Dialogues, implications as rules and definitional reasoning

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"Dialogical foundations of semantics? An assessment")

1. What are dialogues?

Dialogues

A dialogue for $a \to (b \wedge a)$

positions

0.	P	$a \to (b \wedge a)$	
1.	O	a	[0, attack]
2.	P	$b \wedge a$	[1, defense]
3.	O	$\wedge_2$	[2, attack]
4.	P	a	[3, defense]

moves

Argumentation forms

X and Y, where $X \neq Y$, are variables for P and O.

implication $\to$:	assertion:	$X\,A \to B$	
	attack:	$Y\,A$	
	defense:	$X\,B$	
conjunction $\wedge$:	assertion:	$X\,A_1 \wedge A_2$	
	attack:	$Y \wedge_i$	(Y chooses $i = 1$ or $i = 2$)
	defense:	$X\,A_i$	

Dialogues

Dialogue (1)

A *dialogue* is a sequence of moves

(1) where P and O take turns,

(2) according to the argumentation forms,

(3) and P makes the first move.

Dialogue (2)

(A) P may assert an atomic formula only if it has been asserted by O before.

(E) O can only react on the immediately preceding P-move.

(plus some other conditions, specifying the logic)

A dialogue beginning with $P\,A$ is called *dialogue for the formula* A.

Argumentation forms P/O-symmetric.

Asymmetry between proponent P and opponent O due to (A) and (E).

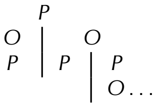
Dialogues and Strategies

P wins a dialogue for a formula A if

(1) the dialogue is finite,
(2) begins with the move $P\,A$ and
(3) ends with a move of P such that O cannot make another move.

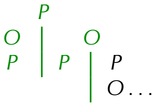
Strategy

A *dialogue tree* contains all possible dialogues for A as paths.



A *strategy* for a formula A is a subtree S of the dialogue tree for A such that

(1) S does not branch at even positions (i.e. at P-moves),
(2) S has as many nodes at odd positions as there are possible moves for O,
(3) all branches of S are dialogues for A won by P.



Strategies correspond to proofs.

2. Implications as rules and dialogues

Implications as rules

Idea: At the logical level, an implication $A \to B$ expresses a rule $\frac{A}{B}$.

Modus ponens viewed as *rule application*: Read $\frac{A \to B \quad A}{B}$ as $A \to B \; \frac{A}{B}$.

Implication introduction = establishing a rule,
Modus ponens = applying a rule.

Differs from implication in Gentzen's sequent calculus:

$$\frac{\Gamma \vdash A \qquad \Delta, B \vdash C}{\Gamma, \Delta, A \to B \vdash C}$$

Based on a different intuition ('implications-as-links'); also underlies standard dialogical interpretation.

As an alternative schema, Schroeder-Heister proposed:

$$\frac{\Gamma \vdash A}{\Gamma, A \to B \vdash B}$$

Expresses notion of *implications-as-rules* in sequent calculus.

Implications as rules: Argumentation forms

assertion:	$O\ A \to B$	
attack:	*no attack*	
defense:	*(no defense)*	

assertion:	$O\ A_1 \wedge A_2$	
attack:	$P\ \wedge_i$	$(i = 1$ or $2)$
defense:	$O\ A_i$	

assertion:		$P\ A \to B$	
question:		$O\ ?$	
choice:	$P\ \lvert A \to B \rvert$	$P\ C$	only if $O\ C \to (A \to B)$ before
attack:	$O\ A$		
defense:	$P\ B$		

assertion:		$P\ A_1 \wedge A_2$	
question:		$O\ ?$	
choice:	$P\ \lvert A_1 \wedge A_2 \rvert$	$P\ C$	only if $O\ C \to (A_1 \wedge A_2)$ before
attack:	$O\ \wedge_i$ $(i = 1$ or $2)$		
defense:	$P\ A_i$		

P/*O*-symmetry of argumentation forms is given up.

Implications as rules: Argumentation forms

assertion:	*O* $A \to B$
attack:	*no attack*
defense:	*(no defense)*

assertion:	$O\ A_1 \wedge A_2$	
attack:	$P\ \wedge_i$	($i = 1$ or 2)
defense:	$O\ A_i$	

assertion:		$P\ A \to B$
question:		O ?
choice:	$P\ \lvert A \to B \rvert$	$P\ C$ only if $O\ C \to (A \to B)$ before
attack:	$O\ A$	
defense:	$P\ B$	

Likewise for atoms a:

assertion:	$P\ a$
question:	O ?
choice:	$P\ C$ only if $O\ C \to a$ before

P/*O*-symmetry of argumentation forms is given up.

Implications as rules: Dialogues and strategies

Dialogues

(A°) P may assert an atomic formula *without* O having asserted it before.

(C) O can question a (complex or atomic) formula A if and only if
(i) A has not yet been asserted by O, or
(ii) A has already been attacked by P.

(E) O can only react on the immediately preceding P-move.

(Strategies defined as before.)

Corresponds to sequent calculus with alternative schema

$$\frac{\Gamma \vdash A}{\Gamma, A \to B \vdash B}$$

Yields dialogical interpretation of implications-as-rules concept.

Implications as rules and Cut

No 'Cut-elimination', but subformula property.

Argumentation form for Cut:

assertion:	$O\ A$	(or O ?, ...)
attack:	$P\ B$	
defense:	$O\ B$	

0.			$P\ a\to((a\to(b\wedge c))\to b)$		
1.			O ?	[0, question]	
2.			$P\ \lvert a\to((a\to(b\wedge c))\to b)\rvert$	[1, choice]	
3.			$O\ a$	[2, attack]	
4.			$P\ (a\to(b\wedge c))\to b$	[3, defense]	
5.			O ?	[4, question]	
6.			$P\ \lvert(a\to(b\wedge c))\to b\rvert$	[5, choice]	
7.			$O\ a\to(b\wedge c)$	[6, attack]	(assuming rule $a\to(b\wedge c)$)
8.			$P\ b\wedge c$	[Cut]	
9.	$O\ b\wedge c$	[Cut]	O ?	[8, question]	
10.	$P\ \wedge_1$	[9, attack]	$P\ a$	[9, choice]	(using rule $a\to(b\wedge c)$)
11.	$O\ b$	[10, defense]			
12.	$P\ b$	[7, defense]			

Implications as rules: Main results

(1) We have developed dialogues for the implications-as-rules interpretation.

(2) Treatment of Cut in dialogues.

(3) Equivalence proof for the corresponding sequent calculus with alternative left implication introduction rule (for intuitionistic logic).

(4) *P*/*O*-symmetry (player independence) of argumentation forms is lost.

This highlights the distinct roles of proponent and opponent.

3. Definitional reasoning and dialogues

Definitional reasoning

Argumentation form

For each atom a defined by $\mathfrak{D} \begin{cases} a \leftarrow \Delta_1 \\ \quad\vdots \\ a \leftarrow \Delta_k \end{cases}$ (Δ_i: conjunction of formulas)

definitional reasoning:	assertion:	$X\,a$	
	attack:	$Y\,\mathscr{D}$	
	defense:	$X\,\Delta_i$	(X chooses $i = 1, \ldots, k$)

('$\mathscr{D}$' special symbol indicating attack.)

Definitional dialogues

Definitional dialogues are dialogues

(1) plus argumentation form of definitional reasoning;

(2) can start with assertion of atomic formula.

Definitional reasoning: Main results

(1) Definitions of atomic formulas can be given by complex formulas.
 We have developed dialogues with an end-rule for complex formulas.

(2) Proof of equivalence to corresponding sequent calculus with complex initial sequents (for intuitionistic logic).

(3) Extension of dialogues to definitional reasoning.

(4) Investigation of substructural definitional dialogues.

(5) Investigation of paradoxes in the framework of definitional dialogues.

Conclusion

Dialogical foundations of semantics have been given for implications as rules and definitional reasoning.

But certain dialogical tenets had to be given up.

In comparison: Proof-theoretic approach is more versatile.