The allocation of tenders using a distance-based extension of Majority Judgment

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LogICCC Final Conference

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Introduction

Allocation of tenders with a Social Choice background

T.H. Chen: "An economic approach to public procurement"
Journal of Public Procurement 8 (2009), pp. 407-430

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- Problem: Ranking paradox

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- Advantages:
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  - Assessment through linguistic terms

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- Disadvantages
  - Evaluation in small committees

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- Possible solution: Balinski and Laraki's Majority Judgment
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  - Assessment through linguistic terms
- Disadvantages
  - Evaluation in small committees
- **Our proposal: Extension of Majority Judgment based on distances**

Ranking paradox: example

- Tenders: A, B y C
- Criteria: Quality and Price
- The grading of the Price criterion is within a mathematical formula

$$\textit{SCORE} = 50 \cdot L/P$$

L: *the lowest price*
P: *the price of the tender being evaluated*

Ranking paradox: example

	Quality score	Price	Price score	Total
A	25	40€	50	75
B	32	50€	40	72
C	50	80€	25	75

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C-tender claims to be the winner because A-tender does not fulfill the requirements

Ranking paradox: example

	Quality score	Price	Price score	Total
A	25	40€	50	75
B	32	50€	40	72
C	50	80€	25	75

If we use the same scores, C-tender will be indeed the winner

	Quality score	Price	Price score	Total
B	32	50€	40	72
C	50	80€	25	75

Ranking paradox: example

	Quality score	Price	Price score	Total
A	25	40€	50	75
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But if we assess the scores again

Ranking paradox: example

	Quality score	Price	Price score	Total
A	25	40€	50	75
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But if we assess the scores again

	Quality score	Price	Price score	Total
B	32	50€	50	82
C	50	80€	31,25	81,25

Ranking paradox: example

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The new winner would be B-tender who was the loser before

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The new winner would be B-tender who was the loser before

Relative scoring → Failure of the independence of irrelevant alternatives

Michel BALINSKI, Rida LARAKI: **MAJORITY JUDGEMENT**

- The Majority Judgement (2007)
  `http://ceco.polytechnique.fr/`
- A theory of measuring, electing and ranking
  Proceedings of the National Academy of Sciences of the United States of America 104, pp. 8720-8725 (2007)
- Election by Majority Judgement: Experimental evidence
  Ecole Polytechnique - Centre National de la Recherche Scientifique, Cahier 2007 - 28 (2007)
- *Majority Judgment. Measuring, Ranking, and Electing*
  The MIT Press, Cambridge MA, 2011

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Political election proposal based on linguistic terms

Median + tie-break

Majority Judgment criticisms

- **W. D. Smith** (2007): On Balinski & Laraki's "Majority Judgement" median-based range-like voting scheme http://rangevoting.org/MedianVrange.html
- **D. S. Felsenthal, M. Machover** (2008): The Majority Judgment voting procedure: A critical evaluation *Homo Oeconomicus* 25 (3), pp. 319-334
- **J. L García-Lapresta, M. Martínez-Panero** (2008): Sorting alternatives into linguistic classes and their aggregation *Computational Intelligence in Decision and Control*, World Scientific, Singapore, pp. 531-536
- **J. L. García-Lapresta, M. Martínez-Panero** (2009): Linguistic-based voting through centered OWA operators *Fuzzy Optimization and Decision Making* 8, pp. 381-393
- **H. Nurmi** (2009): Voting Theory

Notation

- $V = \{1, \ldots, m\}$ set of criteria $(m \geq 2)$
- $X = \{x_1, \ldots, x_n\}$ set of tenders $(n \geq 2)$
- $L = \{l_1, \ldots, l_g\}$ ordered set of linguistic terms $(g \geq 2)$

  $$l_1 < \cdots < l_g$$

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$$l_1 < \cdots < l_g$$

Example

l_1	l_2	l_3	l_4	l_5	l_6
to reject	poor	acceptable	good	very good	excellent

Assignation of the global assessment $l(x_j)$

- A **profile** P is a matrix $m \times n$ with coefficients in L

$$\begin{pmatrix} v_1^1 & \cdots & v_j^1 & \cdots & v_n^1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ v_1^i & \cdots & v_j^i & \cdots & v_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ v_1^m & \cdots & v_j^m & \cdots & v_n^m \end{pmatrix}$$

where $v_j^i \in L$ is the assessment obtened by tender x_j in criterion i

Assignation of the global assessment $l(x_j)$

$$\begin{pmatrix} v_1^1 & \cdots & v_j^1 & \cdots & v_n^1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ v_1^i & \cdots & v_j^i & \cdots & v_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ v_1^m & \cdots & v_j^m & \cdots & v_n^m \end{pmatrix} \longmapsto (l(x_1), \ldots, l(x_j), \ldots, l(x_n))$$

$$l(x_j) = f(v_j^1, \ldots, v_j^m) \qquad j = 1, \ldots, n$$

Assignation of the global assessment $l(x_j)$

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$$l(x_j) = f(v_j^1, \ldots, v_j^m) \qquad j = 1, \ldots, n$$

Our proposal

For each tender x_j, choose $l(x_j)$ such that the vector $(l(x_j), \ldots, l(x_j))$ minimizes the distance between it and the assessments vector $(v_j^1, \ldots, v_j^m)$

Assignation of the global assessment $l(x_j)$

Minkoswki distances $(p \geq 1)$

$d_p : \mathbb{R}^m \times \mathbb{R}^m \rightarrow \mathbb{R}$ defined as

$$d_p\left(\boldsymbol{a}, \boldsymbol{b}\right) = \left(\sum_{i=1}^{m} |a_i - b_i|^p \right)^{\frac{1}{p}}$$

for all $\boldsymbol{a} = (a_1, \ldots, a_m)$, $\boldsymbol{b} = (b_1, \ldots, b_m)$

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Induced Minkoswki distances

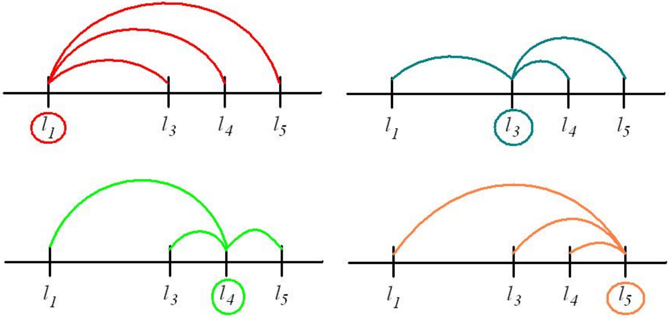
$\bar{d}_p : L^m \times L^m \to \mathbb{R}$ defined as

$$\bar{d}_p\left(\left(l_{a_1}, \ldots, l_{a_m}\right), \left(l_{b_1}, \ldots, l_{b_m}\right)\right) = d_p\left(\boldsymbol{a}, \boldsymbol{b}\right) = \left(\sum_{i=1}^{m} |a_i - b_i|^p \right)^{\frac{1}{p}}$$

Assignation of the global assessment $l(x_j)$

- Select $l_k \in L$ that fulfill

$$\bar{d}_p \left(\left(v_j^1, \ldots, v_j^m \right), \left(l_k, \ldots, l_k \right) \right) \leq \bar{d}_p \left(\left(v_j^1, \ldots, v_j^m \right), \left(l_h, \ldots, l_h \right) \right) \quad \forall l_h \in L$$



Tie-breaking method

To rank the tenders we need to break the ties among the tenders with the same global assessment

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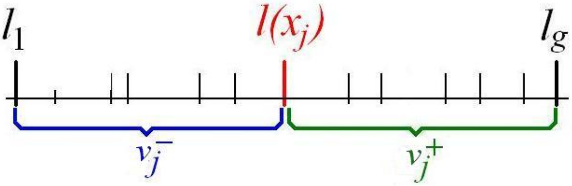

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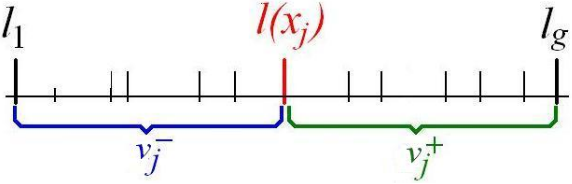

$$
\begin{aligned}
D^+(x_j) &= \bar{d}_p(\boldsymbol{v}_j^+, (l(x_j), \ldots, l(x_j))) \\
D^-(x_j) &= \bar{d}_p(\boldsymbol{v}_j^-, (l(x_j), \ldots, l(x_j)))
\end{aligned}
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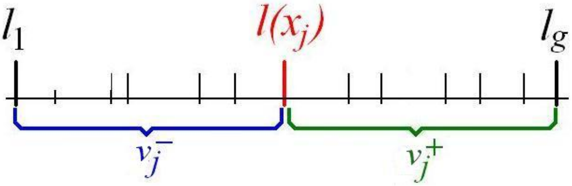

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\end{aligned}
$$

$$D(x_j) = D^+(x_j) - D^-(x_j)$$

Ranking tenders

Consider the triplet $T(\cdot) = (l(\cdot), D(\cdot), E(\cdot))$ for each tender and then proceed lexicographically:

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2. $l(x_j) = l(x_k)$ and $D(x_j) > D(x_k)$
3. $l(x_j) = l(x_k)$, $D(x_j) = D(x_k)$ and $E(x_j) \leq E(x_k)$

where

$$E(x_j) = \bar{d}_p((v_j^1, \ldots, v_j^m), (l(x_j), \ldots, l(x_j))$$
$$E(x_k) = \bar{d}_p((v_k^1, \ldots, v_k^m), (l(x_k), \ldots, l(x_j))$$

Allocation of tenders with the same weight for each criteria

Tenders' grades

	Price	Quality	Warranty	Security
x_1	l_1	l_2	l_5	l_5
x_2	l_3	l_3	l_2	l_1
x_3	l_5	l_4	l_1	l_1

Results for each tender $(p = 1.5)$

	$l(x_j)$	$D^+(x_j)$	$D^-(x_j)$	$D(x_j)$
x_1	l_3	3.17	2.45	0.73
x_2	l_2	1.59	1	0.59
x_3	l_3	2.45	3.17	–0.73

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Final result: $x_1 \succ x_3 \succ x_2$

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Criteria have different weights in the global assessment

How can we show the weights in a qualitative scale?

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We should find the connection within them and replicate each assessment in accordance

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Weight	4	3	2	1

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Arranged and replicated profile

x_1	l_1	l_1	l_1	l_1	l_2	l_2	l_2	l_5	l_5	l_5
x_2	l_1	l_2	l_2	l_3	l_3	l_3	l_3	l_3	l_3	l_3
x_3	l_1	l_1	l_1	l_4	l_4	l_4	l_5	l_5	l_5	l_5

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x_2	l_1	l_2	l_2	l_3	l_3	l_3	l_3	l_3	l_3	l_3
x_3	l_1	l_1	l_1	l_4	l_4	l_4	l_5	l_5	l_5	l_5

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x_2	l_1	l_2	l_2	l_3	l_3	l_3	l_3	l_3	l_3	l_3
x_3	l_1	l_1	l_1	l_4	l_4	l_4	l_5	l_5	l_5	l_5

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x_3	l_1	l_1	l_1	l_4	l_4	l_4	l_5	l_5	l_5	l_5

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x_3	l_1	l_1	l_1	l_4	l_4	l_4	l_5	l_5	l_5	l_5

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x_2	l_1	l_2	l_2	l_3	l_3	l_3	l_3	l_3	l_3	l_3
x_3	l_1	l_1	l_1	l_4	l_4	l_4	l_5	l_5	l_5	l_5

Results for the arranged and replicated profile

	$l(x_j)$
x_1	l_2
x_2	l_3
x_3	l_4

Final result: $x_3 \succ x_2 \succ x_1$

Concluding remarks

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    - $p = 1 \rightsquigarrow$ the median

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    - $p = 2 \rightsquigarrow$ the "mean"

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- Work in progress

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- Work in progress
  - The possibility to assess more than one label when the experts hesitate
  - Different scales depending on the criteria

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