LogICCC Final Conference, Berlin 2011

Vague counterfactuals

Libor Běhounek

Academy of Sciences of the Czech Republic, Prague,
& Vienna University of Technology

Joint work with Ondrej Majer

LoMoReVI

Caveat 1

The aims of logical and linguistic semantic models differ:

Linguistics	Logic
Primarily descriptive	Primarily normative
Description of the use of language	Laws valid in (intuitively plausible) mathematical models
Interested in people's behavior	Regardless of people's behavior

This talk regards logical rather than linguistic analysis

Caveat 2

A degree-theoretical approach to vagueness adopted

Fuzzy plurivaluationism:

Vagueness = graduality + indeterminacy

- Indeterminacy irrelevant here

  (since all possible gradual precisifications can anyway be considered in the intensional semantics of counterfactuals)

- Formal fuzzy logic employed for modeling the graduality

$\Rightarrow$ Vague counterfactuals reduce to fuzzy counterfactuals

Counterfactual conditionals

Counterfactuals are conditionals with false antecedents:

If it were the case that A, it would be the case that C

Their logical analysis is notoriously problematic:

- If interpreted as material implications, they come out always true due to the false antecedent
- However, some counterfactuals are obviously false

$\Rightarrow$ a simple logical analysis does not work

Properties of counterfactuals

Counterfactual conditionals do not obey some standard inference rules of material conditional:

Weakening: $$\frac{A \mathrel{\Box\!\!\rightarrow} C}{A \wedge B \mathrel{\Box\!\!\rightarrow} C}$$

Contraposition: $$\frac{A \mathrel{\Box\!\!\rightarrow} C}{\neg C \mathrel{\Box\!\!\rightarrow} \neg A}$$

Transitivity: $$\frac{A \mathrel{\Box\!\!\rightarrow} B, B \mathrel{\Box\!\!\rightarrow} C}{A \mathrel{\Box\!\!\rightarrow} C}$$

There have been several attempts to propose an adequate semantics for counterfactuals (notably by Nelson Goodman), but the most widely accepted semantics was proposed independently by David Lewis and Robert Stalnaker

Lewis' semantics

Lewis' approach is based on a *similarity relation* which orders possible worlds with respect to their similarity to the actual world:

The counterfactual conditional $A \mathrel{\Box\!\!\rightarrow} C$ is true at a world w wrt a similarity ordering iff either

- There are no A-worlds or
- There is an AC-world which is more similar to w then any $A\neg C$-world

Why a fuzzy semantics for counterfactuals?

Lewis' semantics is based on the notion of similarity
of possible worlds

Similarity relations are prominently studied in fuzzy mathematics
(formalized as axiomatic theories over fuzzy logic)

$\Rightarrow$ Let's see if fuzzy logic can provide a viable semantics for
counterfactuals

Formal fuzzy logic

= logical systems for *gradual* predicates

(eg, *tall*—can be measured in cm's)

The underlying quantities are conventionally normalized to $[0, 1]$ (or another suitable algebra) = degrees of tallness

Certain operations with degrees are defined = 'connectives' and a degree-preserving 'consequence relation' studied

(different operations $\Rightarrow$ different fuzzy logics)

Paradigmatic example: (infinite-valued) Łukasiewicz logic

Formal fuzzy semantics

- Interpret defining formulae in fuzzy (rather than classical) logic
- Use the rules of fuzzy (rather than classical) logic for reasoning about the models

Similarity relations

= fuzzy equivalence relations

Axioms: Sxx, $Sxy \to Syx$, $Sxy \,\&\, Syz \to Sxz$
(NB: interpreted in fuzzy logic!)

Notice: Similarities are *transitive* (in the sense of fuzzy logic), but avoid Poincaré's paradox:

$$x_1 \approx x_2 \approx x_3 \approx \cdots \approx x_n, \text{ though } x_1 \not\approx x_n,$$

since the degree of $x_1 \approx x_n$ can decrease with n,
due to the non-idempotent & of fuzzy logic

Similarity on possible worlds

Σxy . . . the world x is similar to the world y

Axioms of similarity for Σ:

$$\Sigma xx, \quad \Sigma xy \to \Sigma yx, \quad \Sigma xy \,\&\, \Sigma yz \to \Sigma xz$$

NB: Fuzzy logics admit more general scales for similarity degrees than just $[0, 1]$

$\Rightarrow$ The similarity of worlds need not be measured by reals: abstract degrees of similarity are admissible, too

Ordering of worlds by similarity

$x \preccurlyeq_w y$ . . . x is *more or roughly as* similar to w as y

Formally: $x \preccurlyeq_w y \ \equiv_{\mathsf{df}}\ \Sigma wy \lesssim \Sigma wx$
(the similarity degree Σwy is less than or roughly equal to the similarity degree Σwx)

$\lesssim$ rather than $\leq$, by the fuzzy paradigm: when reasonable, use indistinguishability (or similarity) rather than strict equality

Worlds indistinguishable from x (to a large degree) should play a role (to a large degree)

$\Rightarrow$ We also need a similarity relation on degrees

Similarity on degrees (of similarity of worlds)

Axioms for $\sim$:	Def: $\alpha \lesssim \beta \;\equiv\; (\alpha < \beta) \vee (\alpha \sim \beta)$
Similarity:	Fuzzy ordering:
$(\alpha \sim \alpha)$	$(\alpha \lesssim \alpha)$
$(\alpha \sim \beta) \to (\beta \sim \alpha)$	$(\alpha \lesssim \beta)\ \&\ (\beta \lesssim \alpha) \to (\alpha \sim \beta)$
$(\alpha \sim \beta)\ \&\ (\beta \sim \gamma) \to (\alpha \sim \gamma)$	$(\alpha \lesssim \beta)\ \&\ (\beta \lesssim \gamma) \to (\alpha \lesssim \gamma)$
Congruence:	Congruence:
$(\alpha \sim \beta) \to (\alpha \leftrightarrow \beta)$	$(\alpha \lesssim \beta) \to (\alpha \to \beta)$
$(\alpha \leq \beta \leq \gamma)\ \&\ (\alpha \sim \gamma) \to (\alpha \sim \beta)$	$(\alpha \leq \beta \leq \gamma)\ \&\ (\alpha \lesssim \gamma) \to (\alpha \lesssim \beta)$
$(\gamma \leq \beta \leq \alpha)\ \&\ (\alpha \lesssim \gamma) \to (\alpha \lesssim \beta)$	$(\gamma \leq \beta \leq \alpha)\ \&\ (\alpha \lesssim \gamma) \to (\alpha \lesssim \beta)$
Non-triviality:	Non-triviality:
$(\exists \beta \neq \alpha)(\beta \sim \alpha)$	$(\exists \beta \not\leq \alpha)(\beta \lesssim \alpha)$

Fuzzy semantics for counterfactuals

Define: $x \preccurlyeq_w y \ \equiv\ \Sigma wy \lesssim \Sigma wx$

The closest A-worlds: $\text{Min}_{\preccurlyeq_w} A = \{x \mid x \in A \wedge (\forall a \in A)(x \preccurlyeq_w a)\}$

(the properties of minima in fuzzy orderings are well known)

Define: $\|A \mathrel{\Box\!\!\rightarrow} B\|_w \ \equiv\ (\text{Min}_{\preccurlyeq_w} A) \subseteq B$

... the closest A-worlds are B-worlds (fuzzily!)

Properties of fuzzy counterfactuals

Non-triviality: $(A \mathrel{\Box\!\!\rightarrow} B) = 1$ for all B only if $A = \emptyset$

Non-desirable properties are invalid:

$\not\models (A \mathrel{\Box\!\!\rightarrow} B) \mathbin{\&} (B \mathrel{\Box\!\!\rightarrow} C) \rightarrow (A \mathrel{\Box\!\!\rightarrow} C)$

$\not\models (A \mathrel{\Box\!\!\rightarrow} C) \rightarrow (A \mathbin{\&} B \mathrel{\Box\!\!\rightarrow} C)$

$\not\models (A \mathrel{\Box\!\!\rightarrow} C) \rightarrow (\neg C \mathrel{\Box\!\!\rightarrow} \neg A)$

Desirable properties are valid, eg:

$\models \Box(A \rightarrow B) \longrightarrow (A \mathrel{\Box\!\!\rightarrow} B) \longrightarrow (A \rightarrow B)$

+ many more theorems on $\Box\!\!\rightarrow$ easily derivable in higher-order fuzzy logic

However, some of Lewis' tautologies only hold for full degrees

Advantages

- Automatic accommodation of gradual counterfactuals

  "If ants were *large*, they would be *heavy*."

- Accommodation of graduality of counterfactuals

  (some counterfactual conditionals seem to hold to larger degrees than others)

- Standard fuzzy handling of the similarity of worlds

Disadvantages

- Needs non-classical logic for semantic reasoning

  (but a well-developed one $\Rightarrow$ a low cost for experts)

More details

Běhounek L, Majer O: A semantics for counterfactuals based on fuzzy logic. In M Peliš, V Punčochář (eds.): *The Logica Yearbook 2010*, pp. 25–41, College Publications, 2011.

Available online (google "fuzzy counterfactuals"), or ask me by email (behounek@cs.cas.cz)