Probability and logic in psychology: a new form of psychologism?

Niki Pfeifer

Munich Center for Mathematical Philosophy
Ludwig-Maximilians-Universität München

Outline

- Introduction
- Example I: Nonmonotonic reasoning
- Example II: Aristotelian syllogisms
- Example III: Conditionals

What is psychologism? (Kusch, 2007)

- Negative connotation:
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- Neutral connotation
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- Positive connotation
  - right application of psychological techniques to philosophical problems

Experimental philosophy, XΦ

- New philosophical movement (Knobe & Nichols, 2008; Alexander, Mallon, & Weinberg, 2010)
- XΦ supplements traditional tools of analytic philosophy with empirical methods

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  - Cross-cultural intuitions
  - Epistemology
  - Folk morality/psychology
  - Free will
  - Intentional action
  - Metaphilosophy

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  - Metaphilosophy
- Goal: Extending the domain of XΦ to uncertain reasoning

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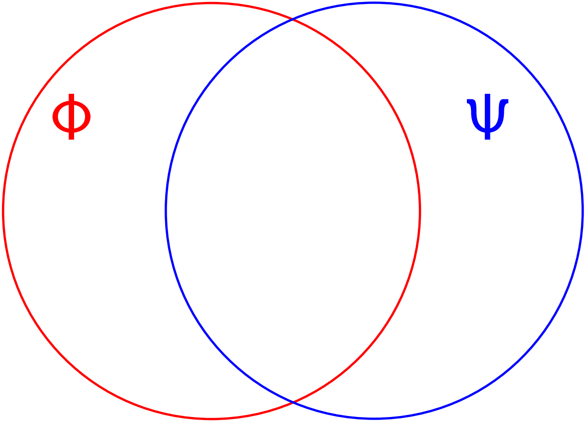
Motivation



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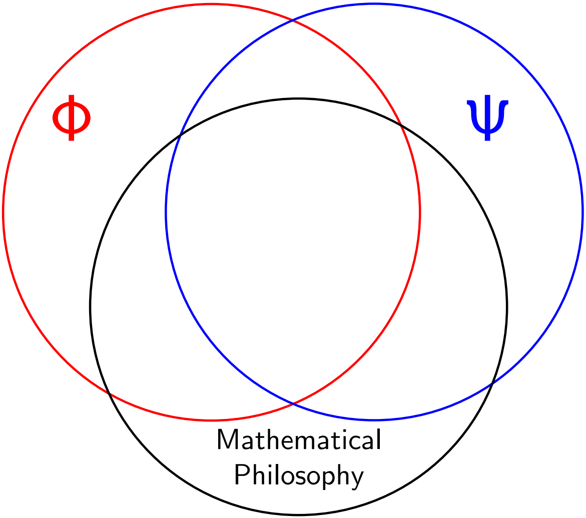

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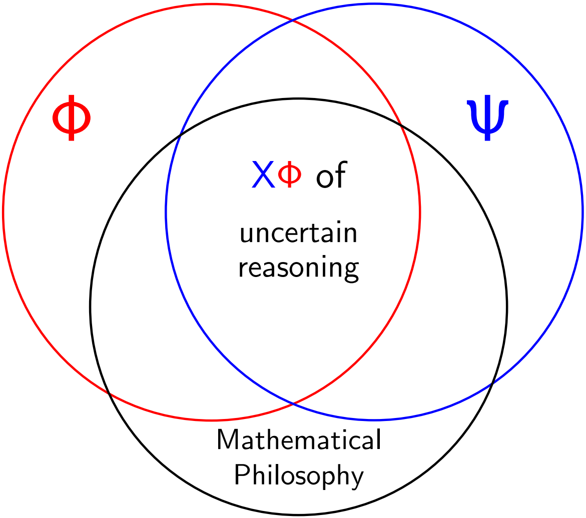

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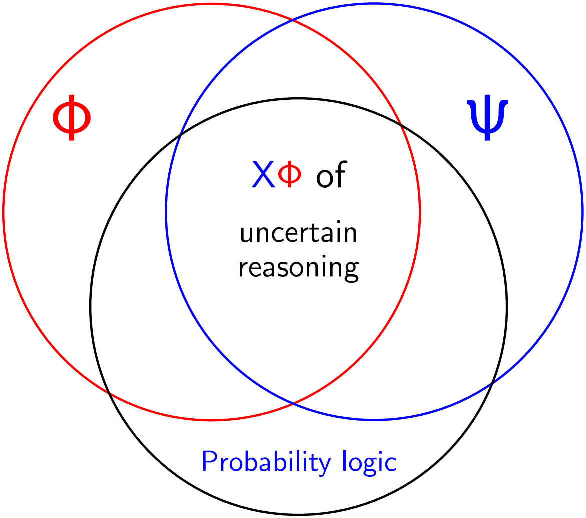

Example I:

Nonmonotonic reasoning

The Tweety problem

The Tweety problem (picture© by L. Ewing, S. Budig, A. Gerwinski; `http://commons.wikimedia.org`)

The Tweety problem (picture© by ytse19; http://mi9.com/flying-tux_35453.html)

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However, there are a priori rationality norms for nonmonotonic reasoning, e.g., System P (Kraus et al., 1990).

NMR fruitfully interacts between formal and empirical work (Pfeifer, in press b):

- empirical data may stimulate new formal theories (e.g., Ford, 2004)
- formal work provides rationality norms
- empirical validation provides external quality criteria beyond purely formal ones (like consistency or completeness)

System P: Rationality postulates for nonmonotonic reasoning (Kraus et al., 1990)

Reflexivity (axiom): $\alpha \mid\!\sim \alpha$

Left logical equivalence:

from $\models \alpha \equiv \beta$ and $\alpha \mid\!\sim \gamma$ infer $\beta \mid\!\sim \gamma$

Right weakening:

from $\models \alpha \supset \beta$ and $\gamma \mid\!\sim \alpha$ infer $\gamma \mid\!\sim \beta$

Or: from $\alpha \mid\!\sim \gamma$ and $\beta \mid\!\sim \gamma$ infer $\alpha \vee \beta \mid\!\sim \gamma$

Cut: from $\alpha \wedge \beta \mid\!\sim \gamma$ and $\alpha \mid\!\sim \beta$ infer $\alpha \mid\!\sim \gamma$

Cautious monotonicity:

from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \wedge \beta \mid\!\sim \gamma$

And (derived rule): from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \mid\!\sim \beta \wedge \gamma$

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$\alpha \mid\!\sim \beta$ is read as If α, $\underbrace{\textit{normally}}_{?}$ β

Semantics for System P

- Normal world semantics (Kraus et al., 1990)
- Possibility semantics: $\alpha \mid\!\sim \beta$ iff $\Pi(A \wedge B) > \Pi(A \wedge \neg B)$ (e.g., Benferhat, Dubois, & Prade, 1997)
  - Empirical support (Da Silva Neves, Bonnefon, & Raufaste, 2002; Benferhat, Bonnefon, & Da Silva Neves, 2005)
- Inhibition nets (Leitgeb, 2001, 2004)
- Probability semantics
  - Infinitesimal: $\alpha \mid\!\sim \beta$ iff $P(\beta|\alpha) = 1 - \epsilon$ (e.g., Adams, 1975)
  - Noninfinitesimal: $\alpha \mid\!\sim \beta$ iff $P(\beta|\alpha) > .5$ (e.g., Gilio, 2002; Biazzo, Gilio, Lukasiewicz, & Sanfilippo, 2005)
    - Empirical support (Pfeifer & Kleiter, 2003, 2005, 2006)
  - ...

Coherence

- de Finetti, and {Coletti, Gilio, Lad, Regazzini, Scozzafava, Walley, . . . }
- degrees of belief
- complete algebra is not required
- conditional probability, $P(B|A)$, is primitive
- zero probabilities are exploited to reduce the complexity
- imprecision
- bridges to possibility, DS-belief functions, fuzzy sets, default reasoning, . . .

Probabilistic version of System P (Gilio, 2002)

Name	*Probability logical version*
Left logical equivalence	$\models(E_1 \equiv E_2), P(E_3\|E_1) = x \ \therefore\ P(E_3\|E_2) = x$
Right weakening	$P(E_1\|E_3) = x, \models(E_1 \supset E_2) \ \therefore\ P(E_2\|E_3) \in [x, 1]$
Cut	$P(E_2\|E_1 \wedge E_3) = x, P(E_1\|E_3) = y$
	$\therefore\ P(E_2\|E_3) \in [xy, 1 - y + xy]$
And	$P(E_2\|E_1) = x, P(E_3\|E_1) = y$
	$\therefore\ P(E_2 \wedge E_3\|E_1) \in [\max\{0, x + y - 1\}, \min\{x, y\}]$
Cautious monotonicity	$P(E_2\|E_1) = x, P(E_3\|E_1) = y$
	$\therefore\ P(E_3\|E_1 \wedge E_2) \in [\max\{0, (x+y-1)/x\}, \min\{y/x, 1\}]$
Or	$P(E_3\|E_1)=x, P(E_3\|E_2)=y$
	$\therefore\ P(E_3\|E_1 \vee E_2) \in [xy/(x+y-xy), (x+y-2xy)/(1-xy)]$
Transitivity	$P(E_2\|E_1) = x, P(E_3\|E_2) = y \ \therefore\ P(E_3\|E_1) \in [0, 1]$
Contraposition	$P(E_2\|E_1) = x \ \therefore\ P(\neg E_1\|\neg E_2) \in [0, 1]$
Monotonicity	$P(E_3\|E_1) = x \ \therefore\ P(E_3\|E_1 \wedge E_2) \in [0, 1]$

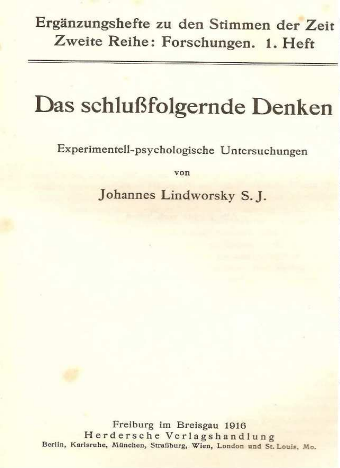
... where $\therefore$ is deductive

Example II:

Aristotelean syllogisms

(joint work with G. Sanfilippo & A. Gilio)

Motivation



Ergänzungshefte zu den Stimmen der Zeit
Zweite Reihe: Forschungen. 1. Heft

Das schlußfolgernde Denken

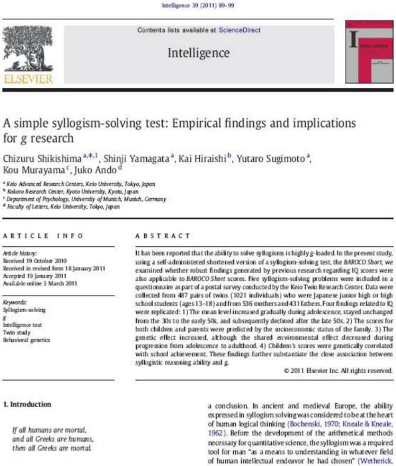
Experimentell-psychologische Untersuchungen

von

Johannes Lindworsky S. J.

Freiburg im Breisgau 1916
Herdersche Verlagshandlung
Berlin, Karlsruhe, München, Straßburg, Wien, London und St. Louis, Mo.

First book on experiments on reasoning (1916)



Intelligence 39 (2011) 89–99

Contents lists available at ScienceDirect

Intelligence

A simple syllogism-solving test: Empirical findings and implications for g research

Chizuru Shikishima[a,*,1], Shinji Yamagata[a], Kai Hiraishi[b], Yutaro Sugimoto[a], Kou Murayama[c], Juko Ando[d]

[a] Keio Advanced Research Centers, Keio University, Tokyo, Japan
[b] Kokoro Research Center, Kyoto University, Kyoto, Japan
[c] Department of Psychology, University of Munich, Munich, Germany
[d] Faculty of Letters, Keio University, Tokyo, Japan

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g
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ABSTRACT

It has been reported that the ability to solve syllogisms is highly g-loaded. In the present study, using a self-administered shortened version of a syllogism-solving test, the BAROCO Short, we examined whether robust findings generated by previous research regarding IQ scores were also applicable to BAROCO Short scores. Five syllogism-solving problems were included in a questionnaire as part of a postal survey conducted by the Keio Twin Research Center. Data were collected from 487 pairs of twins (1021 individuals) who were Japanese junior high or high school students (ages 13–18) and from 536 mothers and 431 fathers. Four findings related to IQ were replicated: 1) The mean level increased gradually during adolescence, stayed unchanged from the 30s to the early 50s, and subsequently declined after the late 50s. 2) The scores for both children and parents were predicted by the socioeconomic status of the family. 3) The genetic effect increased, although the shared environmental effect decreased during progression from adolescence to adulthood. 4) Children's scores were genetically correlated with school achievement. These findings further substantiate the close association between syllogistic reasoning ability and g.

1. Introduction

If all humans are mortal,
and all Greeks are humans,
then all Greeks are mortal.

a conclusion. In ancient and medieval Europe, the ability expressed in syllogism solving was considered to be at the heart of human logical thinking (Bochenski, 1970; Kneale & Kneale, 1962). Before the development of the arithmetical methods necessary for quantitative science, the syllogism was a required tool for man "as a means to understanding in whatever field of human intellectual endeavor he had chosen" (Wetherick,

Paper on syllogisms (2011)

Syllogistic types of propositions and figures

Name of Proposition Type	*PL formula*
Universal affirmative (A)	$\forall x(Sx \supset Px) \wedge \exists x Sx$
Particular affirmative (I)	$\exists x(Sx \wedge Px)$
Universal negative (E)	$\forall x(Sx \supset \neg Px) \wedge \exists x Sx$
Particular negative (O)	$\exists x(Sx \wedge \neg Px)$

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	Figure name			
	1	2	3	4
Major premise	M–P	P–M	M–P	P–M
Minor premise	S–M	S–M	M–S	M–S
Conclusion	S–P	S–P	S–P	S–P

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Minor premise	S–M	S–M	M–S	M–S
Conclusion	S–P	S–P	S–P	S–P

256 possible syllogisms, 24 Aristotelianly-valid, 9 require $\exists x Sx$

Example: Modus Barbara

All philosophers are mortal.
All members of the Vienna Circle are philosophers.

All members of the Vienna Circle are mortal.

Example: Modus Barbara

$$\frac{\begin{array}{c}\text{All } M \text{ are } P\\ \text{All } S \text{ are } M\end{array}}{\text{All } S \text{ are } P}$$

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$$\frac{\begin{array}{lcl}\forall x(Mx \supset Px) & \wedge & \exists x Mx \\ \forall x(Sx \supset Mx) & \wedge & \exists x Sx\end{array}}{\forall x(Sx \supset Px)}$$

Example: Modus Barbari

All M are P
All S are M

At least one S is P

$$\frac{\begin{array}{lcl}\forall x(Mx \supset Px) & \wedge & \exists x Mx \\ \forall x(Sx \supset Mx) & \wedge & \exists x Sx\end{array}}{\exists x(Sx \wedge Px)}$$

Acceptability conditions

Conj-Formalization:

All S are P: $p(S \wedge \neg P) = 0$ and EI

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CondEv-Formalization:

All S are P:	$p(P\|S) = 1$	and EI
Almost-all S are P:	$p(P\|S) \gg .5$	and EI
Most S are P:	$p(P\|S) > .5$	and EI
At least one S is P:	$p(S \wedge P) > 0$	

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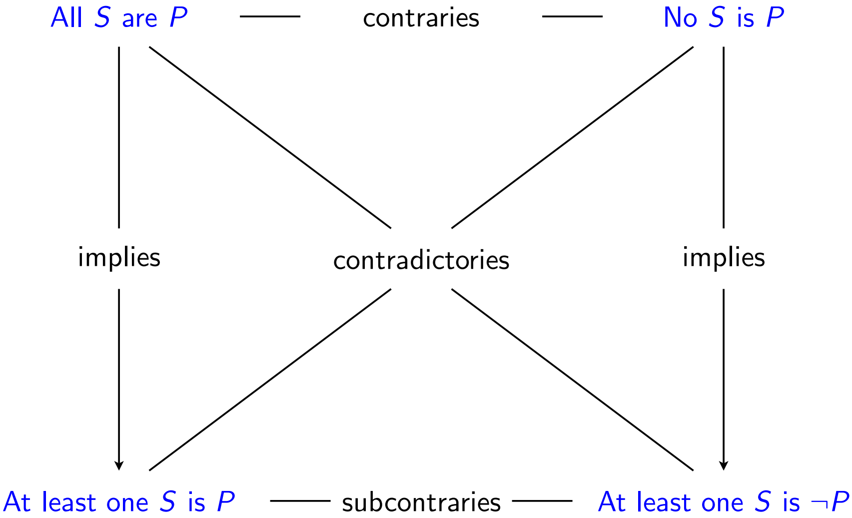
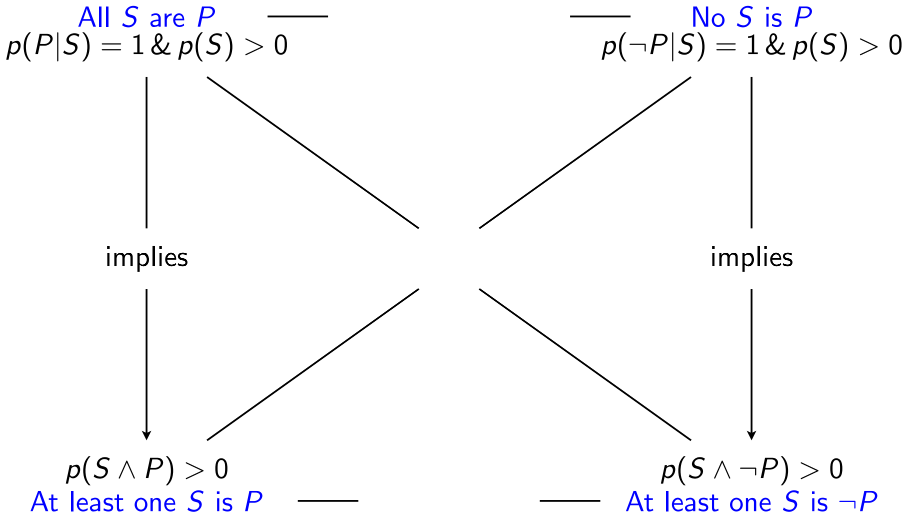
Most S are P:	$p(P\|S) > .5$	and EI
At least one S is P:	$p(S \wedge P) > 0$	

$p(S \wedge P) > 0$ if, and only if $p(P|S) > 0$ and $p(S) > 0$

Traditional square of oppositions



Towards a probabilistic square of oppositions



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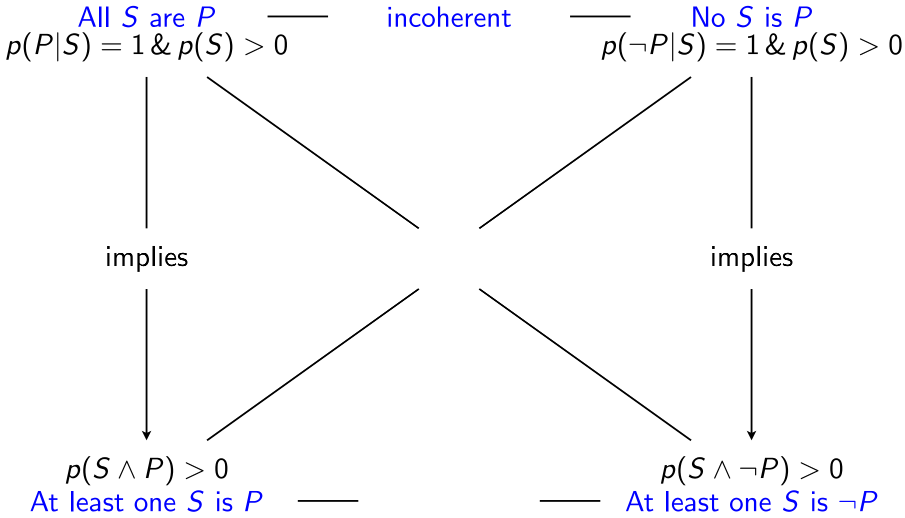

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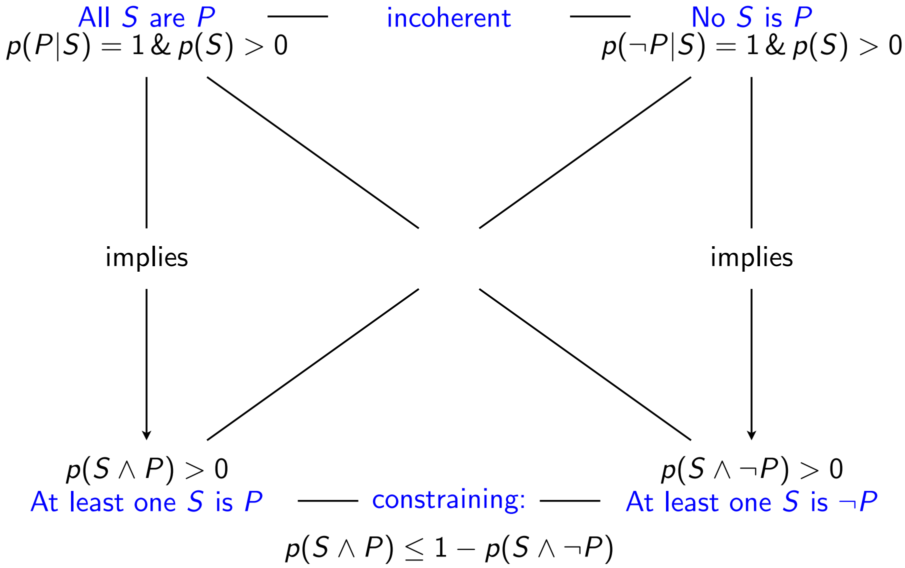

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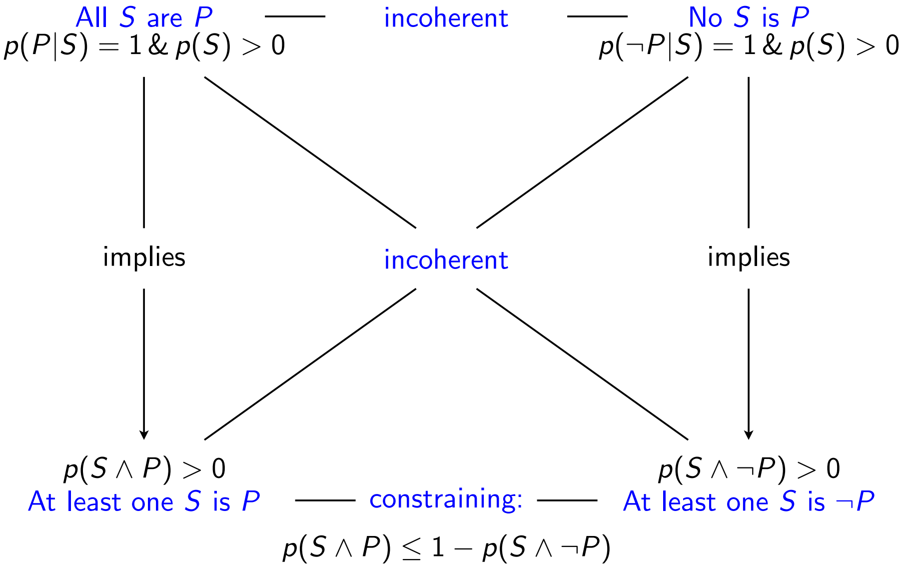

Example 1 (CondEv): Probabilistic Modus Barbara

All M are P	$p(P\|M) = 1$
All S are M	$p(M\|S) = 1$
All S are P	$0 \leq p(P\|S) \leq 1$

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All M are P	$p(P \mid M) = 1$
(Existential import: M	$p(M) > 0$)
All S are M	$p(M \mid S) = 1$
Existential import: S	$p(S) > 0$
All S are P	$p(P \mid S) = 1$

Example 1 (CondEv): Probabilistic Modus Barbara

All *M* are *P*	$p(P\|M) = 1$
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All *M* are *P*	$p(P\|M) = 1$
(Existential import: *M*	$p(M) > 0$)
All *S* are *M*	$p(M\|S) = 1$
Existential import: *S*	$p(S) > 0$
All *S* are *P*	$p(P\|S) = 1$

If $p(S) = \gamma$ and $p(M|S) = 1$, then $\gamma \leq p(M) \leq 1$

Example 2 (CondEv): Probabilistic Modus Barbari

All M are P	$p(P\|M) = 1$
All S are M	$p(M\|S) = 1$
At least one S is P	$0 \leq p(S \wedge P) \leq 1$

Example 2 (CondEv): Probabilistic Modus Barbari

All M are P	$p(P\|M) = 1$
All S are M	$p(M\|S) = 1$
At least one S is P	$0 \leq p(S \wedge P) \leq 1$

All M are P	$p(P\|M) = 1$
(Existential import: M	$p(M) > 0$)
All S are M	$p(M\|S) = 1$
Existential import: S	$p(S) > 0$
At least one S is P	$0 < p(S \wedge P) \leq 1$

Existential Import: Different options

- Replacing the first premise by a logical constraint, e.g.:

$$\frac{\begin{array}{c}\models (M \supset P) \\ p(M|S) = 1\end{array}}{p(P|S) = 1}$$

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- Strengthening the antecedent of the first premise, e.g.:

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- Positive probability of the conditioning event, e.g.:

All S are P: $p(S) > 0$

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- Positive probability of the conditioning event, e.g.:

All S are P: $p(S) > 0$

- Positive probability of each conditioning event, given the disjunction of all conditioning events ("conditional event EI"):

$$\frac{\begin{array}{l}p(P|M) = 1 \\ p(M|S) = 1 \\ p(S|S \vee M) > 0 \\ p(M|S \vee M) > 0 \text{ (irrelevant)}\end{array}}{p(P|S) = 1}$$

Example: Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	$[0, \ 1]$

Example: Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	[0, 1]
1	1	$t > 0$	[1, 1]

Example: Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	$[0,\ 1]$
1	1	$t > 0$	$[1,\ 1]$
1	y	$t > 0$	$[y,\ 1]$

Example: Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	[0, 1]
1	1	$t > 0$	[1, 1]
1	y	$t > 0$	$[y, 1]$
.9	1	1	[.9, .9]
.9	1	.5	[.8, 1]
.9	1	.2	[.5, 1]

Example: Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	$[0,\ 1]$
1	1	$t > 0$	$[1,\ 1]$
1	y	$t > 0$	$[y,\ 1]$
.9	1	1	$[.9,\ .9]$
.9	1	.5	$[.8,\ 1]$
.9	1	.2	$[.5,\ 1]$
(major)	(minor)		

Example: Figure 1, conditional event EI

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$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
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x	y	0	[0, 1]
1	1	$t > 0$	[1, 1]
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.9	1	1	[.9, .9]
.9	1	.5	[.8, 1]
.9	1	.2	[.5, 1]
(major)	(minor)		

$$z' = \max\left\{0, xy - \frac{(1-t)(1-x)}{t}\right\}$$
$$z'' = \min\left\{1, (1-x)(1-y) + \frac{x}{t}\right\}$$

Example III:

Conditionals

How people interpret indicative conditionals

- Material conditional $A \supset B$; explicit mental model (Johnson-Laird & Byrne, 2002)

$$\begin{array}{rr} A & B \\ \neg A & B \\ \neg A & \neg B \end{array}$$

How people interpret indicative conditionals

- Material conditional $A \supset B$; explicit mental model (Johnson-Laird & Byrne, 2002)

$$\begin{array}{cc} A & B \\ \neg A & B \\ \neg A & \neg B \end{array}$$

- Conjunction $A \wedge B$; implicit mental model (Johnson-Laird & Byrne, 2002)

$$\begin{array}{cc} A & B \\ \multicolumn{2}{c}{\ldots} \end{array}$$

How people interpret indicative conditionals

- Material conditional $A \supset B$; explicit mental model (Johnson-Laird & Byrne, 2002)

$$\begin{array}{rr} A & B \\ \neg A & B \\ \neg A & \neg B \end{array}$$

- Conjunction $A \wedge B$; implicit mental model (Johnson-Laird & Byrne, 2002)

$$\begin{array}{cc} A & B \\ & \ldots \end{array}$$

- Conditional event $B|A$ (e.g., Evans & Over, 2004; Oaksford & Chater, 2009; Pfeifer & Kleiter, 2009)

A priori arguments against the material conditional interpretation of $A \rightarrow B$

Paradoxes of the material conditional, e.g.,

(Paradox 1)	(Paradox 2)
$\dfrac{B}{A \supset B}$	$\dfrac{\neg A}{A \supset B}$

A priori arguments against the material conditional interpretation of $A \to B$

Paradoxes of the material conditional, e.g.,

(Paradox 1)

$$\frac{P(B) = x}{x \leq P(A \supset B) \leq 1}$$

(Paradox 2)

$$\frac{P(\neg A) = x}{1 - x \leq P(A \supset B) \leq 1}$$

probabilistically informative

A priori arguments against the material conditional interpretation of $A \to B$

Paradoxes of the material conditional, e.g.,

$$\text{(Paradox 1)} \quad \frac{P(B) = x}{0 \leq P(B|A) \leq 1} \qquad \text{(Paradox 2)} \quad \frac{P(\neg A) = x}{0 \leq P(B|A) \leq 1}$$

probabilistically non-informative

A priori arguments against the material conditional interpretation of $A \to B$

Paradoxes of the material conditional, e.g.,

(Paradox 1)
$$\frac{P(B) = x}{0 \leq P(B|A) \leq 1}$$

(Paradox 2)
$$\frac{P(\neg A) = x}{0 \leq P(B|A) \leq 1}$$

probabilistically <u>non</u>-informative

Special case not covered in the standard approach to probability:
If $P(B) = 1$, then $P(A \wedge B) = P(A)$.

A priori arguments against the material conditional interpretation of $A \to B$

Paradoxes of the material conditional, e.g.,

(Paradox 1)
$$\frac{P(B) = x}{0 \leq P(B|A) \leq 1}$$

(Paradox 2)
$$\frac{P(\neg A) = x}{0 \leq P(B|A) \leq 1}$$

probabilistically <u>non</u>-informative

Special case not covered in the standard approach to probability:
If $P(B) = 1$, then $P(A \wedge B) = P(A)$. Thus,
$P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{P(A)}{P(A)} = 1$, if $P(A) > 0$.

Negating conditionals

Aristotle's Theses

AT #1: $\neg(\neg A \rightarrow A)$

AT #2: $\neg(A \rightarrow \neg A)$

Aristotle's Theses

AT #1: $\neg(\neg A \to A)$

$$\neg(\neg A \supset A)$$

AT #2: $\neg(A \to \neg A)$

$$\neg(A \supset \neg A)$$

Aristotle's Theses

AT #1: $\neg(\neg A \to A)$

$$\neg(\neg A \supset A) \quad \equiv \quad \neg A \wedge \neg A \quad \equiv \quad \neg A$$

AT #2: $\neg(A \to \neg A)$

$$\neg(A \supset \neg A) \quad \equiv \quad A \wedge A \quad \equiv \quad A$$

Aristotle's Theses: Probability logical predictions (Pfeifer, in press a)

AT #1: $\neg(\neg A \rightarrow A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$

Aristotle's Theses: Probability logical predictions (Pfeifer, in press a)

AT #1: $\neg(\neg A \rightarrow A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$
- $P(\neg(\neg A \wedge A)) = 1$

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AT #1: $\neg(\neg A \to A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$
- $P(\neg(\neg A \wedge A)) = 1$
- $P(A|\neg A) = 0$, its negation: $P(\neg A|\neg A) = 1$

Aristotle's Theses: Probability logical predictions (Pfeifer, in press a)

AT #1: $\neg(\neg A \rightarrow A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$
- $P(\neg(\neg A \wedge A)) = 1$
- $P(A|\neg A) = 0$, its negation: $P(\neg A|\neg A) = 1$

AT #2: $\neg(A \rightarrow \neg A)$

- $P(\neg(A \supset \neg A)) = P(A)$
- $P(\neg(A \wedge \neg A)) = 1$
- $P(\neg A|A) = 0$, its negation: $P(\neg\neg A|A) = P(A|A) = 1$

Aristotle's Theses: Probability logical predictions (Pfeifer, in press a)

AT #1: $\neg(\neg A \rightarrow A)$

- $P(\neg(\neg A \supset A)) = P(\neg A)$
- $P(\neg(\neg A \wedge A)) = 1$
- $P(A|\neg A) = 0$, its negation: $P(\neg A|\neg A) = 1$

AT #2: $\neg(A \rightarrow \neg A)$

- $P(\neg(A \supset \neg A)) = P(A)$
- $P(\neg(A \wedge \neg A)) = 1$
- $P(\neg A|A) = 0$, its negation: $P(\neg\neg A|A) = P(A|A) = 1$

Complete uncertainty of A: $0 \leq P(A) \leq 1$ is coherent.

Experiment 1: Abstract version, Aristotle's Thesis #1

The letter "A" denotes a sentence, like "It is raining".

There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:

- "A and not-A" is guaranteed to be false.
- "A or not-A" is guaranteed to be true.

There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "A" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If not-A, then A.

The sentence in the box is guaranteed to be false	☐
The sentence in the box is guaranteed to be true	☐
One cannot infer whether the sentence is true or false	☐

Experiment 1: Abstract version, Aristotle's Thesis #2

The letter "A" denotes a sentence, like "It is raining".

There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:

- "A and not-A" is guaranteed to be false.
- "A or not-A" is guaranteed to be true.

There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "A" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

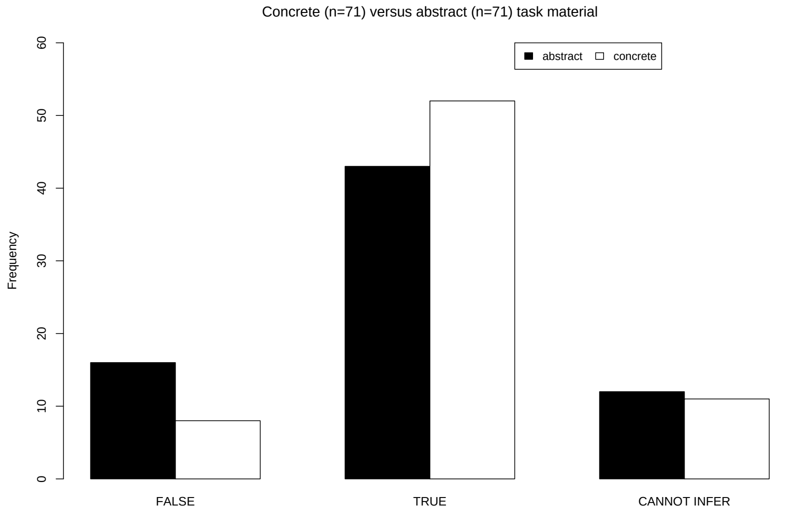
Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If A, then not-A.

The sentence in the box is guaranteed to be false	☐
The sentence in the box is guaranteed to be true	☐
One cannot infer whether the sentence is true or false	☐

Experiment 1: Sample (Pfeifer, in press a)

- $N = 141$
- all psychology students
- 91% third semester
- 78% female
- median age: 21 (1st Qu. = 20, 3rd Qu. =23)


Concrete (n=71) versus abstract (n=71) task material
abstract
concrete
Frequency
0
10
20
30
40
50
60
FALSE
TRUE
CANNOT INFER

Scope ambiguities

(W) Negating the conditional: $\neg \underbrace{(A \rightarrow \neg A)}_{\text{wide scope}}$

(N) Negating the consequent: $(A \rightarrow \neg \underbrace{\neg A)}_{\text{narrow scope}}$

Scope ambiguities

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(W) and (N) are well defined for $\wedge$ and $\supset$.

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(W) and (N) are well defined for $\wedge$ and $\supset$. Conditional events, $B|A$, are usually negated by (N), $P(\neg B|A)$.

$\neg(\neg A|A)$ could mean that $\neg A|A$ is completely rejected.

Scope ambiguities

(W) Negating the conditional: $\neg \underbrace{(A \to \neg A)}_{\text{wide scope}}$

(N) Negating the consequent: $(A \to \neg \underbrace{\neg A)}_{\text{narrow scope}}$

(W) and (N) are well defined for $\wedge$ and $\supset$. Conditional events, $B|A$, are usually negated by (N), $P(\neg B|A)$.

$\neg(\neg A|A)$ could mean that $\neg A|A$ is completely rejected.

$$\neg(B|A) \quad \text{iff} \quad 0 \leq P(B|A) \leq 1$$

Experiment 2: Design (Pfeifer, in press a)

Between participants: Explicit ($n_1 = 20$) vs. implicit negation ($n_2 = 20$)
Within participants: 12 Tasks

Task	Name	Argument form
1	Aristotle's Thesis 1	$\neg(A \to \neg A)$
2	Negated Reflexivity	$\neg(A \to A)$
3	Aristotle's Thesis 2	$\neg(\neg A \to A)$
4	Reflexivity	$A \to A$
5	Contingent Arg. 1	$A \to B$
6	Contingent Arg. 2	$\neg(A \to B)$
7-10	4 Probabilistic truth-table tasks	
11	Paradox 1	from B infer $A \to B$
12	Neg. Paradox 1	from B infer $A \to \neg B$

Experiment 2: Predictions

Argument form		Scope		
		wide	narrow	
	$\cdot\vert\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \to \neg A)$	T	CT	T	T
$\neg(A \to A)$	F	F	CT	CT
$\neg(\neg A \to A)$	T	CT	T	T
$A \to A$	T	T	T	CT
$A \to B$	CT	CT	CT	CT
$\neg(A \to B)$	CT	CT	CT	CT
from B infer $A \to B$	U	H		U
from B infer $A \to \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Experiment 2: Predictions $\cdot|\cdot$ against wide vs. narrow scope of $\cdot \supset \cdot$

Argument form		Scope		
		wide	narrow	
	$\cdot|\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \to \neg A)$	T	CT	T	T
$\neg(A \to A)$	F	F	CT	CT
$\neg(\neg A \to A)$	T	CT	T	T
$A \to A$	T	T	T	CT
$A \to B$	CT	CT	CT	CT
$\neg(A \to B)$	CT	CT	CT	CT
from B infer $A \to B$	U	H		U
from B infer $A \to \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Experiment 2: Aristotle's Thesis #1, implicit version

[. . .]

Hans expects to be visited by Thea and Ida. He is sitting in his room. Suddenly someone knocks at the door. Hans is absolutely certain, that either Thea or Ida is knocking.

Experiment 2: Aristotle's Thesis #1, implicit version

[...]

Hans expects to be visited by Thea and Ida. He is sitting in his room. Suddenly someone knocks at the door. Hans is absolutely certain, that either Thea or Ida is knocking.

Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If Ida knocks, then Thea knocks.

The sentence in the box is guaranteed to be false	☐
The sentence in the box is guaranteed to be true	☐
One cannot infer whether the sentence is true or false	☐

Experiment 2: Aristotle's Thesis #1, explicit version

[...]

Hans expects to be visited by Thea and Ida. He is sitting in his room. Suddenly someone knocks at the door. Hans is absolutely certain, that either Thea or Ida is knocking.

Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If Ida knocks, then Ida does not knock.

The sentence in the box is guaranteed to be false	☐
The sentence in the box is guaranteed to be true	☐
One cannot infer whether the sentence is true or false	☐

Experiment 2: Sample (Pfeifer, in press a)

- $N = 40$
- no psychology students
- individual tested, 5 € for participation
- 50% female
- median age: 22 (1st Qu. = 21, 3rd Qu. =23)

Experiment 2: Results (Pfeifer, in press a)

Argument form		Scope			Responses in percent		
		wide	narrow				
	$\cdot\vert\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$	T	F	CT
$\neg(A \to \neg A)$	T	CT	T	T	78	18	5
$\neg(A \to A)$	F	F	CT	CT	10	88	2
$\neg(\neg A \to A)$	T	CT	T	T	80	13	8
$A \to A$	T	T	T	CT	93	3	5
$A \to B$	CT	CT	CT	CT	0	13	88
$\neg(A \to B)$	CT	CT	CT	CT	20	3	78
from B infer $A \to B$	U	H		U	40	0	60
from B infer $A \to \neg B$	U	H		L	5	30	65

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Experiment 2: Results (Pfeifer, in press a)

Argument form		Scope			Responses in percent		
		wide	narrow				
	$\cdot\vert\cdot$	$\cdot\supset\cdot$	$\cdot\supset\cdot$	$\cdot\wedge\cdot$	T	F	CT
$\neg(A \rightarrow \neg A)$	T	CT	T	T	78	18	5
$\neg(A \rightarrow A)$	F	F	CT	CT	10	88	2
$\neg(\neg A \rightarrow A)$	T	CT	T	T	80	13	8
$A \rightarrow A$	T	T	T	CT	93	3	5
$A \rightarrow B$	CT	CT	CT	CT	0	13	88
$\neg(A \rightarrow B)$	CT	CT	CT	CT	20	3	78
from B infer $A \rightarrow B$	U	H		U	40	0	60
from B infer $A \rightarrow \neg B$	U	H		L	5	30	65

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Outline

- Introduction
- Example I: Nonmonotonic reasoning
- Example II: Aristotelian syllogisms
- Example III: Conditionals

Interaction of formal and empirical work (Pfeifer, in press b)

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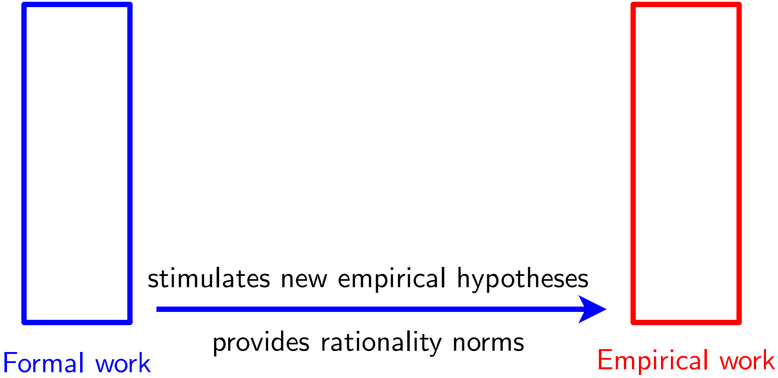

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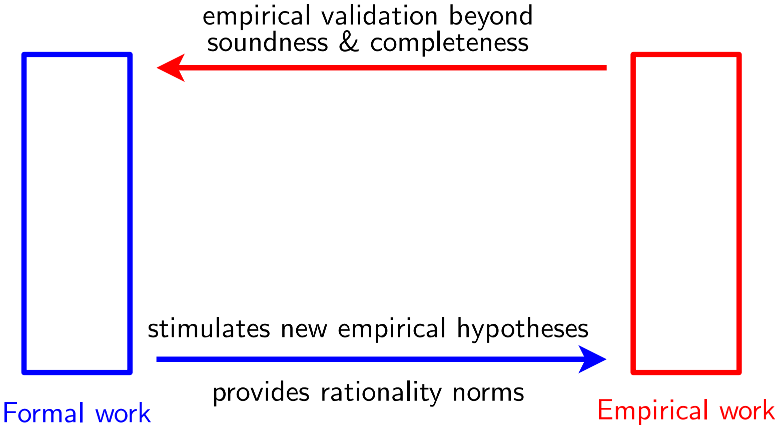

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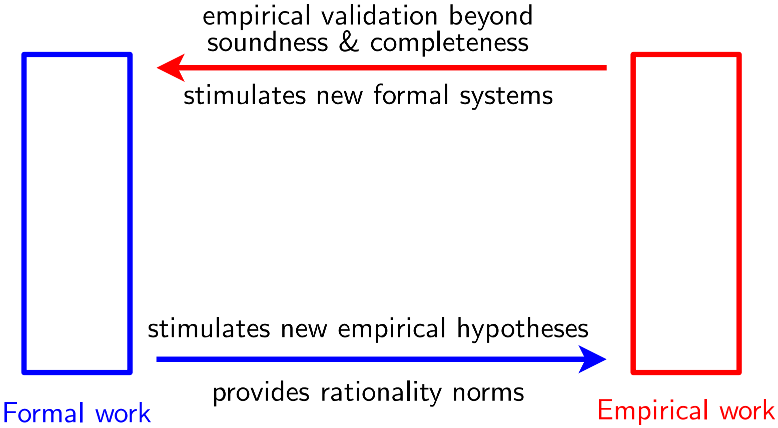

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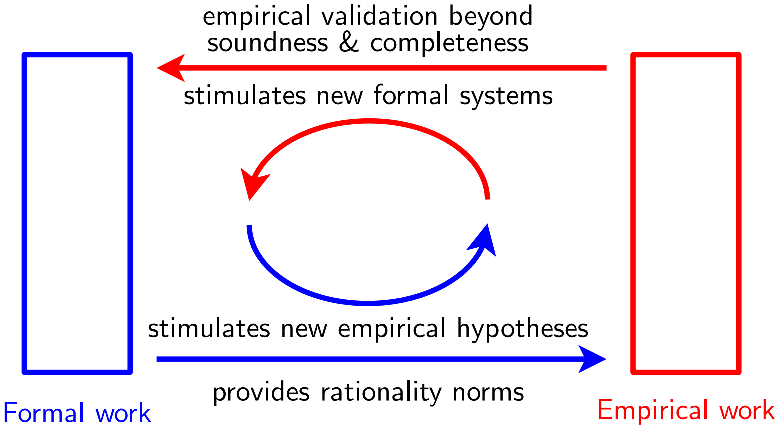

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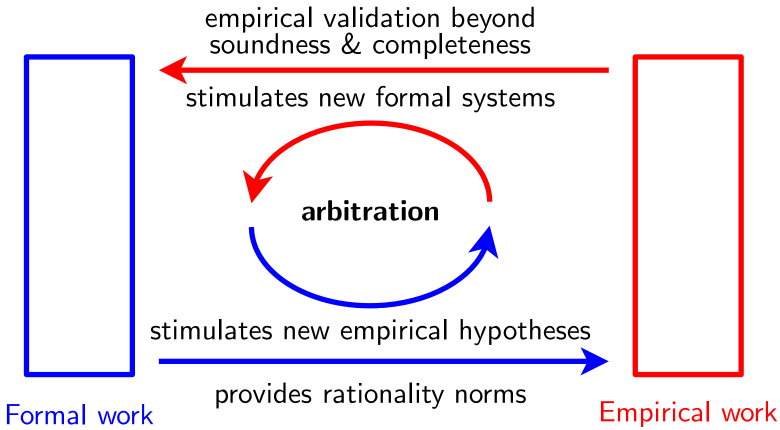

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