ESF project LogiCCC FP014

S: Social
S: Software for
E: Elections,
A: Aggregation of tenders, and
C: Coalition formation

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Independence of Irrelevant Alternatives (IIA)

Elections:

Suppose 9 voters have the following preferences over 3 alternatives a, b, c:

$$\begin{aligned} &4\text{: } a\ b\ c \\ &3\text{: } b\ c\ a \\ &2\text{: } c\ b\ a \end{aligned}$$

Under the plurality rule, a will be elected. The resulting social preference is: $a\ b\ c$.

Now suppose c is not available; then b would be elected. So, the choice between a and b depends on the irrelevant (i.e. not available) alternative c.
In other words: Plurality rule is not IIA.

Most election mechanisms are not IIA.

Tenders: Suppose a tender with price and quality as criteria. The one with the lowest price gets the maximum number of 50 points. Ten percent higher price means ten percent (5) less points. Price and quality count equally.

Firm	Price	Price-score	Quality-score	Sum
A	100	50	10	60
B	150	25	37	62
C	160	20	41	61
D	170	15	46	61

B is the winner. However, suppose that A turns out not to be able to do the project. Then the situation would be as follows:

Firm	Price	Price-score	quality	Total
B	150	50	37	87,0
C	160	46,7	41	87,7
D	170	43,3	46	89,3

D may go to court and argue that he should be the winner. So, the choice between B and D depends on the irrelevant alternative A. I.e., the allocation of tenders is not IIA.

Arrow (1950): Every social preference rule satisfying IIA and some other straightforward conditions (Pareto and Transitivity) yields a dictator.

Since then practically all social choice scientists work in the framework of rank orderings of the alternatives by the voters or judges.

Balinski and Laraki's (2006): Instead of asking voters their rank orderings of the alternatives, one should ask them their **evaluations** of each alternative, using a common grading system. The final grade of each alternative should be the median value of the grades given by the voters.
They call this the **Majority Judgement**. It is IIA: the choice between 2 alternatives does not depend on a third (irrelevant) alternative.

Example (Jerome Renault) with 7 voters, 2 candidates A and B, and as common grading language {Very Good, Good, Mildly Good, Passable, Insufficient, Reject}.

A's scores: Reject, Very Good, Good, Passable, Mildly Good, Good, Reject.
B's scores: Very Good, Mildly Good, Insufficient, Insufficient, Mildly Good, Very Good, Reject.

One starts by sorting the scores of each candidate.
A: Very Good, Good, Good, Mildly Good, Passable, Reject, Reject.
B: Very Good, Very Good, Mildly Good, Mildly Good, Insufficient, Insufficient, Reject.

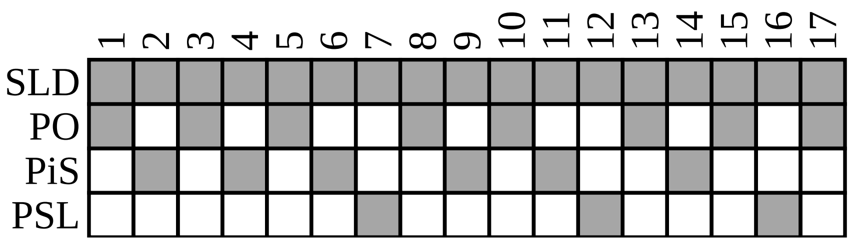
The "majority score" (median) for both candidates is "Mildly Good", a tie. To decide between them one removes one "Mildly Good" from each to get:
A: Very Good, Good, Good, Passable, Reject, Reject.
B: Very Good, Very Good, Mildly Good, Insufficient, Insufficient, Reject.
Thus the "majority score of rank one" of A is Passable, while that of B is Insufficient, therefore A is elected.

The RelVieW software

RelVieW is software developed by Rudolf Berghammer (Kiel) and others to deal with relations. Both input and output can be visualized on the screen. The programs consist of relational equations. It was succesfully applied to coalition formation as designed by Agnieszka Rusinowska.

Coalition formation: Let $N = \{1, \ldots, n\}$ be a set of parties. A *coalition* S is a subset of N. Let P be a set of policies. A *government* g is a pair (S, p) consisting of a coalition S and a policy p. G denotes the set of all governments.

Input for RelVieW: *government-membership relation* M as Boolean 4 x 17 matrix: For instance, with 4 parties and 17 governments:



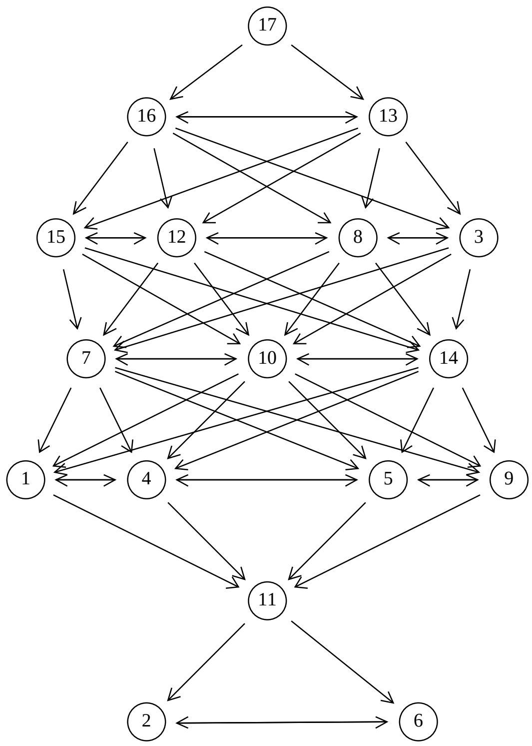
$M : N \leftrightarrow G$ where $M_{i,g} \leftrightarrow i \in S$ if $g = (S, p)$.

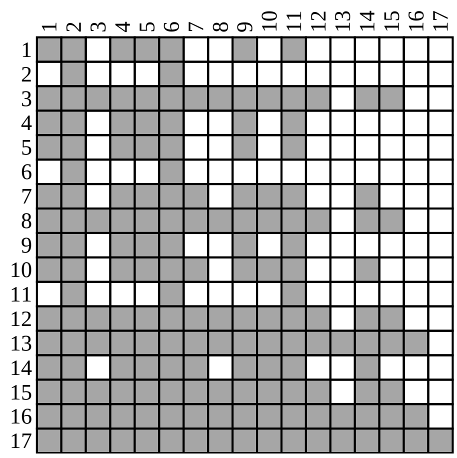
We assume that each party i has a utility $U_i(g)$ for each government g. These utilities can be determined by the McBeth software.

Input for RelVieW: for each party i the *utility relation* $R^i : G \leftrightarrow G$ or rather the *Comparison relation* $C : N \leftrightarrow G \times G$ defined by

$$R^i_{h,g} \leftrightarrow U_i(h) \geq U_i(g) \text{ and } C_{i,<h,g>} := R^i_{h,g}$$

For instance, R^i may be represented in RelVieW graphically as follows, expressing that $U_i(g_{17}) \geq U_i(g_{16})$, etc.:





$h = (S, p)$ **dominates** g, denoted by $h \succ g$, iff

$\forall i \in S[U_i(h) \geq U_i(g)] \wedge \exists i \in S[U_i(h) > U_i(g)]$.

In relation-algebraic terms: $dominance(M, C)$

$$= \overline{(\pi; M^{\mathsf{T}} \cap \overline{C}^{\mathsf{T}}); \mathsf{L}} \cap (\pi; M^{\mathsf{T}} \cap E; \overline{C}^{\mathsf{T}}); \mathsf{L} \qquad (1)$$

g is **stable** := there is no government dominating g, i.e., $\neg\exists h \in G[h \succ g]$.

In relation-algebraic terms:

$$stable(M, C) = \overline{\rho^{\mathsf{T}}; dominance(M, C)}. \qquad (2)$$

Given inputs M and C, RelVieW computes the $dominance(M, C)$ and the $stable(M, C)$ relation and can give a graphical representation of them.

Deliverables and Milestones:

NL a) The description of a fair election system for choosing a president, committee members and possibly elections for parliament (year 1 and 2);
b) next software incorporating these procedures, making use of RelVieW (year 3);
c) Ph.D.thesis (year 4).

Es a) The description of a fair procedure for the allocation of tenders in the new framework (year 1 and 2);
b) next software incorporating this procedure for the allocation of tenders, making use of RelVieW (year 3);
c) Ph.D. thesis (year 4).

Fr a) The description of fair procedures for coalition and alliance formation (year 1 and 2);
b) next software incorporating these procedures (year 3 and 4).

De a) Software for elections and for the allocation of tenders (year 2 and 3);
b) Software for coalition and/or alliance formation, incorporating the existing MacBeth and RelVieW software (year 2 and 3);
c) Ph.D. thesis (year 4).

Fi a) Insight in the significance of different metrics in devising voter support systems (year 1);
b) Insight in the significance of aggregation paradoxes to results on spatial voting (year 2);
c) Insight in which topologies are compatible with the finite and infinite languages at hand (year 3).

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