The ESF project LogiCCC FP014: SSEAC Social Software for Elections, Allocation of tenders and Coalition formation Some Highlights

Partners

- **Partner** Hannu Nurmi, Turku, Finland
  Topic: Voting methods, power measures
  presentation: Saturday, Sept 17, 14:30 - 14:50 in session 4

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- **Associated partner**: Agnieszka Rusinowska, Lyon, France
  Topic: Coalition formation, networks
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  Topic: the RelViewsoftware tool
  presentation: Sunday, Sept 18, 09:00 - 09:30 in the session on Logic and Games

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  Topic: the RelViewsoftware tool
  presentation: Sunday, Sept 18, 09:00 - 09:30 in the session on Logic and Games
- **Associated partner** Harrie de Swart, Tilburg, Netherlands
  Topic: Majority judgment (adapted to choosing a parliament)

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- Both input and output can be visualized on the screen. It has been shown that this software tool is extremely useful for computing and visualizing solution concepts for problems from social choice and game theory.
- Although problems from social choice and game theory are mostly exponential, due to the BDD implementation of RelView, computations are feasible for examples which appear in practice.
- Due to special plug-ins recently developed it can deal with computationally complex problems from real-life, such as computing the (Banzhaf) power indices of the different parties in German parliament.

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- The game $(N, \mathcal{W})$ is monotone if for all $S, T \in 2^N$, if $S \in \mathcal{W}$ and $S \subseteq T$, then $T \in \mathcal{W}$.

Leading Example

- In the period 2006 - 2010 the city council of the municipality of Tilburg (NL) consists of the 10 parties

  PvdA, CDA, SP, LST, VVD, GL, D66, TVP, AB, VSP

  with respectively 11, 7, 5, 5, 4, 3, 1, 1, 1, 1 seats.

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- The total number of seats is 39.
- Decisions are taken by simple majority.
- Parties typically vote en bloc.
- So, in this example N is the set of the 10 parties just mentioned and a coalition is winning if the total number of seats of the parties in the coalition is at least 20. A simple game like this is called a *weighted majority game* and usually denoted by $[20; 11, 7, 5, 5, 4, 3, 1, 1, 1, 1]$.

Relation algebraic representation of simple games

There are two obvious ways to model simple games $(N, \mathcal{W})$ relation-algebraically:

- By a vector $v : 2^N \leftrightarrow \mathbf{1}$ that describes the set $\mathcal{W}$ as subset of 2^N, so that v_S iff $S \in \mathcal{W}$. We call v the vector model of the game.

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- By a relation $M : N \leftrightarrow \mathcal{W}$, with $M_{k,S}$ iff $k \in S$. We call M the membership model of the game.
- Given a simple game, in one can show that if $v : 2^N \leftrightarrow \mathbf{1}$ is the vector model, then $\mathsf{E}\, inj(v)^{\mathsf{T}} : N \leftrightarrow \mathcal{W}$ is the membership model, and conversely, if $M : N \leftrightarrow \mathcal{W}$ is the membership model, then $syq(\mathsf{E}, M)\mathsf{L} : 2^N \leftrightarrow \mathbf{1}$ (with $\mathsf{L} : \mathcal{W} \leftrightarrow \mathbf{1}$) is the vector model, where E, $inj(v)$ and $syq(\mathsf{E}, M)$ are defined as follows.

Membership relation, injective mapping, symmetric quotient

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- If $v : 2^N \leftrightarrow \mathbf{1}$ describes the set $\mathcal{W}$, then the injective mapping $inj(v) : \mathcal{W} \leftrightarrow 2^N$ is defined by $inj(v)_{S,T}$ iff $S = T$.
- For $R : N \leftrightarrow N_1$ and $S : N \leftrightarrow N_2$, the symmetric quotient $syq(R,S) : N_1 \leftrightarrow N_2$ is defined by $syq(R,S)_{U,V}$ iff for all $x \in N$ we have $R_{x,U}$ iff $S_{x,V}$.

Minimal winning, swinger, vulnerable winning

Let $(N, \mathcal{W})$ be a monotone simple game.

Definition

- Coalition S is minimal winning if $S \in \mathcal{W}$, but $T \notin \mathcal{W}$ for all $T \subset S$.

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- Player k is a swinger of coalition S if $S \in \mathcal{W}$, $k \in S$, but $S \setminus \{k\} \notin \mathcal{W}$.
- Coalition S is vulnerable winning if $S \in \mathcal{W}$ and it contains a swinger.

Example (continued)

- The 3-parties coalition

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  with 21 seats is a minimal winning coalition: every proper subset of it has less than 20 seats.
- In addition, every player in this coalition S is a swinger of S.
- As another example, the winning 4-parties coalition

$$S' = \{\text{PvdA, SP, VVD, GL}\}$$

  with 23 seats is not minimal, because without GL (3 seats) it is still winning, but it is vulnerable because it contains the swingers PvdA (11 seats), SP (5 seats) and VVD (4 seats).

Theorem

Let $v : 2^N \leftrightarrow \mathbf{1}$ be the vector model of the simple game $(N, \mathcal{W})$.

- Then the vector

$$\mathrm{minwin}(v) := v \cap \overline{(\mathsf{S} \cap \bar{\mathsf{I}})^{\mathsf{T}} v} : 2^N \leftrightarrow \mathbf{1}$$

describes the set $\mathcal{W}_{\min}$ of all minimal winning coalitions.

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- Assuming additionally that the simple game $(N, \mathcal{W})$ is monotone and $\mathsf{L} : N \leftrightarrow \mathbf{1}$, for the relation

$$\mathrm{Swingers}(v) := \mathsf{E} \cap \mathsf{L} v^{\mathsf{T}} \cap \mathrm{rel}(\mathsf{R}\,\overline{v}) : N \leftrightarrow 2^N$$

it holds $\mathrm{Swingers}(v)_{k,S}$ iff k is a swinger of S.

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- The vector

$$\mathrm{vulwin}(v) := \mathrm{Swingers}(v)^{\mathsf{T}} \mathsf{L} : 2^N \leftrightarrow \mathbf{1}$$

describes the set of all vulnerable winning coalitions.

Identity-, set-inclusion- and removal relation

- I is the identity relation and the relation $\mathsf{S} := \overline{\mathsf{E}^{\mathsf{T}}\,\overline{\mathsf{E}}} : 2^N \leftrightarrow 2^N$ defines set inclusion, i.e., $\mathsf{S}_{S,T}$ iff $S \subseteq T$.

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- $\mathsf{R} : N \times 2^N \leftrightarrow 2^N$ specifies the removal of an element from a set: for all $\langle k, S\rangle \in N \times 2^N$, $T \in 2^N$, $\mathsf{R}_{\langle k,S\rangle,T}$ iff $S \setminus \{k\} = T$.

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- If $w : N \times 2^N \leftrightarrow \mathbf{1}$, then $\mathrm{rel}(w) : N \leftrightarrow 2^N$ is defined by $\mathrm{rel}(w)_{k,S}$ iff $w_{(k,S)}$.

Specification of minimal winning into RelView

- The RelView program for the column-wise enumeration of all minimal winning coalitions:

```
Minwin(E,v)
  DECL S, I,
  BEG  S = -(E^ * -E);
       I = I(S);
       m = v & -((S & -I)^ * v)
       RETURN E * inj(m)^
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   END.
```
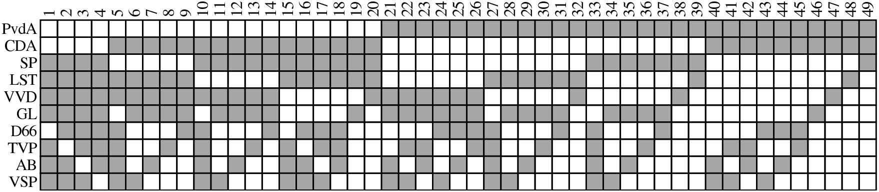

- $E : N \leftrightarrow 2^N$ and $v : 2^N \leftrightarrow \mathbf{1}$ are input. Then the relations $S : 2^N \leftrightarrow 2^N$, $I : 2^N \leftrightarrow 2^N$ and $\mathrm{minwin}(v) : 2^N \leftrightarrow \mathbf{1}$ are computed. The RETURN-clause column-wisely enumerates the set $\mathrm{minwin}(v)$.

Example continued

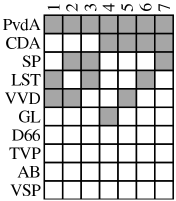
In case of our running example of the city council of Tilburg, the above program yields as output the following picture with all 49 minimal winning coalitions described by the 49 columns of the 10×49 matrix.



So, e.g., the first column describes the coalition consisting of the parties SP, LST, VVD, GL, TVP, AB and VSP.

Example continued

- The following 10×7 RelView matrix shows that the minimum number of parties needed to form a winning coalition is 3 and there are exactly 7 possibilities to form such 3-parties winning coalitions.



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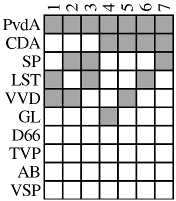

- RelView also computes 417 vulnerable winning coalitions, where the largest number of critical players in such a coalition is 7 (and 6 such cases exist).

Central player

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- Given a simple game $(N, \mathcal{W})$ and a policy order of the players in the form of a linear strict order relation $P : N \leftrightarrow N$, a player $k \in N$ is central if the connected coalition $\{j \in N \mid P_{j,k}\}$ "to the left of k" as well as the connected coalition $\{j \in N \mid P_{k,j}\}$ "to the right of k" are not winning, but both can be turned into winning coalitions when k joins them.

Theorem

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- Let a simple game $(N, \mathcal{W})$ with a policy order $P : N \leftrightarrow N$ be given and assume that $v : 2^N \leftrightarrow \mathbf{1}$ is the game's vector model. Using $Q := P \cup \mathsf{I}$ as reflexive closure of P, the vector $\mathrm{central}(v, P) :=$

$$\overline{syq(P, \mathsf{E})v} \cap \overline{syq(P^{\mathsf{T}}, \mathsf{E})v} \cap syq(Q, \mathsf{E})v \cap syq(Q^{\mathsf{T}}, \mathsf{E})v$$

of type $[N \leftrightarrow \mathbf{1}]$ describes the set of all central players.

Example continued

- Assume that the left-to-right strict order relation $<$ of the parties of the city council of Tilburg is as follows:

$$\text{GL}(3) < \text{SP}(5) < \text{PvdA}(11) < \text{D66}(1) < \text{CDA}(7) < \text{AB}(1) < \text{VSP}(1) < \text{VVD}(4) < \text{LST}(5) < \text{TVP}(1)$$

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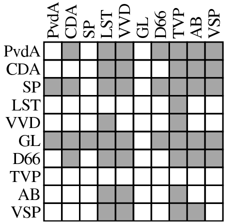
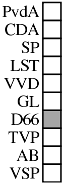
- Then D66 with 1 seat is the central player, because the parties to the left of it have only 19 seats and also the parties to the right of it have only 19 seats.

Example continued

The left picture below shows the strict order relation P, as depicted by RelView. The RelView program resulting from the above relation-algebraic specification to the membership model of our running example and P yields the right one of the pictures below.





Dominant players

Let $(N, \mathcal{W})$ be a simple game.

- The player $k \in N$ dominates the coalition $S \in 2^N$, written as $k \gg S$, if $k \in S$ and there exists $U \in 2^N$ such that $U \cap S = \emptyset$, $U \cup \{k\} \in \mathcal{W}$, but $U \cup (S \setminus \{k\}) \notin \mathcal{W}$, and for all $U \in 2^N$ with $U \cap S = \emptyset$, if $U \cup (S \setminus \{k\}) \in \mathcal{W}$, then $U \cup \{k\} \in \mathcal{W}$.

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- If k dominates S, then k can form a winning coalition with players outside of S while $S \setminus \{k\}$ is not able to do this.
- The player $k \in N$ is dominant if there exists a coalition $S \in \mathcal{W}$ such that $k \gg S$.
- Peleg proved that in weak simple games and weighted majority games at most one dominant player may occur.

Example continued

In our running example the party PvdA with 11 seats dominates the coalition

$$\{\text{PvdA, CDA, GL}\}$$

and hence is a dominant player:

- there is a coalition, viz.

  $$U = \{\text{LST, VVD}\},$$

  having 9 seats which together with PvdA (11 seats) is winning, but not together with CDA (7) and GL (3).

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  having 9 seats which together with PvdA (11 seats) is winning, but not together with CDA (7) and GL (3).
- for any coalition U not containing PvdA, CDA, GL, if $U \cup \{\text{CDA, GL}\}$ is winning, then also $U \cup \{\text{PvdA}\}$ is winning, since PvdA has one more seat than CDA (7 seats) and GL (3 seats) together.

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- PvdA dominates 140 coalitions, 50 of them winning;
- CDA dominates 48 coalitions, none of them winning;

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- VVD dominates 16 coalitions, none of them winning;

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- VVD dominates 16 coalitions, none of them winning;
- GL dominates 11 coalitions, none of them is winning;

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- CDA dominates 48 coalitions, none of them winning;
- SP and LST each dominate 22 coalitions, none of them winning;
- VVD dominates 16 coalitions, none of them winning;
- GL dominates 11 coalitions, none of them is winning;
- D66, TVP, AB and VSP each dominate 1 coalition, none of them winning.

Example continued

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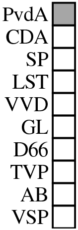
- Hence, PvdA, the party with the highest number of seats, is the (only) dominant player.

Example continued

The RelView-picture for the vector of dominant players is below:



Banzhaf power indices

- The absolute (non-normalized) Banzhaf index $\overline{B}(k)$ of player k is the probability that player k is decisive for the outcome, that is the number of times that k is a swinger in a winning coalition, divided by the number (2^{n-1} if there are n players) of coalitions he belongs to, assuming that all coalitions are equally likely and that each player votes yes or no with probability $\frac{1}{2}$.

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- Let $(N, \mathcal{W})$ be a monotone simple game and $k \in N$. Then the absolute Banzhaf index $\overline{B}(k)$ and the normalized Banzhaf index $B(k)$ of k are defined as follows, where n is the number of players:

$$\overline{B}(k) := \frac{|\{S \in \mathcal{W} \mid k \text{ swinger of } S\}|}{2^{n-1}} \qquad B(k) := \frac{\overline{B}(k)}{\sum_{j \in N} \overline{B}(j)}$$

Theorem

- For $R : X \leftrightarrow Y$ and $x \in X$, below $|R|$ denotes the number of 1-entries of R and $|R|_x$ the number of 1-entries of the x-row of R.

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- Assume a monotone simple game $(N, \mathcal{W})$ with n players and its vector model $v : 2^N \leftrightarrow \mathbf{1}$. Then we have for all players $k \in N$:

$$\overline{B}(k) = \frac{|\mathrm{Swingers}(v)|_k}{2^{n-1}} \qquad B(k) = \frac{|\mathrm{Swingers}(v)|_k}{|\mathrm{Swingers}(v)|}$$

Example continued

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- Using all this, RelView computes in case of our running example the following normalized Banzhaf indices:

$$\text{PvdA: } \frac{332}{988} \quad \text{CDA: } \frac{160}{988} \quad \text{SP: } \frac{116}{988} \quad \text{LST: } \frac{116}{988} \quad \text{VVD: } \frac{96}{988}$$

$$\text{GL: } \frac{88}{988} \quad \text{D66: } \frac{20}{988} \quad \text{TVP: } \frac{20}{988} \quad \text{AB: } \frac{20}{988} \quad \text{VSP: } \frac{20}{988}$$

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- Notice that although the number of seats of PvdA is about 1.5 times that of CDA, the power of PvdA expressed by the Banzhaf index is more than twice the power of CDA.