Logic for Interaction

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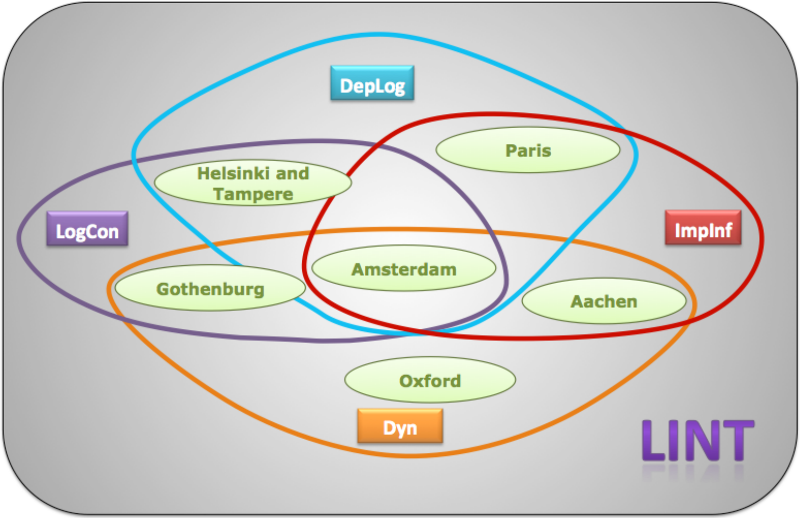
Outline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Highlights

Two highlights:

- Logical constants.
- Logic of dependence and independence concepts.

Outline

A new take on the problem of Logical Constants

The ‘classical’ problem of Logical Constants:

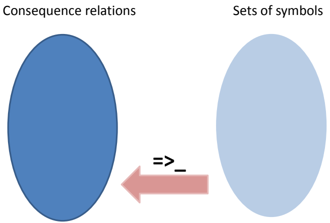
- first, give a principled characterization of the class of LC
- and then define logical consequence
(in semantic, proof-theoretic or game-theoretic terms)

Go the other way around and do some reverse engineering:

- start with a given consequence relation
- extract those symbols that are constants wrt that relation

The intuition for extracting is that constant symbols are those that are *essential to the validity of inferences*.

Forth

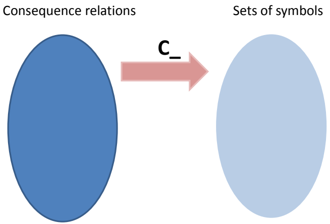


Given a language L with a fixed interpretation I_L,
sets of constants generate consequence relations:

Definition

$\Gamma \Rightarrow_X \phi$ iff for all interpretations J that agree with I_L on all symbols in X, $J \models_L \Gamma$ implies $J \models_L \phi$.

Back

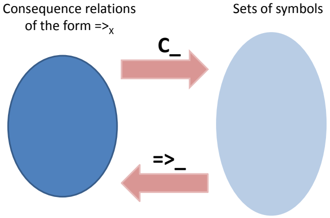


Given a consequence relation $\vdash$,
$C_\vdash$ extracts its constants:

Definition

$u \in C_\vdash$ iff there are Γ, ϕ, and u' such that
$\Gamma \vdash \phi$ but $\Gamma[u/u'] \nvdash \phi[u/u']$.

Back and Forth



Theorem (Bonnay & Westerståhl)

$C\_$ and $\Rightarrow\_$ form a Galois connection which is perfect on the left and whose right kernel consists in 'minimal' sets of symbols

minimal = all proper subsets generate a smaller consequence relation

Broadening the picture

- $C_{\_}$ can be shown to extract the expected sets of constants when it is applied to usual logical consequence relations.
- Introducing classes of interpretations, $C_{\_}$ can be shown to extract precisely the symbols whose denotations do not vary freely among interpretations respecting the consequence relation.
- For consequence relations $\vdash$ which are not of the form $\Rightarrow_X$, $\Rightarrow_{C_\vdash}$ properly extends $\vdash$
- This may be seen as a shortcoming if one is looking not just for symbols that are constants according to $\vdash$ but for a subset consisting of *logical* constants.
- Another extraction operation may be defined in terms of symbols whose denotations do not vary at all among interpretations respecting the consequence relation.

Outline

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Innovation

- Dependence and independence (as they occur in computer science, statistics, experimental science, etc) can be treated as atoms in logic, like their cousin identity.
- They are logical concepts.
- Like identity, they can be axiomatized.
- First we had just dependence. Isolating independence is a real LINT-product. It was an eye-opener, and turned out to have been around in disguised form.
- Erich Grädel, Fredrik Engström, Pietro Galliani, Fan Yang, J.V. and others.

Interaction

- What is the connection of dependence and independence to interaction?
- In our project their background is game theory:
  - A move of a player is (typically) dependent on previous moves in the game.
  - If the player is playing a strategy his or her moves are even determined by previous moves.
  - A move of a player may be independent of a particular previous move in the game, especially if the player does not know the move.

Examples

- Let us look at some examples of dependence and independence.
  - Not in game theory.
  - Not in database theory.
  - Not in statistics.
  - But in experimental science.

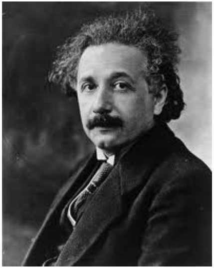
Balls of identical size but different weights are dropped from different heights.

Aristotle: The heavier the ball, the shorter the time of descent.
Galileo: The time t of descent is completely determined by the height h but completely independent of the weight w.

Height (m)	Weight (kg)	Time (s)
20	1.0	2.0
20	1.2	2.0
20	1.4	2.0
30	1.0	2.5
30	1.2	2.5
30	1.4	2.5
40	1.0	2.8
40	1.2	2.8
40	1.4	2.8

Species	Sex chromosomes	Sex
human	XY	male
human	XX	female
horse	XY	male
horse	XX	female
fruit fly	XY	male
fruit fly	XX	female

Aristotle: The sex of the offspring is determined by species, the environment and the nutrients.
C. E. McClung 1902: Sex is completely determined by the XY-chromosomes, independently of the species, environment and the nutrients.



- The speed of light in vacuum, measured by a non-accelerating observer, is independent of the motion of the observer or the source.
- Sun rises every morning independently of whether I rise from my bed or not.
- **Lesson:** Being a constant is a form of independence.

Outline

Armstrong's Axioms for functional dependence

We use

$$=(\vec{x}, \vec{y})$$

to denote the atomic formula with the intuitive interpretation "the values of the variables $\vec{y}$ are completely determined by the values of the variables $\vec{x}$".

Armstrong's Axioms:

1. $=(\vec{x}, \vec{x})$.
2. If $=(\vec{y}, \vec{x})$ and $\vec{y} \subseteq \vec{z}$, then $=(\vec{z}, \vec{x})$.
3. If $\vec{y}$ is a permutation of $\vec{z}$, $\vec{u}$ is a permutation of $\vec{x}$, and $=(\vec{z}, \vec{x})$, then $=(\vec{y}, \vec{u})$.
4. If $=(\vec{y}, \vec{z})$ and $=(\vec{z}, \vec{x})$, then $=(\vec{y}, \vec{x})$.

Axioms for independence

We use

$$\vec{x} \perp \vec{y}$$

to denote the atomic formula with the intuitive interpretation "the values of the variables $\vec{x}$ are completely independent of the values of the variables $\vec{y}$".

Axioms (Geiger-Paz-Pearl):

1. If $\vec{y} \perp \vec{x}$, then $\vec{y} \perp \vec{x}$.
2. If $\vec{y} \perp \vec{x}$ and $\vec{z} \subseteq \vec{y}$, then $\vec{z} \perp \vec{x}$.
3. If $\vec{y}$ is a permutation of $\vec{z}$, $\vec{u}$ is a permutation of $\vec{x}$, and $\vec{z} \perp \vec{x}$, then $\vec{y} \perp \vec{u}$.
4. If $\vec{y} \perp \vec{z}$ and $\vec{y}\vec{z} \perp \vec{x}$, then $\vec{y} \perp \vec{z}\vec{x}$.

Note: $=(\vec{x})$ is equivalent to $\vec{x} \perp \vec{x}$.

Inclusion logic

We use

$$\vec{x} \subseteq \vec{y}$$

to denote the atomic formula with the intuitive interpretation "every value of $\vec{x}$ occurs as a value of $\vec{y}$".

- Axiomatized by Casanova-Fagin-Papdimitriou.
- Mitchell, Chandra-Vardi: Inclusion and dependence atoms together cannot be axiomatized.

Exclusion logic

We use

$$\vec{x} | \vec{y}$$

to denote the atomic formula with the intuitive interpretation “no value of $\vec{x}$ occurs as a value of $\vec{y}$".

- Axiomatized by Casanova-Vania-Vidal.

Conditional independence

We use

$$\vec{x}\perp_{\vec{z}}\vec{y}$$

to denote the atomic formula with the intuitive interpretation "the values of $\vec{x}$ are independent of the values of $\vec{y}$, **if** the value of $\vec{z}$ is kept fixed".

- Cannot be axiomatized.
- $=(\vec{x},\vec{y})$ is equivalent to $\vec{y}\perp_{\vec{x}}\vec{y}$.

- Whatever dependence/independence atoms we have, we can coherently add logical operations $\wedge, \vee, \forall, \exists$ in front of the atoms also $\neg$.
- Subtlety: the logical operations have variants. The differences do not manifest themselves in first order logic, only in connection with the new atoms.

Novelty: Team semantics

- A team is a set of assignments (or a table, tree, database, etc)
- The point (W. Hodges): The dependence - independence phenomena do not manifest themselves in the presence of only one assignment. Teams called "Higher dependence models" in "Modal foundations for predicate logic" by J. van Benthem.
- With teams we can give meaning to formulas involving $\wedge, \vee, \forall, \exists, \neg$ and the new atoms.

Definition

A team X satisfies $=(\vec{x}, \vec{y})$ if

$$\forall s, s' \in X(s(\vec{x}) = s'(\vec{x}) \rightarrow s(\vec{y}) = s'(\vec{y})).$$

Dependence as a non-logical symbol occurs in "Generalized quantification as substructural rule" by N. Alechina and M. van Lambalgen.

Definition

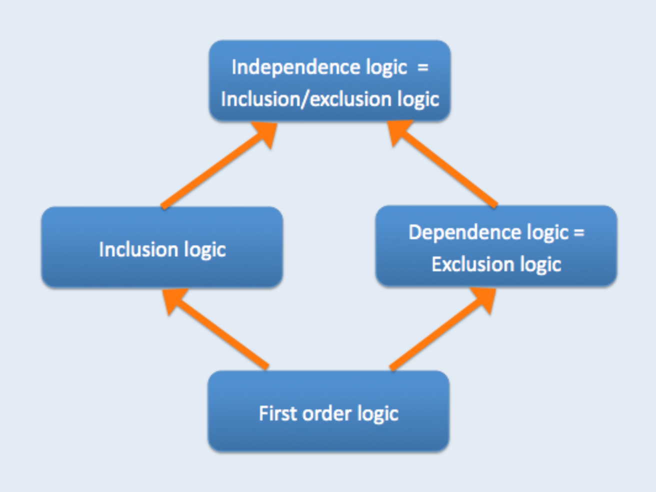
A team X satisfies the atomic formula $\vec{y} \perp_{\vec{x}} \vec{z}$ if for all $s, s' \in X$ such that $s(\vec{x}) = s'(\vec{x})$ there exists $s'' \in X$ such that $s''(\vec{x}) = s(\vec{x})$, $s''(\vec{y}) = s(\vec{y})$, and $s''(\vec{z}) = s'(\vec{z})$.

Similarly inclusion $\vec{x} \subseteq \vec{y}$ and exclusion $\vec{x} | \vec{y}$.

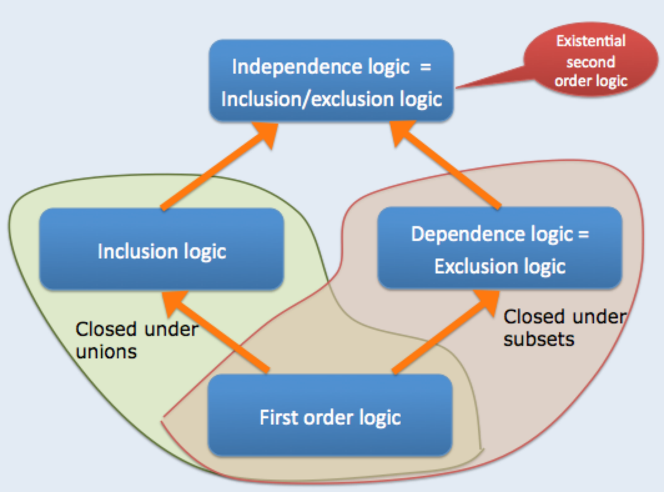
Definition

A team X satisfies $\phi \vee \psi$ if $X = Y \cup Z$ such that Y satisfies ϕ and Z satisfies ψ.

Pietro Galliani, others



Pietro Galliani, others



Outline

- Probabilistic semantics has been developed (Galliani, Sandu-Sevenster, Galliani-Mann).
- Proof theory has been developed (Galliani, Väänänen).
- It turns out that intuitionistic implication has a natural and important role, although it leads to full second order logic (Abramsky-Väänänen, Yang).
- On finite domains computational complexity of these logics has been studied leading to interesting hierarchy results in NP ((Jarmo) Kontinen, Durand-(Juha) Kontinen)
- Generalized quantifiers in dependence and independence logics (Engström).
- Modal dependence logic (Sevenster, Väänänen, Yang)
- Epistemic, dynamic, belief revision (Galliani)
- Compositionality (Galliani)
- 2-variable fragments (Kontinen-Kuusisto-Lohmann-Virtema)

Future

- DAAD (Hannover-Helsinki), "Complexity Theoretic Aspects of Dependence Logic", 2010-2012 (Kontinen, Väänänen, Vollmer).
- A Dagstuhl Seminar "Dependence Logic: Theory and Applications" will take place 2013 (Abramsky, Kontinen, Väänänen, Vollmer)