Complex comparatives
— in theory and in practice

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A simple argument form

x is *more* P and Q than y

$\therefore$ x is *more* P than y and x is *more* Q than y

Example

premise:

x is *more* expensive and time consuming than *y*

conclusion:

x is *more* expensive than *y* and x is *more* time consuming than *y*

Set up

- *Introduction*
- *Empirical data*
- *What some theories predict*
- *Another theory*
- *More data*
- *Complications*

Survey: instructions

- Please read carefully the following paragraph and then answer the short questions following it.

- Siam and Vole are two methods to test the condition of car wheels. Official authorities highly recommend that car owners test their cars with one of these methods at least once every five years.

- Both take 30 minutes.
- Vole costs 15 Euros, and Siam costs 40 Euros.

Survey: type of questions

. Price AND Time

I agree that...

1. Siam is more TIME-CONSUMING AND EXPENSIVE than Vole is

- ○ Yes
- ○ No

2. Siam is less TIME-CONSUMING AND EXPENSIVE than Vole is

- ○ Yes
- ○ No

3. Siam and Vole are equally TIME-CONSUMING AND EXPENSIVE

- ○ Yes
- ○ No

97% answered like this (n=74)

. Price AND Time

I agree that...

1. Siam is more TIME-CONSUMING AND EXPENSIVE than Vole is

○	Yes
●	No

2. Siam is less TIME-CONSUMING AND EXPENSIVE than Vole is

○	Yes
●	No

3. Siam and Vole are equally TIME-CONSUMING AND EXPENSIVE

○	Yes
●	No

The experiment

- *Languages:* Hebrew, English
- *Adjectives:* tall, bald, fat, expensive, time consuming
- *Number of participants:* 109
- various types of question ordering

Theory 1: Kleinean approach

Consider:

A *small* elephant is a *large* animal

Basic idea

The interpretation *I(P,* ***c****)* of a gradable adjective *P* depends on a *comparison class* ***c***.

I(P, ***c****)* is a partial function from objects in ***c*** to truth values 0 and 1.

Crosscontextual Constraints

No reversal

If there is some $\boldsymbol{c}$ such that $I(P,\boldsymbol{c})(x)=1$ and $I(P,\boldsymbol{c})(y)=0$, then there is no $\boldsymbol{c}$ such that $I(P,\boldsymbol{c})(y)=1$ and $I(P,\boldsymbol{c})(x)=0$.

(+ -)

Let $\boldsymbol{c} \subset \boldsymbol{b}$. There are x $\in \boldsymbol{c}$ such that $I(P,\boldsymbol{c})(x)=1$ and $I(P,\boldsymbol{c})(x)=0$ only if there are $y \in \boldsymbol{b} \sim \boldsymbol{c}$ such that $I(P,\boldsymbol{c})(y)=1$.

Comparative

Definition

$V(\mathit{more}\,P)(x)(y) = 1$ iff there is some $\boldsymbol{c}$ such that $I(P, \boldsymbol{c})(x) = 1$, whereas $I(P, \boldsymbol{c})(y) = 0$.

Given the cross contextual constraints one can prove that this way the comparative *more P* has all the logical properties it should have: it's asymmetric and transitive.

For complex predicates $\lambda u\varphi$ we have the same definition of *more* $\lambda u\varphi$.

This does not work

It is very well possible that

$$V(more\ \lambda u(Pu \& Qu))(x)(y) = 1$$

while

$$V(more\ P)(x)(y) = 0.$$

Given the definition of the comparative, for *x* to be more *P* and *Q* than *y* it suffices that there is a comparison class ***c*** in which *x* is both *P* and *Q* while *y* is *either* not *P* *or* not *Q.*

Theory 2: The fuzzy approach

Basic idea

The truth value $V(\phi)$ of a sentence ϕ can be any real number between 0 and 1.

More specifically, the interpretation $I(P)$ of a one place predicate is a function from objects to the interval [0,1].

If $I(P)(x)=1$, then x is definitely P,

If $I(P)(x)=0$, then x is definitely not P,

but x can also be something in between.

Comparative

To define the comparative, the first thing that comes to mind is this:

$$V(\textit{more}\,P)(x)(y) = 1 \text{ iff } I(P)(x) > I(P)(y).$$

Given this, does $\textit{more}\ \lambda u\,(Pu \,\&\, Qu))(x)(y)$ imply $\textit{more}\,P(x)(y)$?

To sort this out we first have to define meaning of &. One way — but not the only one — to do this is the following.

$$V(\phi \,\&\, \psi) = \min\{V(\phi), V(\psi)\}$$

This does not work either

It is very well possible that

$$V(more\,\lambda u\,(Pu\ \&\ Qu))\,(x)\,(\mathrm{y}) = 1$$

while

$$V(more\,P)\,(x)\,(y) = 0.$$

Given the definition of the comparative, for x to be more P & Q than y it suffices that $\min\{I(P)(x), I(Q)(x)\} > \min\{I(P)(y), I(Q)(y)\}$. But from this it does not follow that $I(P)(x) > I(P)(y)$.

There are other fuzzy ways to define the comparative, and conjunction, but they give rise to similar problems.

Alternative: Degree based approach

Every gradable adjective *P* comes with

– a *degree function f* which assigns to every object *x* a degree $f(x)$.

– a *membership norm*, which is some degree $n(f)$ in the range of *f*.

$I(P)$ is the function from D into {0,1} given by

$$I(P)(x) = 1 \text{ iff } f(x) > n(f).$$

$$I(\textit{more}\,P)(x)(y) = 1 \text{ iff } f(x) > f(y)$$

Conjoining gradable adjectives *P* and *Q*

- Seems impossible because there is no degree function combining the degree functions of *P* and *Q*.
- In other words: $\mathit{more}\,\lambda u(Pu\&Qu)$ cannot get an interpretation.
- Solution: We can “lift” conjunction and keep working with both degree functions separately.

Conjoining gradable adjectives *P* and *Q*

Thus we get as the interpretation of this lifted 'and':

$$\lambda P \lambda Q \lambda M \lambda y \lambda x (M(P)(x)(y) \;\&\; M(Q)(x)(y))$$

Conjoining gradable adjectives *P* and *Q*

Example

'*Siam is more time consuming and expensive than Vole'* becomes

$\lambda P\ \lambda Q\ \lambda M\ \lambda x\ \lambda y\ (M(P)(x)(y) \& M(Q)(x)(y))(time\ consuming)(expensive)(more)(Vole)(Siam)$

which is equivalent to

more(*time consuming)(Vole)(Siam)* & *more*(*expensive)(Vole)(Siam).*

In the same way people used lifting to explain why
'*John warmly hugs and kisses Mary'* is equivalent to
'John warmly hugs Mary and John warmly kisses Mary'.

Complication: Multidimensional adjectives

Some gradable adjectives come with several degree functions, each with its own norm.

Consider for example ‘*healthy*’. One can be healthy in some respects but not in other. To count as healthy one has to be healthy in *all* (contextually relevant) respects; however, to count as unhealthy it is sufficient to not be healthy in *some* respects.

Survey with multidimensional adjectives

- For a new job, IBM is looking for graduate students of computer science who were successful in their studies and/or have gained actual market experience as programmers.

- Mary worked as a programmer for 4 years and Sue worked as a programmer for 1 year.

- Both have studied computer science in the same university with the same average grade (85 on a 100 point scale) and were similarly successful at their first year of work (rate 5.5 on a 7-point scale).

Only 26% answered like this (n=50)

Success in studies and experience

I agree that...

1. Mary is more SUCCESSFUL AND EXPERIENCED than Sue

- ○ Yes
- ● No

2. Mary is less SUCCESSFUL AND EXPERIENCED than Sue

- ○ Yes
- ● No

3. Mary is exactly as SUCCESSFUL AND EXPERIENCED as Sue

- ○ Yes
- ● No

72% gave exactly one ‘yes’ (n=50)

Success in studies and experience

I agree that...

1. Mary is more SUCCESSFUL AND EXPERIENCED than Sue
   - Yes 80%
   - No
2. Mary is less SUCCESSFUL AND EXPERIENCED than Sue
   - Yes 3%
   - No
3. Mary is exactly as SUCCESSFUL AND EXPERIENCED as Sue
   - Yes 17%
   - No

Diagnosis 1

If there are more dimensions, one can find some of them more important than other. One can ignore some and compare the objects only in all *relevant* respects. This could explain the data.

(But we don’t know for sure that this is what happens)

Diagnosis 2: multi-dimensional adjectives are used in two different ways

One way:

Think of ‘*healthy*’ as

‘*healthy in respect 1* & ... *& healthy in respect n’*

Then, by our earlier strategy (typeshifting).‘healthier’ gets the meaning

‘*healthier in respect 1 & ... & healthier in respect n’*

In other words, this way ‘*healthier*’ would mean

‘*healthier in all (relevant) respects’*

Diagnosis 2: multi-dimensional adjectives are used in two different ways

The other way:

Think of ‘healthy’ as being associated with the degree function *f* given by

$$f(x) = |\{ d \in \mathrm{dim}(\mathrm{healthy}) : g_d\,(\mathrm{x}) > n(g_d)\}|$$

In other words, *x*’s degree of health equals the number of respects *d* such that x is healthy in respect *d*.

Given what we said earlier, the norm $n(f)$ associated with this degree function would be the number of all (relevant) respects.

By the interpretation of the comparative in the degree approach ‘*healthier*’ then gets the meaning ‘*healthy in more respects*’.

Diagnosis 2

Other experiments (than the ones reported here) suggest that people use multidimensional adjectives in both ways. This could also play a role in the experiment discussed here.

Conclusion: More experimentation needed!

Thank you!