ATL and extensions

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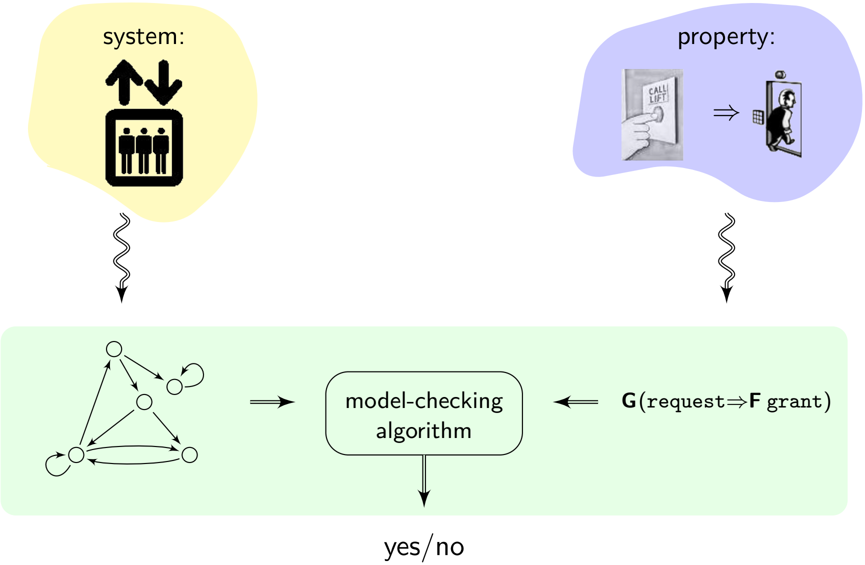
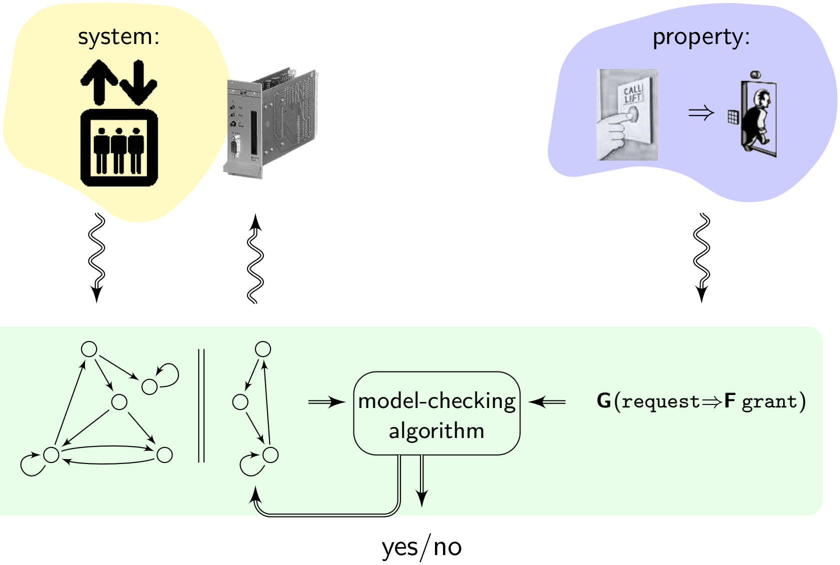
Model checking



Model checking and control



Computation-Tree Logic (CTL) [CE81,QS82]

Definition

$$\mathrm{CTL} \ni \varphi ::= \bigcirc \mid \varphi \vee \varphi \mid \neg \varphi \mid \mathbf{E}\,\mathbf{X}\,\varphi \mid \mathbf{E}\,\mathbf{G}\,\varphi \mid \mathbf{E}\varphi\;\mathbf{U}\;\varphi$$

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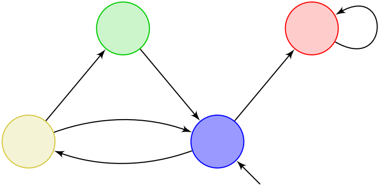

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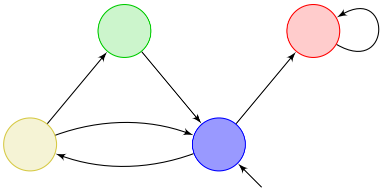

- ✓ $\mathbf{E}(\texttt{true}\ \mathbf{U}\ \bigcirc) \equiv \mathbf{EF}\ \bigcirc$
- ✓ $\mathbf{EG}\ \neg\bigcirc \equiv \neg(\mathbf{AF}\ \neg\bigcirc)$

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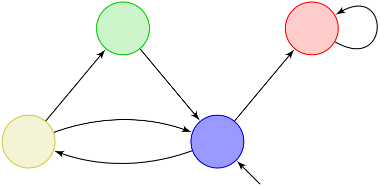

- ✓ $\mathbf{E}(\mathtt{true}\ \mathbf{U}\ {\color{red}\bigcirc}) \equiv \mathbf{E\,F}\,{\color{red}\bigcirc}$
- ✓ $\mathbf{E\,G}\,\neg{\color{green}\bigcirc} \equiv \neg(\mathbf{A\,F}\,\neg{\color{green}\bigcirc})$
- × $\mathbf{E}(\neg{\color{olive}\bigcirc}\ \mathbf{U}\ {\color{green}\bigcirc})$

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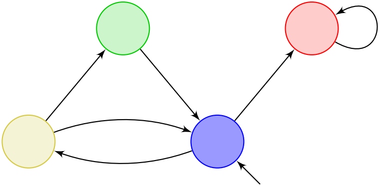

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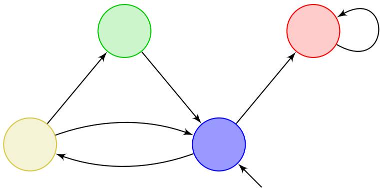

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Theorem

CTL model checking is PTIME*-complete.*

Alternating-time Temporal Logic (ATL) [AHK97]

Definition

ATL extends CTL with *strategy quantifiers*:

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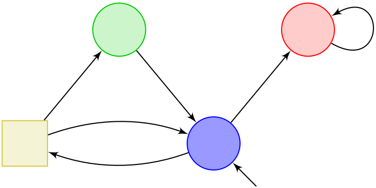

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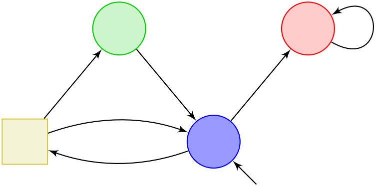

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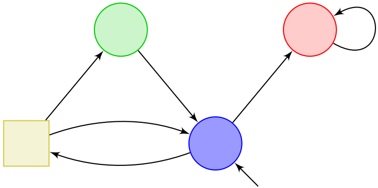

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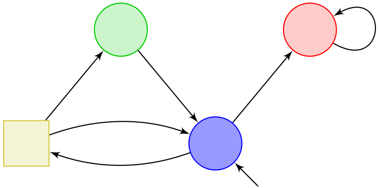

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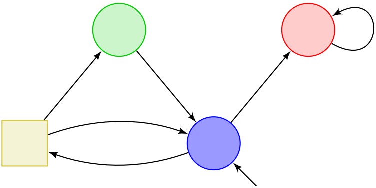

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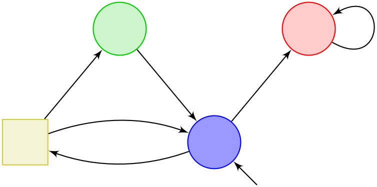
ATL with strategy contexts [BDLM09]

ATL$_{sc}$ has the same syntax as ATL, but different semantics:



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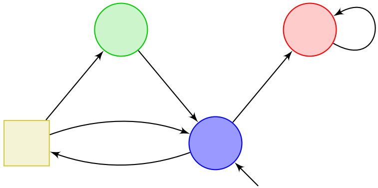

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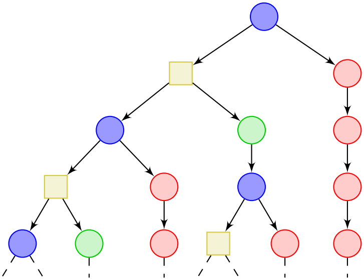

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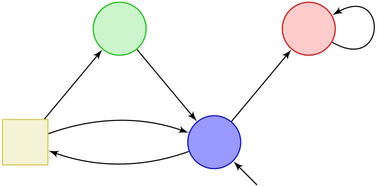

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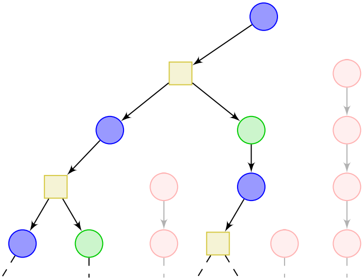

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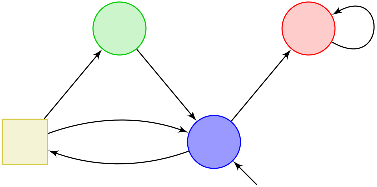

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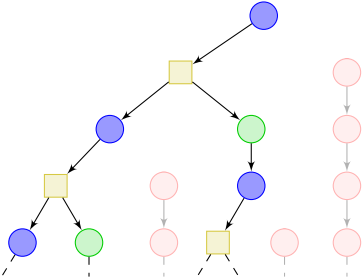

Evaluate the formula on the execution tree:

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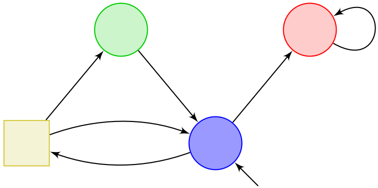

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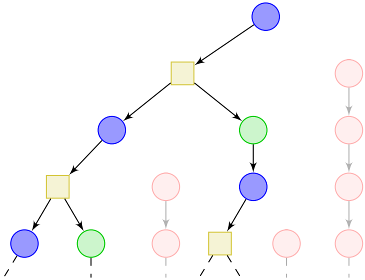

Evaluate the formula on the execution tree:

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- Existence of Nash equilibria:

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- Existence of dominating strategy:

$$\langle\!\cdot A \cdot\!\rangle\ [\cdot B \cdot] \left(\neg \varphi \Rightarrow [\cdot A \cdot]\ \neg \varphi \right)$$

Verifying ATL$_{sc}$ properties

Theorem

Given a CGS $\mathcal{C}$, a state ℓ_0 and an ATL$_{sc}$ formula φ, we can build a Büchi tree automaton $\mathcal{A}$ s.t.

$$\mathcal{L}(\mathcal{A}) \neq \varnothing \quad \Leftrightarrow \quad \mathcal{C}, \ell_0 \models_\varnothing \varphi.$$

$\mathcal{A}$ has size d-exponential, where d is the maximal number of nested quantifiers in φ.

Checking whether $\mathcal{C}, \ell_0 \models_\varnothing \varphi$ is in d-EXPTIME*.*

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Proposition

Checking whether $\mathcal{C}, \ell_0 \models_{\varnothing} \varphi$ is (d−1)-EXPSPACE*-hard.*

Conclusions and research directions

ATL_{sc} has a natural semantics:

- it can express many interesting properties (especially non-zero-sum);
- this expressiveness comes with a cost (in terms of model-checking complexity);
- we also studied bounded-memory strategies.

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We keep on exploring ATL_{sc}:

- characterize behavioural equivalence for ATL_{sc};
- randomized strategies;
- find interesting sublogics, with more efficient model-checking algorithm;
- study satisfiability of ATL_{sc}.