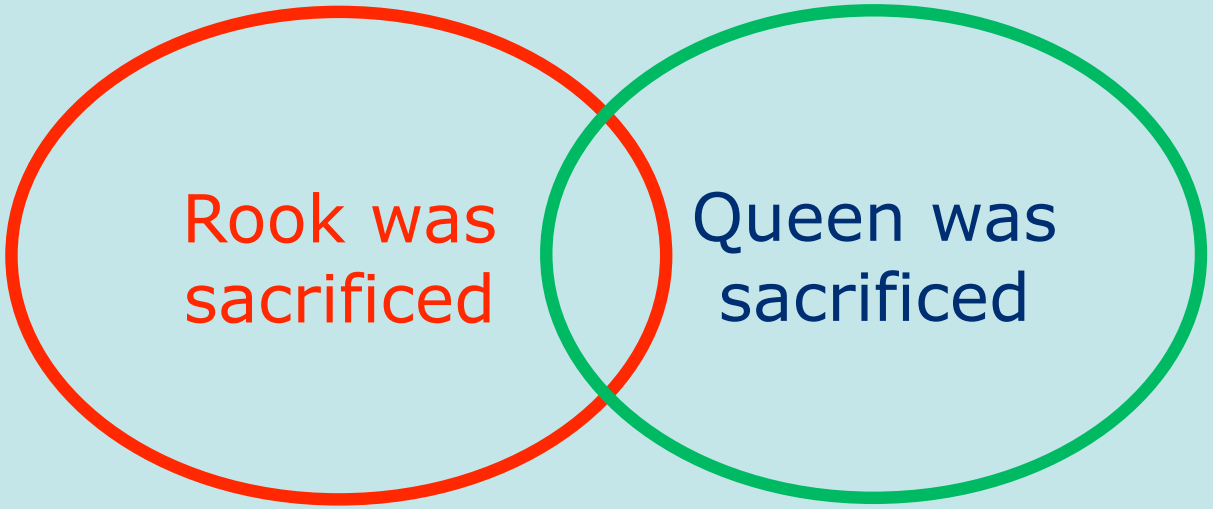

Let us investigate
Dependence Logic

- Dependence does not manifest itself in a **single** event or observation.
- We need semantics where the basic concept is a **set** of observations
- Such sets are called **teams**.

The Intuition of Teams

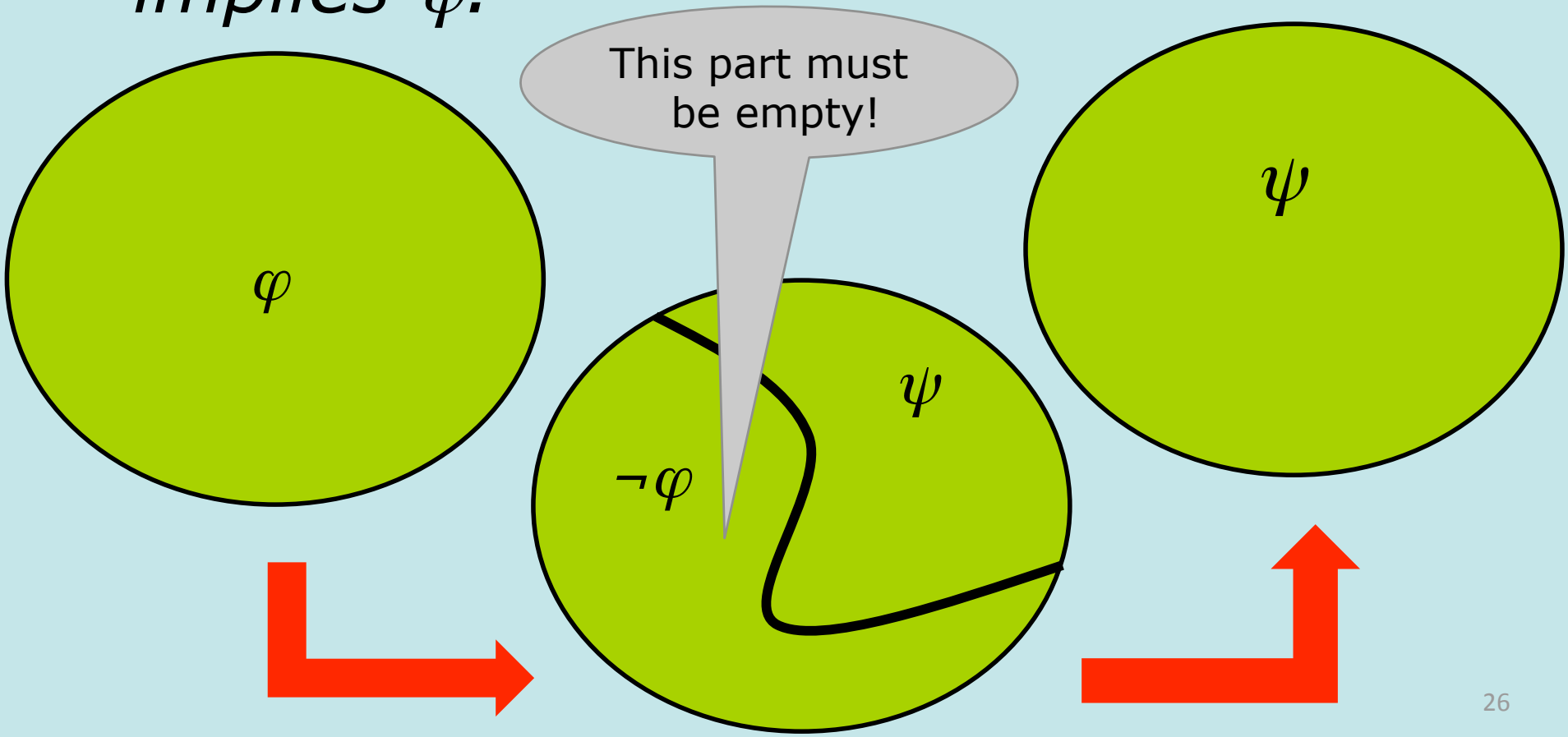
- Teams accomplish tasks by
  - Every member doing the same
  - Dividing into subteams (skills)
  - Supplementing a new feature, (a skill)
  - Duplicating along a feature, (gender)
- Teams manifest dependence by e.g.
  - Letting rank, not gender, determine salary

Plays where rook **or** queen was sacrificed:



Example

- If $\neg\varphi \vee \psi$ is valid then φ *logically implies* ψ.



Team X

Finnish driver
Swedish author
Norwegian skier

Duplicted team

Finnish male driver
Finnish female driver
Swedish male author
Swedish female author
Norwegian male skier
Norwegian female skier

- Dependence logic has features related to
  - Game theory: games of imperfect information
  - Intuitionism: dependence on evidence
  - Linear logic: dependence on resources

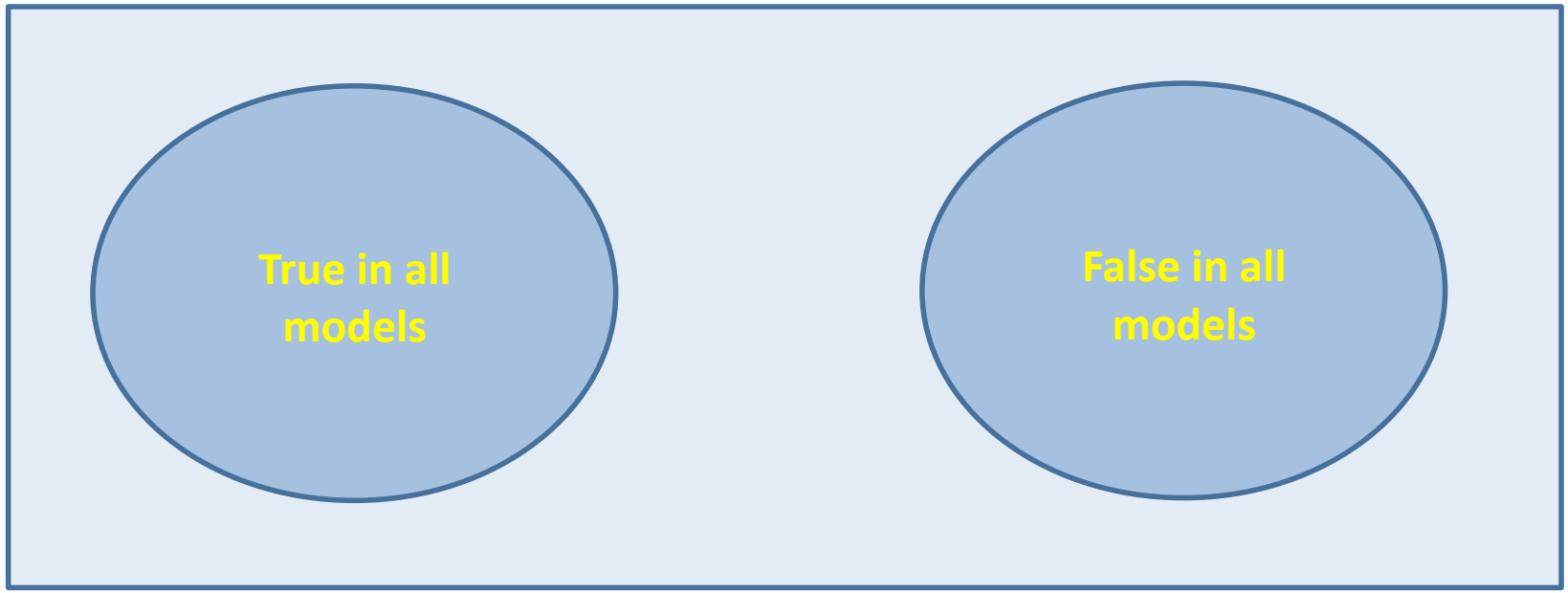
Unpublished joint work with Abramsky

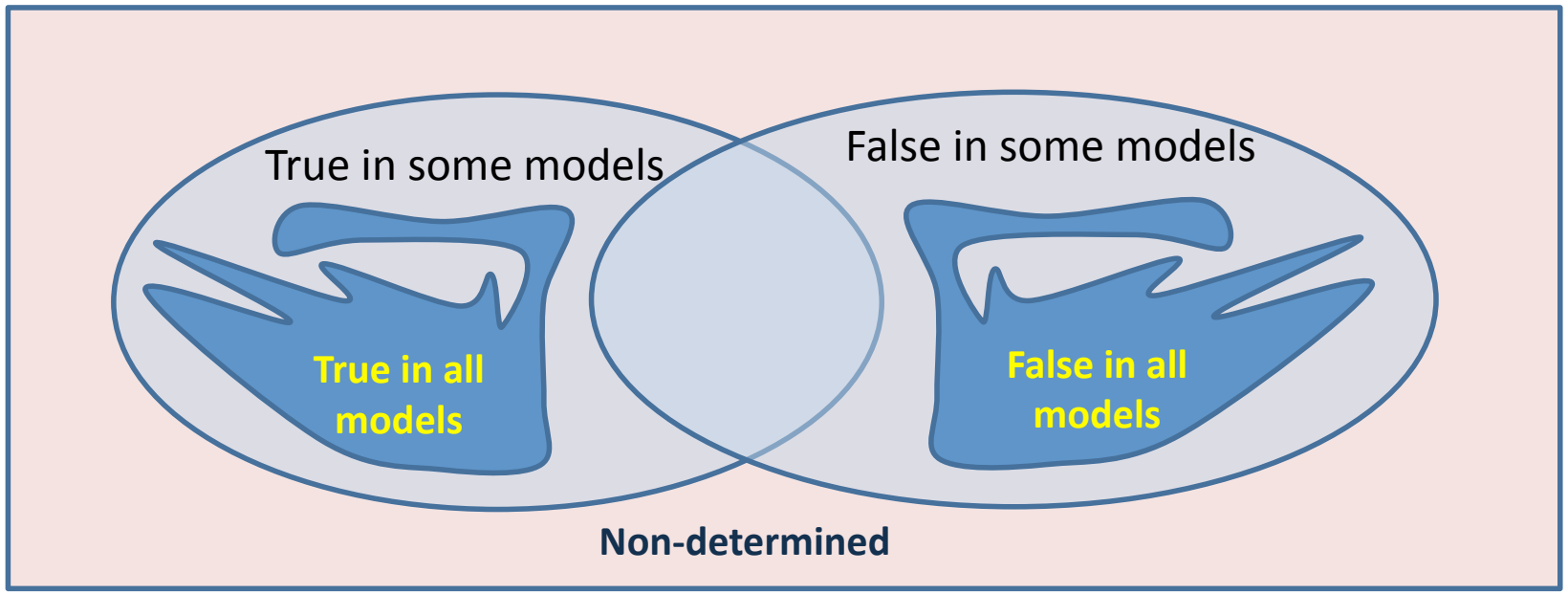
Conclusion

There is a mathematical theory of dependence with applications to games, logic, computer science, linguistics, economics, etc.

The price

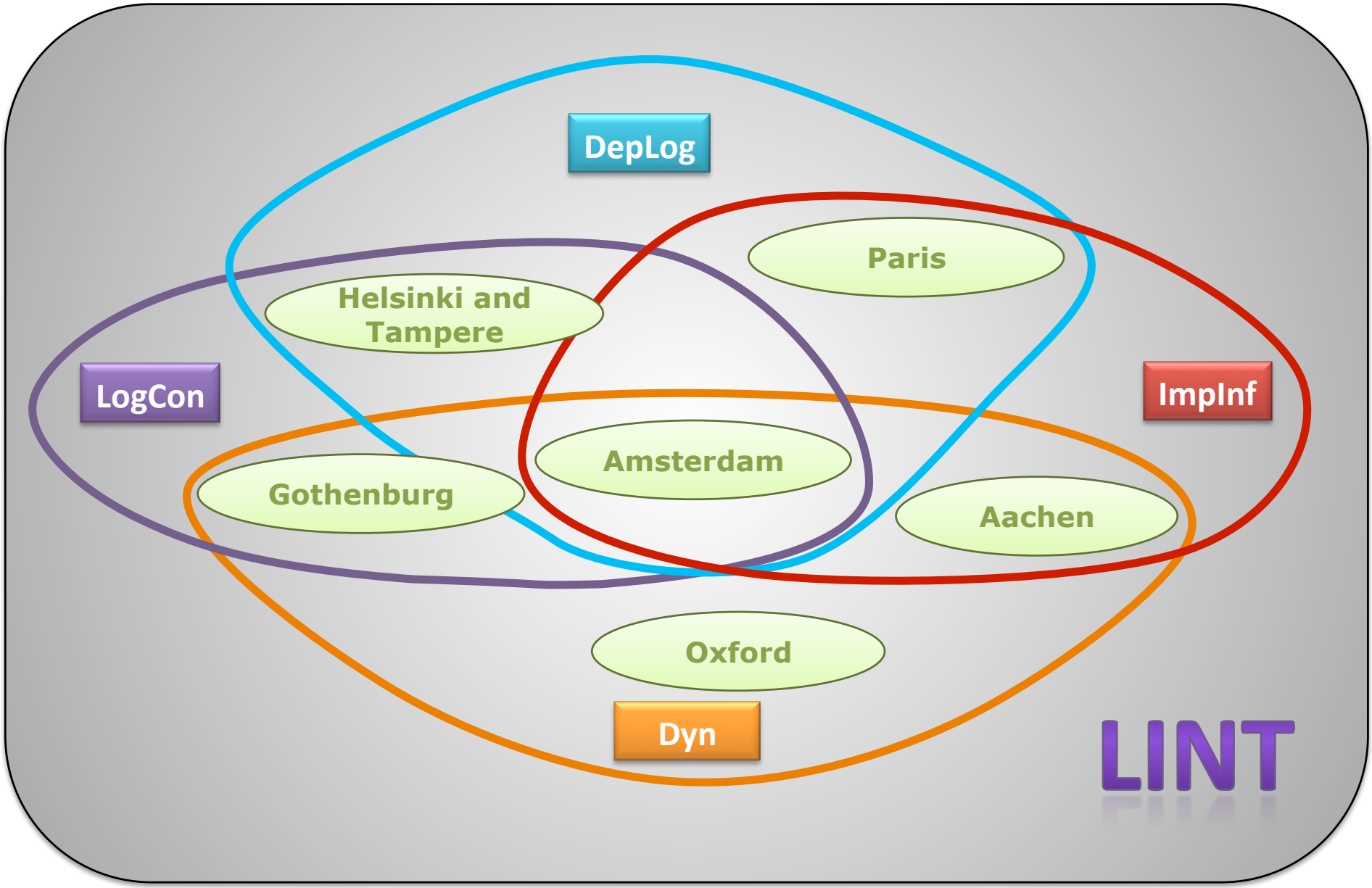
- An exponential complexity jump.
- Logarithmic compression.


True in all models
False in all models


True in some models
False in some models
True in all models
False in all models
Non-determined

Summary

- There is a natural logic of dependence based on the lift from individuals to teams.
- Concepts can be clarified: compositionality, imperfect information, non-determinacy.
- A basic **Logic for Interaction**.


DepLog
Paris
Helsinki and Tampere
LogCon
ImpInf
Amsterdam
Gothenburg
Aachen
Oxford
Dyn
LINT

LINT subprojects

- **DepLog** (*Amsterdam*) To engage in a thorough investigation of the new dependence logic, to find its axiomatization, its fine-structure, and its relation to modal dependence theories.
- **ImpInf** (*Aachen*) To develop a uniform logical and operational framework for handling imperfect information in logical games and other interactive systems.
- **Dyn** (*Oxford*) To relate and merge the two major existing approaches to the logic of interaction: 'local' and 'global' dynamics.
- **LogCon** (*Gothenburg*) To apply existing methods from logic and mathematics for characterizing the standard logical constants for proof and truth to logical frameworks specifically designed to deal with interaction.

- Pricipal Investigators
  - **Grädel**, Aachen
    - Algorithmic model theory, fixed-point logics, algorithmic theory of games
  - **Väänänen**, Amsterdam
    - Game-theoretic mehods in logic, dependence logic
  - **Westerståhl**, Gothenburg
    - Model-theoretic semantics, generalized quantifiers in natural language
  - **Hella**, Tampere
    - Finite model theory, generalized quantifiers
- Associated Partners
  - **Abramsky**, Oxford
    - Semantics of computation, game semantics
  - **Sandu**, Paris
    - Evaluation games, game theoretic semantics

Modelling intelligent interaction - Logic in the Humanities, Social and Computational sciences (LogICCC)

What does LINT have to do with the other projects?

- Dialogical Foundations of Semantics
  - Incorporating interaction into logical semantics
- The Logic of Causal and Probabilistic Reasoning in Uncertain Environments
  - Probability logic, causality, conditional independence
- Logical Models of Reasoning with Vague Information
  - Imperfect information, data extraction
- Games for Analysis and Synthesis of Interactive Computational Systems
  - Game-theoretic formalizations of interactive computational systems
- Vagueness, Approximation, and Granularity
  - "Is he tall?" "Depends on what you mean by tall!"
- Computational Foundations of Social Choice
  - Logic-based languages for modeling and reasoning about choice problems and preference structures
- SOCIAL SOFTWARE for elections, the allocation of tenders and coalition/ alliance formation
  - Social software