Skeptical Agents

Ondrej Majer
Marta Bílková, Michal Peliš, Greg Restall

Outline

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Knowledge

- knowledge = true justified belief
- belief = set of propositions
- any set of propositions?

No, our agent shall be *rational!*
Rational beliefs should be:

- consistent (not to belief A and not$A$)
- complete (to believe A or not A for any A)
- closed with respect to
  - conjunctions, disjunctions
  - logical consequence

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Knowledge - modal representation

Classical solution – possible worlds semantics

- belief sets are represented by means of possible worlds
- it is usually required
  – what she knows is true (truth)
  – if she knows something, she knows, that she knows it, (positive introspection)
  – other properties (negative introspection...)
- we get fully introspective logically omniscient agents with complete and consistent sets of beliefs
- Isn‘t it too perfect ??

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Conditions a belief set has to satisfy depend on the kind of agent we have in mind.

- our prototypical agent works with collections of data
- data are typically incomplete and might be inconsistent
- she might accept some of them as knowledge
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- two kinds of information:
  experimental data – inputs and outputs of experiments/observations ('facts')
  An α particle hits the surface.
- generalizations extracted from these data ('laws')
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Representation

- 'facts' - atomic formulas and their (weak) conjunctions and disjunctions
- 'laws' - implication
- incomplete/inconsistent states (not possible words)
- no omniscience, no introspection
- reasonable implication (no 'paradoxes')
- substructural epistemic logic

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Relational semantics

Information states
'Local' data available to the agent

- sets of propositions
- might be incomplete (neither $s \Vdash \varphi$ nor $s \Vdash \neg\varphi$ for some φ
- or/and inconsistent (both $s \Vdash \varphi$ and $s \Vdash \neg\varphi$ for some φ

Involvement
relation representing evolution of information states

- persistence – all the information from the past states is preserved
- partial order

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Relational semantics – lattice connectives

Local combinations of data

- (lattice) conjunctions
  $x \Vdash \varphi \wedge \psi$ iff $x \Vdash \varphi$ and $x \Vdash \psi$
- (lattice) disjunctions
  $x \Vdash \varphi \vee \psi$ iff $x \Vdash \varphi$ or $x \Vdash \psi$

Relational semantics – implication

Relevance

ternary relation R reponsible for *implication*
$R(x, y, z)$ connects different sources of data

- y 'antecedent state' – initial data of an experiment,
- z 'consequent state' – resulting data of the experiment.
- implication – empirical rule: if I observe at x, that an observation of φ at any antecedent state y is followed by observation of ψ in the consequent state, then I accept 'ψ follows φ' as a rule.

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Formally:

$$x \Vdash \varphi \to \psi \text{ iff for all } y, z, Rxyz \text{ and } y \Vdash \varphi \text{ implies } z \Vdash \psi$$

$\varphi \to \psi$ holds everywhere in the R-neighborhood of s
($< y, z >$ such that $R(s, y, z)$)

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Properties of the relation R

- $Rxyz$ and $x' \leq x, y' \leq y, z' \geq z$ implies $Rx'y'z'$
  monotonicity
- $Rxyz$ implies $Ryxz$
  exchange
- $Rxxx$
  contraction
- $R^2(xy)zw$ implies $R^2(xz)yw$
  associativity

Relational semantics – negation

Compatibility
binary relation C responsible for **negation**

- compatible states are collections of data our scientist wants to be consistent with

- before accepting a negative claim the agent 'looks around' – if nobody claims that φ she can accept $\neg\varphi$ as a piece of data

- asymmetry of positive and negative data

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Properties of the relation C

Compatibility is in general neither reflexive nor transitive.

- xCy, $x_1 \leq x$, and $y_1 \leq y$, imply $x_1 C y_1$ monotonicity
- xCy implies yCx
  symmetry – one negation
- $(\forall x)(\exists y)(xCy)$ directedness – $\neg\top \vdash \bot$
- convergence
  $(\forall x)(\exists y(xCy)$ implies $(\exists x^*)(xCx^*$ and $\forall z(xCz$ implies $z \leq x^*)))$
- $x \leq y$ implies $y^* \leq x^*$
- $x^{**} \leq x$

Relational semantics

Logical states
$L \subseteq W$ a set of states responsible for the definition of **truth** in a relevant frame (model).

$$\mathcal{F} \Vdash \varphi \text{ iff } (\forall x \in L)(x \Vdash \varphi) \tag{1}$$

– if require truth in all states, we get very weak system (e.g. $(\alpha \to \alpha)$) and the Modus Ponens fail)

– we require truth only in logically 'well behaved' states

Relevant frame is a tuple $F = (W, L, \leq, C, R)$,

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Epistemic frames - motivation

- data can be accepted as knowledge only if they are *confirmed* by a *source*
- we explicitly represent the relation of *being a source* by a new binary relation S on the set of states W
- we define our epistemic modality K:

$$x \Vdash K\varphi \text{ iff } s \Vdash \varphi \quad \text{for some } s \quad \text{such that} \quad sSx \tag{2}$$

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A source shall be:

- *compatible* with the current state
- *preceding* the current state in the involvement ordering
- *persistent* with respect to the involvement relation (once you have a source, you don't lose it)

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Classic frames

'Independent' confirmation – a source state should strictly precede the current state (a state should not count as a source for itself)

Classic frames, $\mathcal{F}_c$
satisfy strict precedence, compatibility and persistency

$$sSx \text{ iff } s < x \text{ and } sCx \tag{3}$$

$$sSx \text{ and } x \leq x' \text{ then } (\exists s')(s \leq s' \wedge s'Sx') \tag{4}$$

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$\mathcal{F}_g$ (*General frames*) – even weaker condition, we replace 'iff' in the other condition with 'only if':

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For $\mathcal{F}_g$ we provide an axiomatisation .

Every classic (weak classic) frame is a general frame, we have

$$\mathcal{F}_c \subseteq \mathcal{F}_g \text{ and } \mathcal{F}_{wc} \subseteq \mathcal{F}_g$$

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Positive properties

Factivity

$$K\varphi \to \varphi$$

Strong factivity

$$\neg\varphi \wedge K\varphi \to \bot$$

(not only information warranted at a state can be known, but that anything 'diswarranted' at a state is excluded from knowledge)

Monotonicity

$$\frac{\varphi \to \psi}{K\varphi \to K\psi}$$

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Negative properties

K-axiom

$$\not\models K(\alpha \to \beta) \to (K\alpha \to K\beta)$$

Necessitation rule

$$\frac{\varphi}{K\varphi}$$

Modal Modus Ponens

$$\frac{K\alpha \quad K(\alpha \to \beta)}{K\beta}$$

do not hold.

Negative properties

Introspection corresponds to a 'second order confirmation' (if α is confirmed then the confirmation of α is confirmed as well, similarly for the negative introspection).

Positive introspection

$$K\alpha \rightarrow KK\alpha$$

fails in $\mathcal{F}_g$ and $\mathcal{F}_c$, while it holds in $\mathcal{F}_{wc}$.

Negative introspection fails for all frames:

$$\not\models \neg K\alpha \rightarrow K\neg K\alpha$$

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Axiomatisation

Hilbert style axiomatisation of the background (distributive) substructural logic (e.g. Restall 2000),
+ axioms for t and $\top$
+ axioms for K:

- $K\varphi \to \varphi$ (factivity)
- $\neg\varphi \wedge K\varphi \to \bot$ (strong factivity)
- $K(\varphi \vee \psi) \to K\varphi \vee K\psi$ (distribution)

and the rules:

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi} \qquad \frac{\varphi \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi \to \psi}{K\varphi \to K\psi}$$

Soundness and completeness

Proven for the background logic being distributive relevant logic *R* of Belnap and Anderson

Theorem (Soundness)

Any formula provable in RK *is valid in all general frames.*

Theorem (Strong Completeness)

The axiomatization RK *is strongly complete with respect to the class* $\mathcal{F}_g$ *of general frames.*

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a 'forward looking' modality I adjoint to K.

ψ is implicitly known in a state s iff
ψ is true in all the states, for which s is a source

$x \Vdash I\psi$ iff $y \Vdash \psi$ for all y such that xSy

$$\frac{\varphi \to I\psi}{K\varphi \to \psi}$$

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Properties of I

$$\varphi \to I\varphi$$

(everything what is true in the current state is implicitly known) hence

$$I\varphi \to II\varphi$$

(positive introspection)

$$\varphi \to IK\varphi$$

(all that holds in a state is at least implicitly known there)

$$KI\varphi \to \varphi$$

(nothing else can be known to be implicit then facts true in the state)

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$$I\varphi \to II\varphi$$

(positive introspection)

$$\varphi \to IK\varphi$$

(all that holds in a state is at least implicitly known there)

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(nothing else can be known to be implicit then facts true in the state)

Strong knowledge

dual of (diamond-like) K – box-like backwards looking modality .

φ is strongly confirmed in φ iff it is true in all its source states (if any).
$x \Vdash \blacksquare\varphi$ iff for any s if sSx then $s \Vdash \varphi$

Further research

- proof system
- motivation for weaker systems
- non-distributive frames