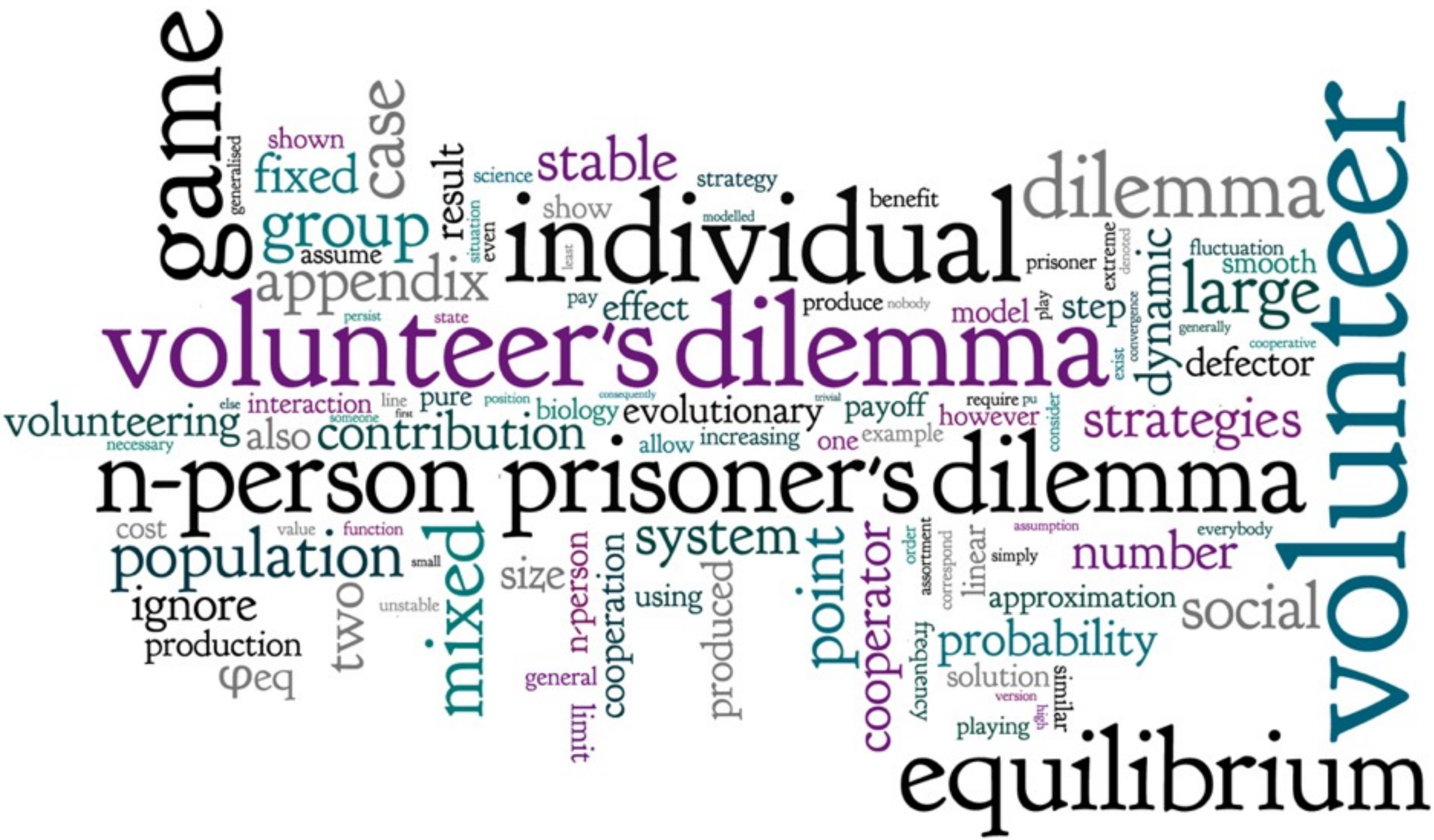
Marco Archetti

Department of Organismic and Evolutionary Biology
Harvard University



Cooperation in Public Goods Games

1. Volunteer's Dilemma
2. Public Goods
3. No Relatedness | Iterations
4. Most Social Dilemmas are VD, not PD
5. Practical Ways to Increase Cooperation

1. Volunteer’s Dilemma

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5. Practical Ways to Increase Cooperation

1 volunteer
required to produce a public good

raising the alarm when a predator approaches

Volunteer's Dilemma

(Dieckmann 1985, Archetti 2009)

1 volunteer

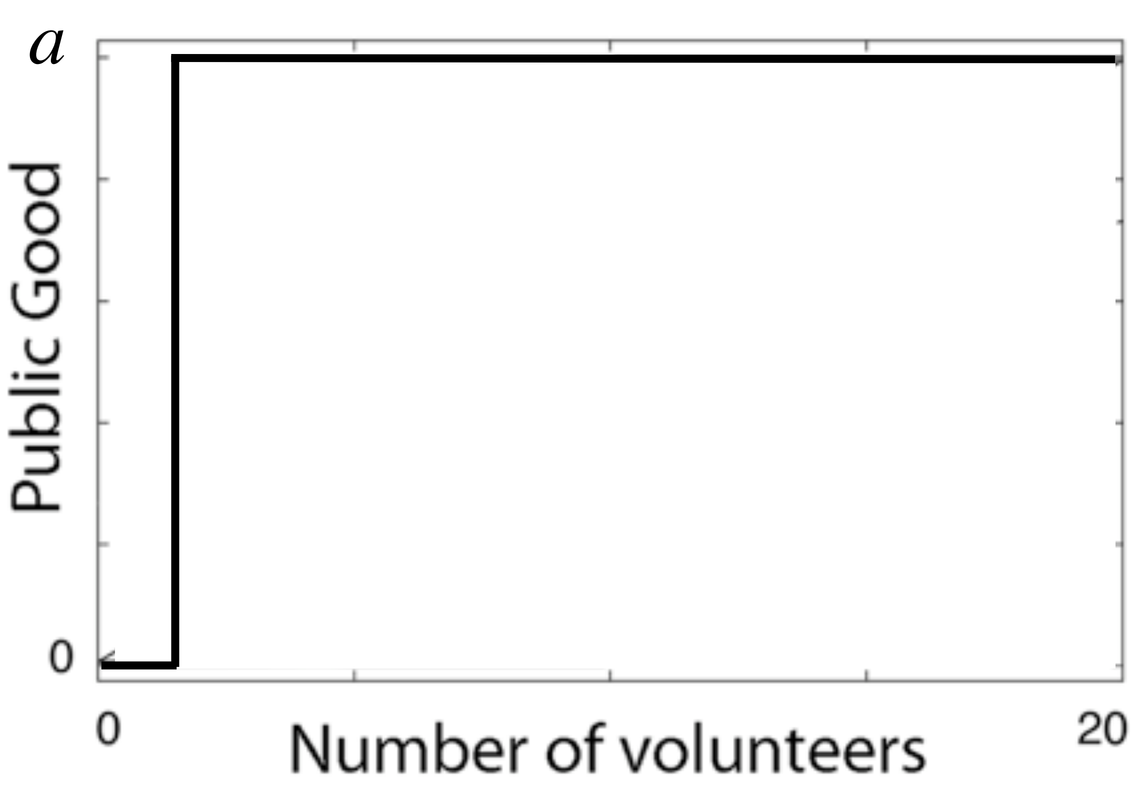
required to produce a public good

Benefit if someone gives the alarm: a

Cost of giving the alarm: $c<a$

Number of witnesses: N

Probability of ignoring: γ



FITNESS:

volunteer: $1 - c$

ignore: $\gamma^{N-1}(1-a) + (1-\gamma^{N-1})$

MIXED EQUILIBRIUM

$$\gamma_{\text{eq}} = (c/a)^{1/(N-1)}$$

$\boxed{k}$ volunteers

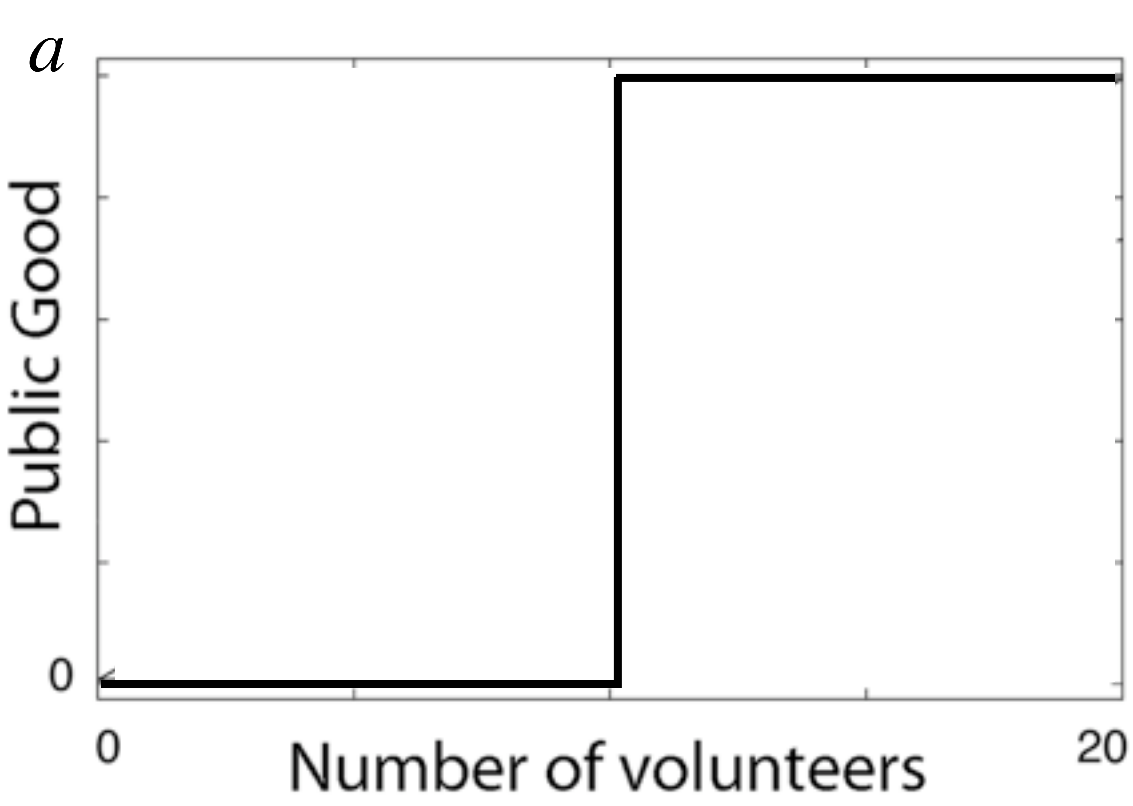
required to produce a public good

Benefit if someone gives the alarm: a

Cost of giving the alarm: $c<a$

Number of witnesses: N

Probability of ignoring: γ



N=20

k=10

Public good

non-rivalrous:

consumption does not reduce availability for others

non-excludable:

no one can be excluded from using it

BIOLOGY

- alarm calls in social vertebrates
- capture of large preys by predators
- replication enzymes in viruses
- adhesive polymers in bacteria
- invertase in yeast
- fruiting bodies in social amoebas

...

SOCIAL SCIENCES

- reporting a crime
- voting
- open-source software
- not downloading music

...

Public good

non-rivalrous:
consumption does not reduce availability for others

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no one can be excluded from using it

The problem with public goods

free-riding (Olson 1965, Hardin 1968)

BIOLOGY

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The problem with public goods

free-riding (Olson 1965, Hardin 1968)

The solutions

relatedness: kin selection

iterations: reciprocity | reputation | punishment | ...

Science

Clutton-Brock et al. 2009

Alarm calls

Explanations of the evolution of sentinel behavior have frequently relied on kin selection or reciprocal altruism...

... the same behaviour, however, is observed among non-relatives and in the absence of reciprocation

Public good
non-rivalrous:
consumption does not reduce availability for others
non-excludable:
no one can be excluded from using it

The problem with public goods
free-riding (Olson 1965, Hardin 1968)

The alleged solutions
relatedness: kin selection
iterations: reciprocity | reputation | punishment | …

The problems with these solutions
1. **empirical**:
   cooperation without relatedness | iterations
2. **practical**:
   solution can't rely on relatedness | iterations

The New York Times

1964

Kitty Genovese

For more than half an hour 38 respectable, law-abiding citizens in Queens watched a killer stab a woman… Nobody called the police.

How to increase the probability that someone gives the alarm
?

Science

Clutton-Brock et al. 2009

Alarm calls

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… the same behaviour, however, is observed among non-relatives and in the absence of reciprocation

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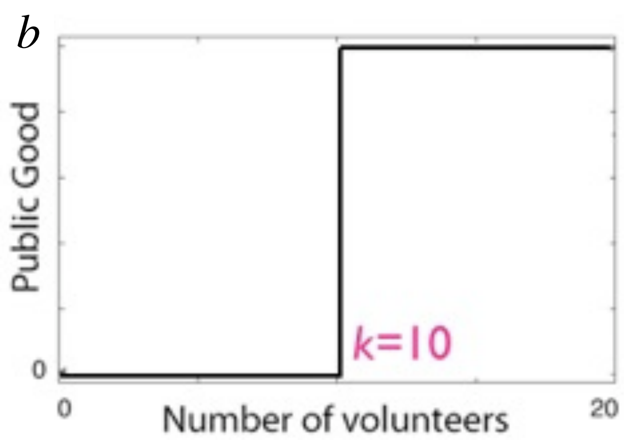
Public Goods Games as a Volunteer's Dilemma

Number of witnesses: N
Number of actual volunteers: j
Number of volunteers required: k

Cost of giving the alarm: c
Benefit due to the public good:

$$B_I(j)=\begin{cases} b & \text{if } j \geq k \\ 0 & \text{if } j < k \end{cases}$$

$$B_V(j)=\begin{cases} b & \text{if } j \geq k-1 \\ 0 & \text{if } j < k-1 \end{cases}$$



probability that j of the other N-1 individuals play *Volunteer*

$$f_j = \binom{N-1}{j} \phi^j (1-\phi)^{N-1-j}$$

FITNESS:

volunteer: $\sum_{j=0}^{N-1} f_j \, B_V(j) - c$

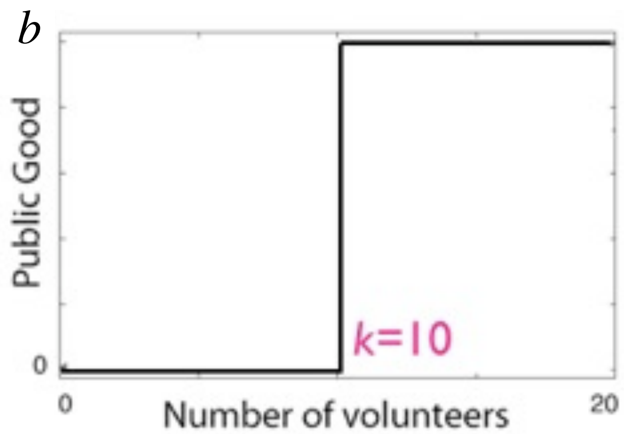
ignore: $\sum_{j=0}^{N-1} f_j \, B_I(j)$

MIXED EQUILIBRIUM

$$c/b = \binom{N-1}{N-k} \gamma^{N-k} (1-\gamma)^{N-1-(N-k)}$$

γ = probability of ignoring:

k volunteers required to produce a public good



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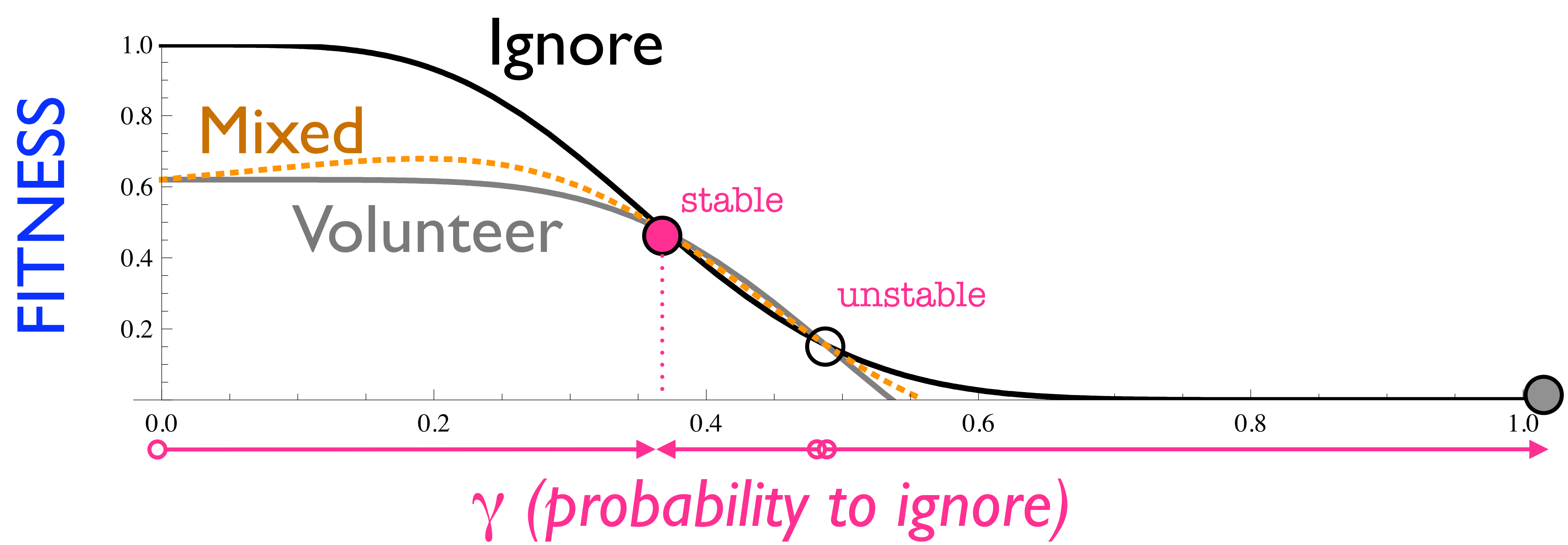
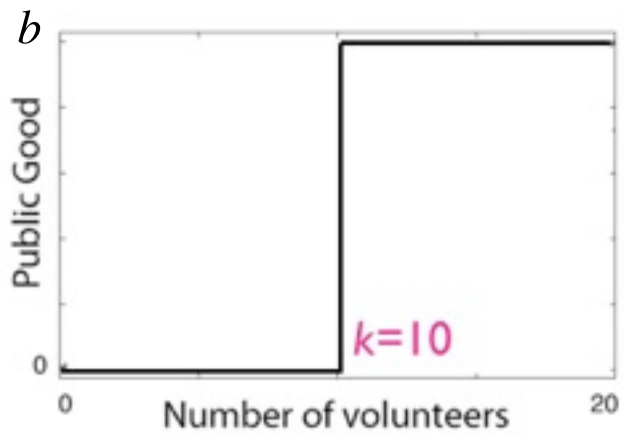


***k* volunteers** required to produce a public good



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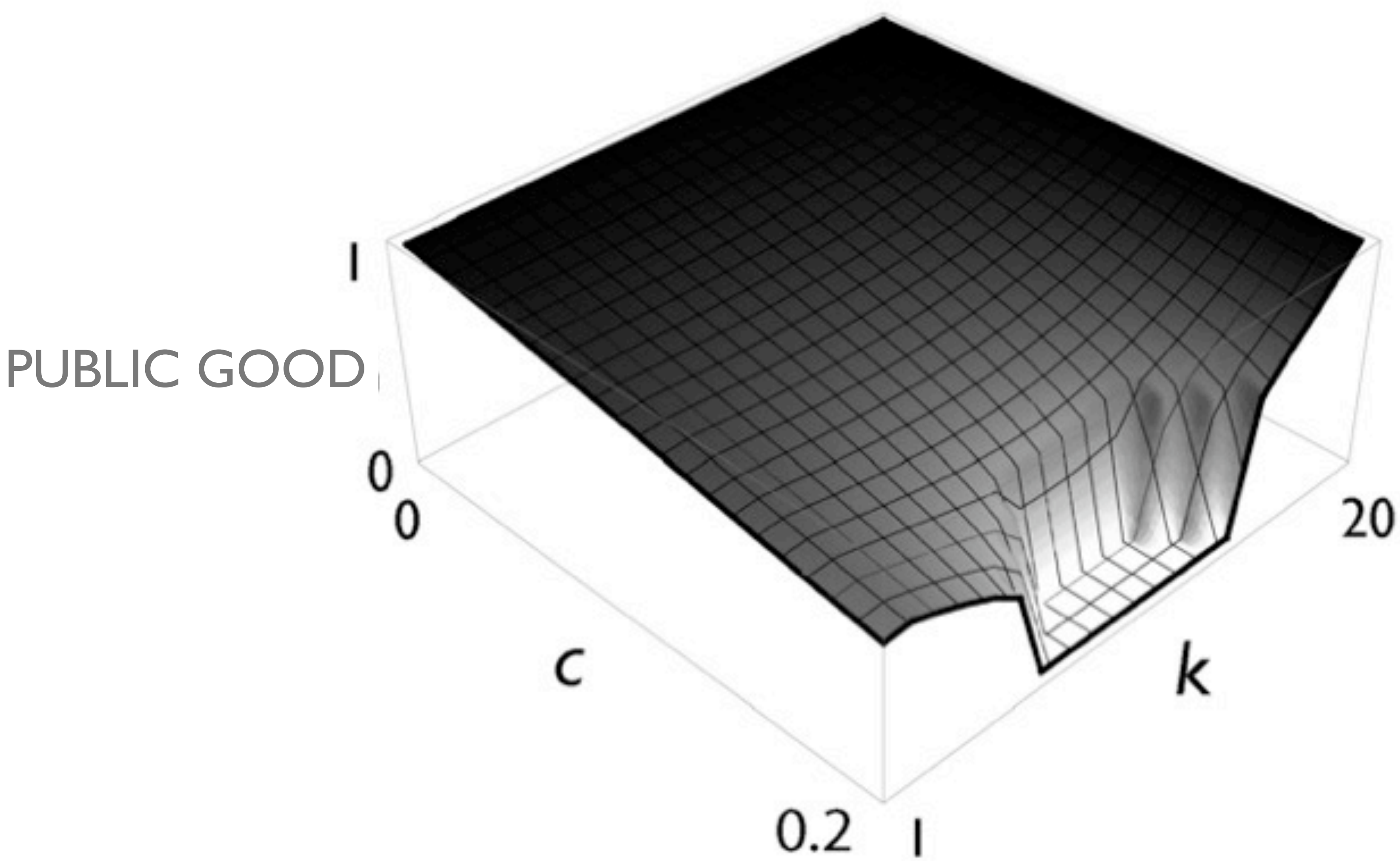

N=20

k=1 to 20

An approximate solution for large groups

Evolutionary dynamics

$$\frac{d\phi}{dt} = G(\phi) = \phi(1-\phi)\left[b\binom{N-1}{k-1}\phi^{k-1}(1-\phi)^{N-k} - c\right]$$

Stirling series approximation
Limit for N>>0
Taylor expansion
2nd order approximation

$$\phi_{eq}^{(s)} = \phi^{*} + \sqrt{\frac{2\phi^{*}(1-\phi^{*})}{N-1}\left[1-\frac{c}{bF(\phi^{*})}\right]}$$

For very large populations ($N \rightarrow \infty$)

$\phi = (k\text{-}1)/(N\text{-}1)$

probability of volunteering

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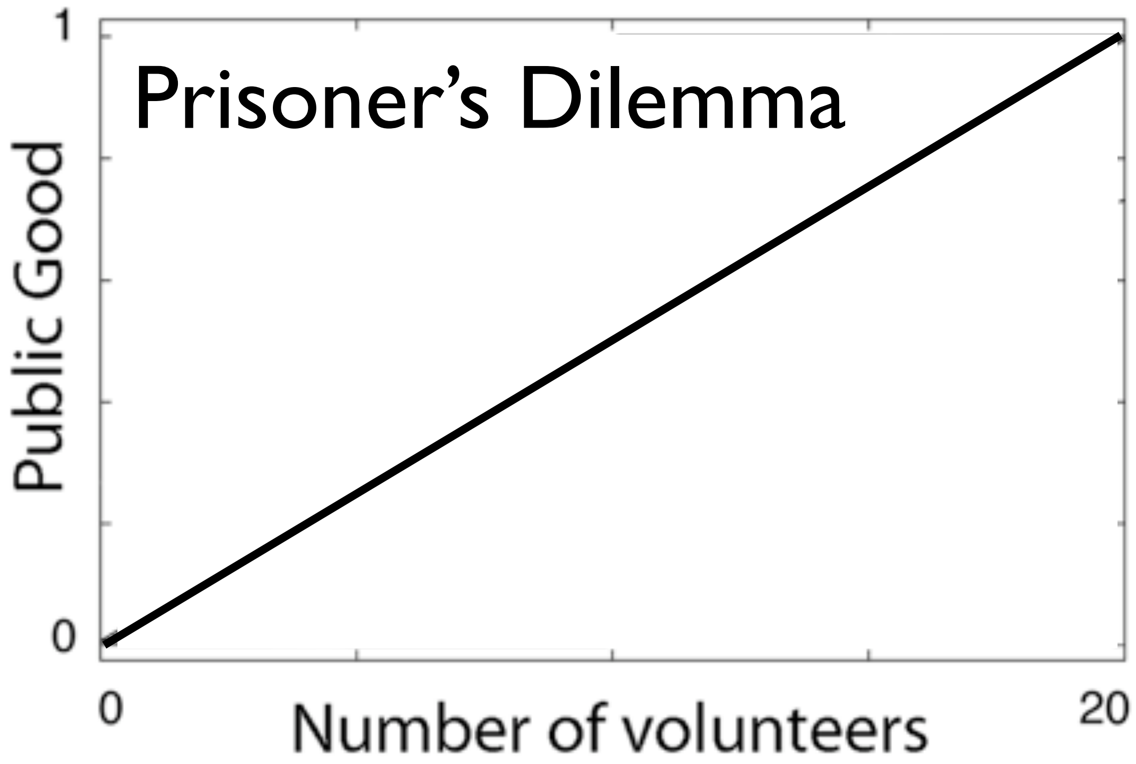
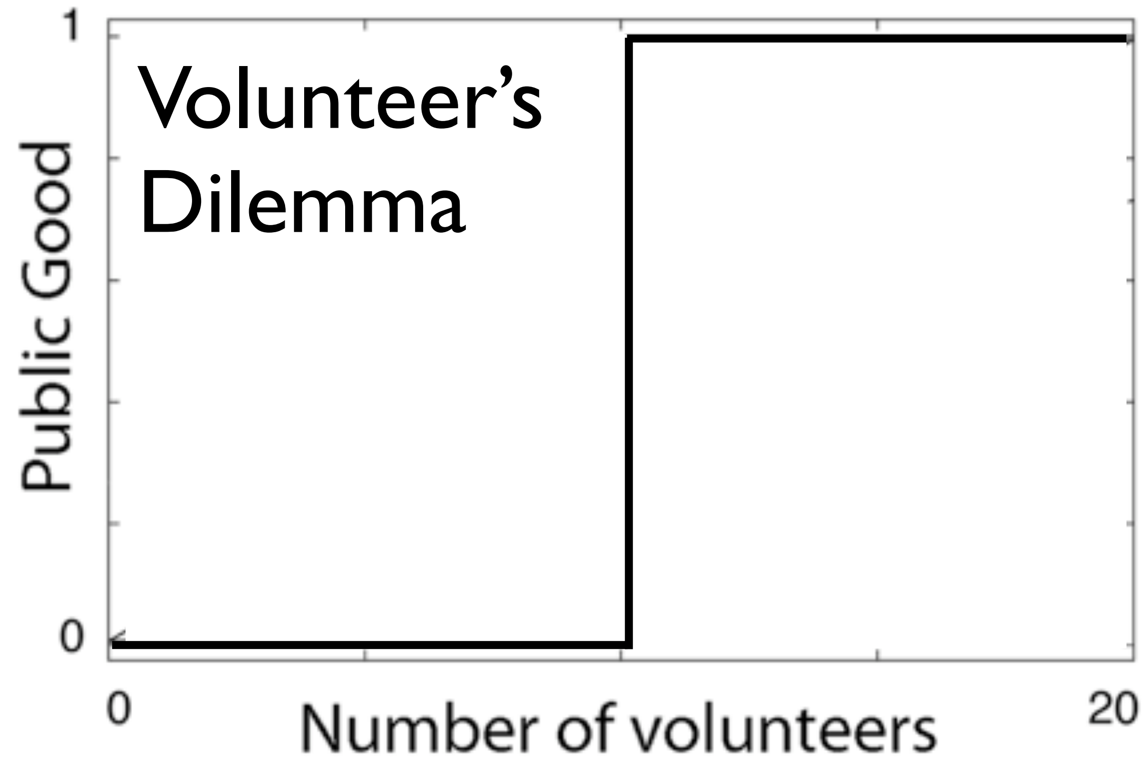




NO MIXED EQUILIBRIUM

Only Defectors

unless:

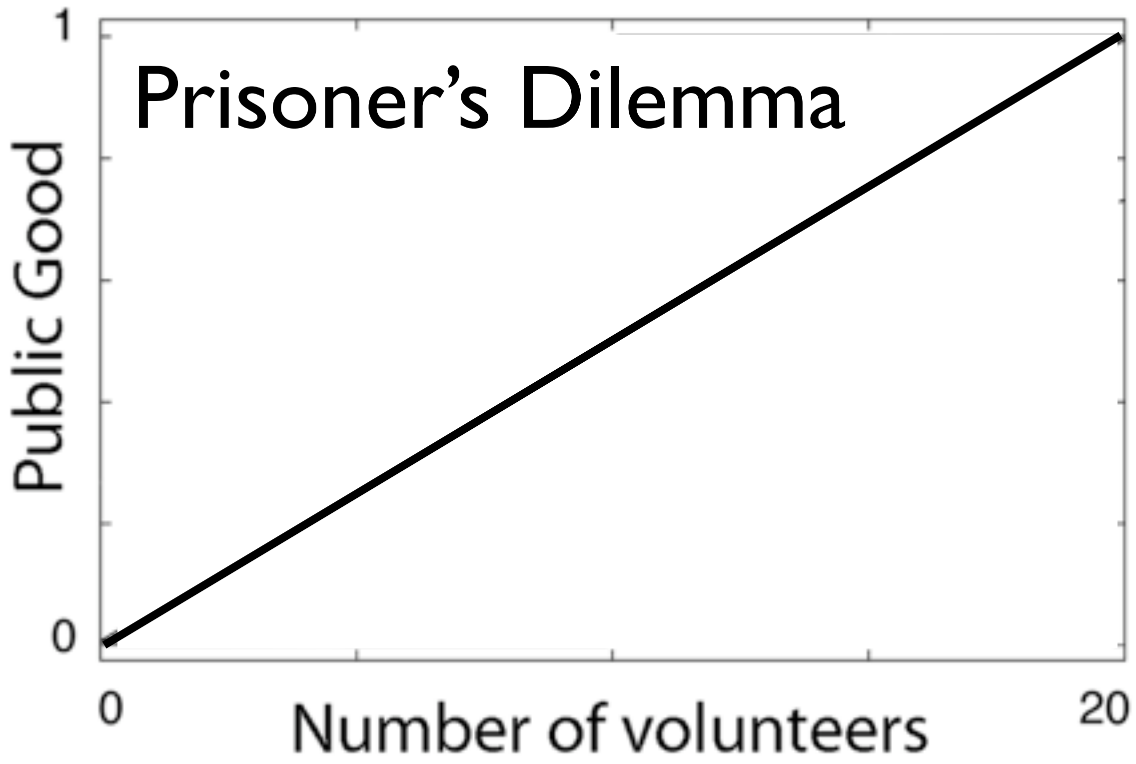
relatedness | **iterations**

MIXED EQUILIBRIUM

Coexistence of Cooperators + Defectors

N-person Prisoner's Dilemma

Volunteer's Dilemma



- individuals can be cooperators or defectors
- cooperators pay a contribution c
- sum of contributions multiplied by a factor
- redistributed to all (cooperators and defectors)

N-person Prisoner's Dilemma

(Hamburger 1973, Fox & Guyer 1978)

in Biology

Review TRENDS in Ecology and Evolution Vol.22 No.12 ScienceDirect

The tragedy of the commons in evolutionary biology

Rankin, Bargum& Kokko
Trends in Ecology and Evolution 2007

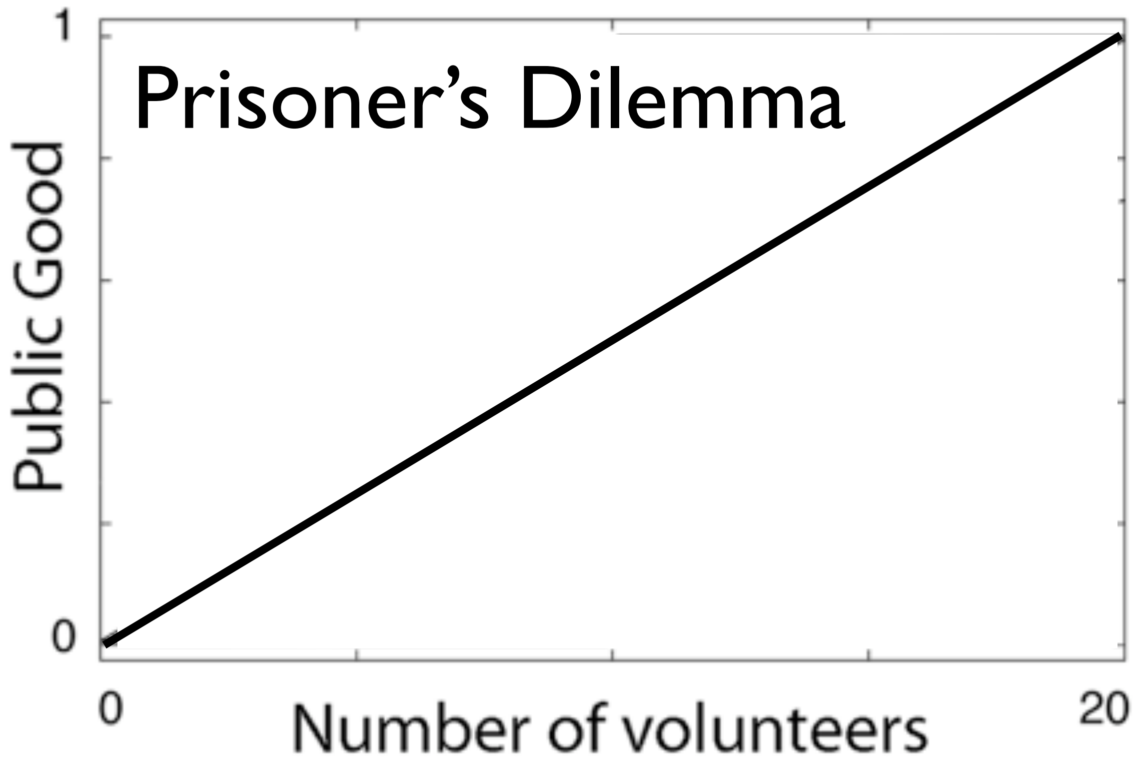
The tragedy of the commons encompasses what social scientists call a public goods game, or an N-person Prisoner's Dilemma

in the Social Sciences

Social dilemmas: the anatomy of cooperation

Kollock
Annual Review of Sociology 1998

A common misunderstanding is the assumption that all N-person dilemmas have the structure of an N-person Prisoner's Dilemma



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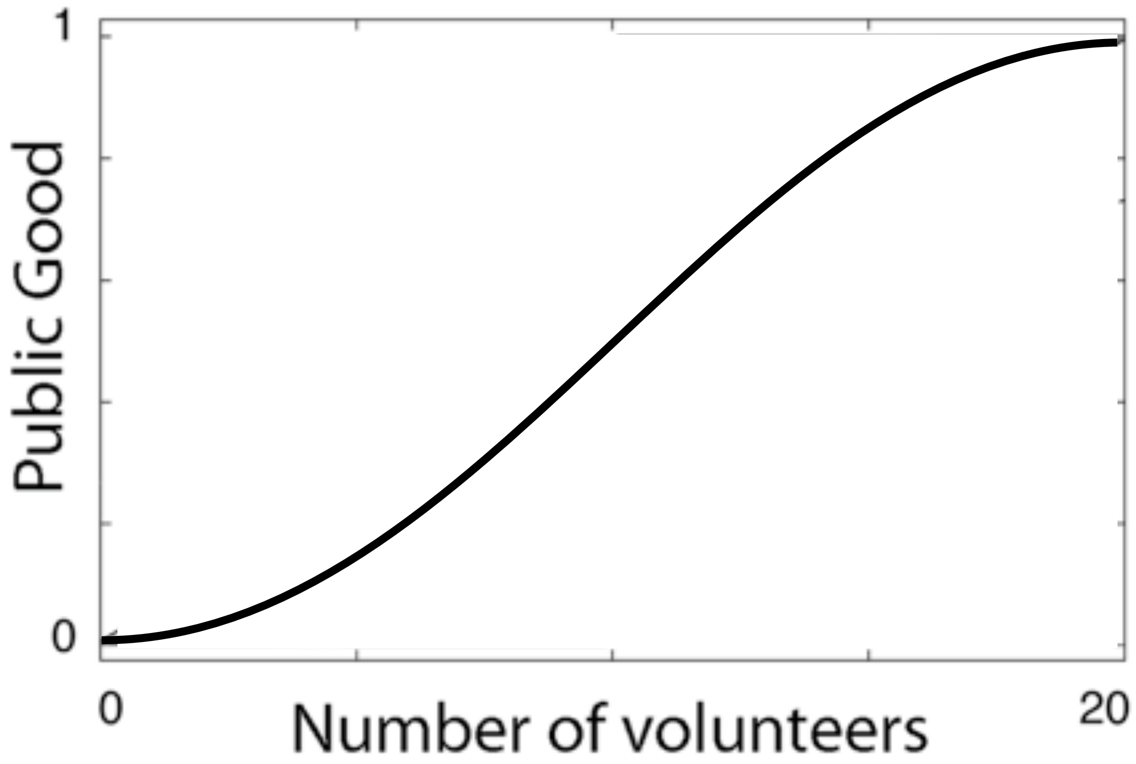
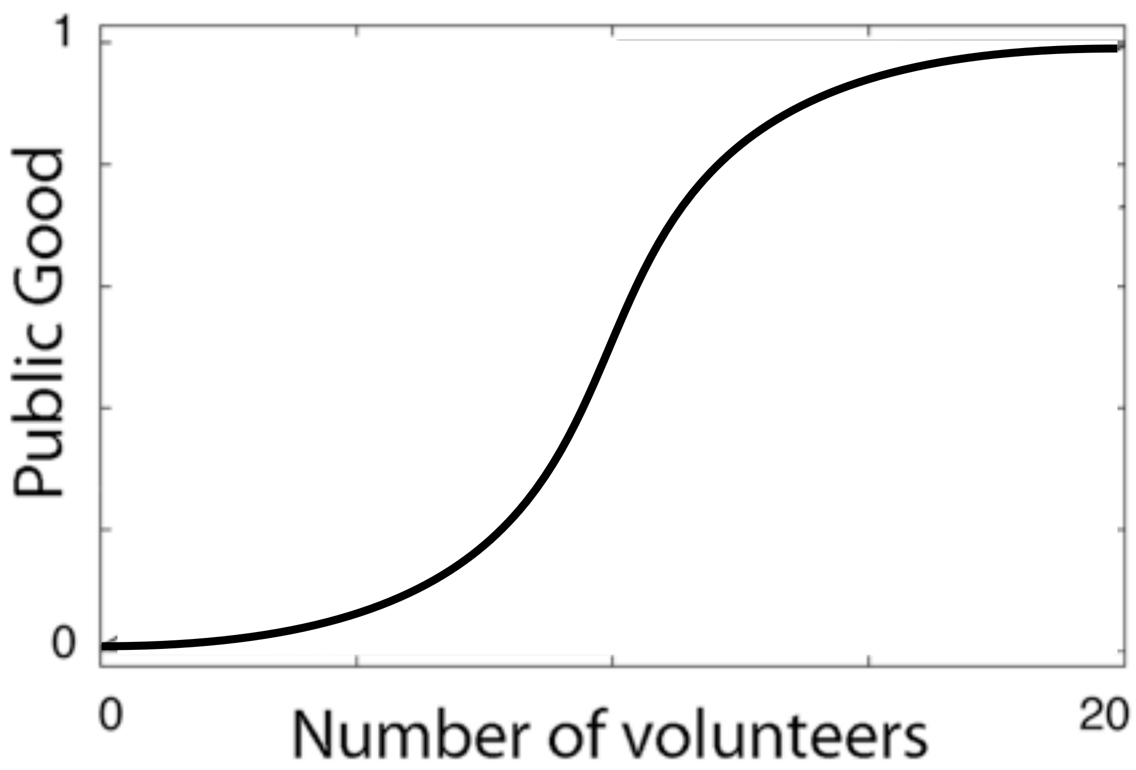


1
0
Public Good
0
20
Number of volunteers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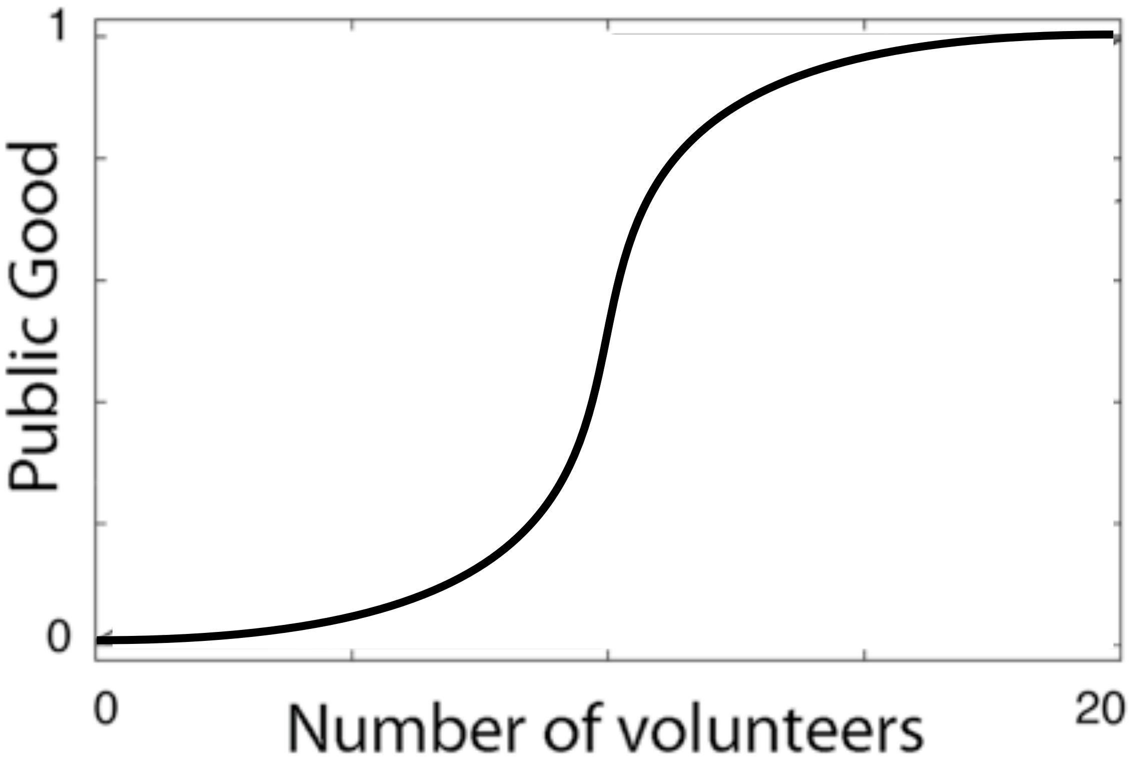
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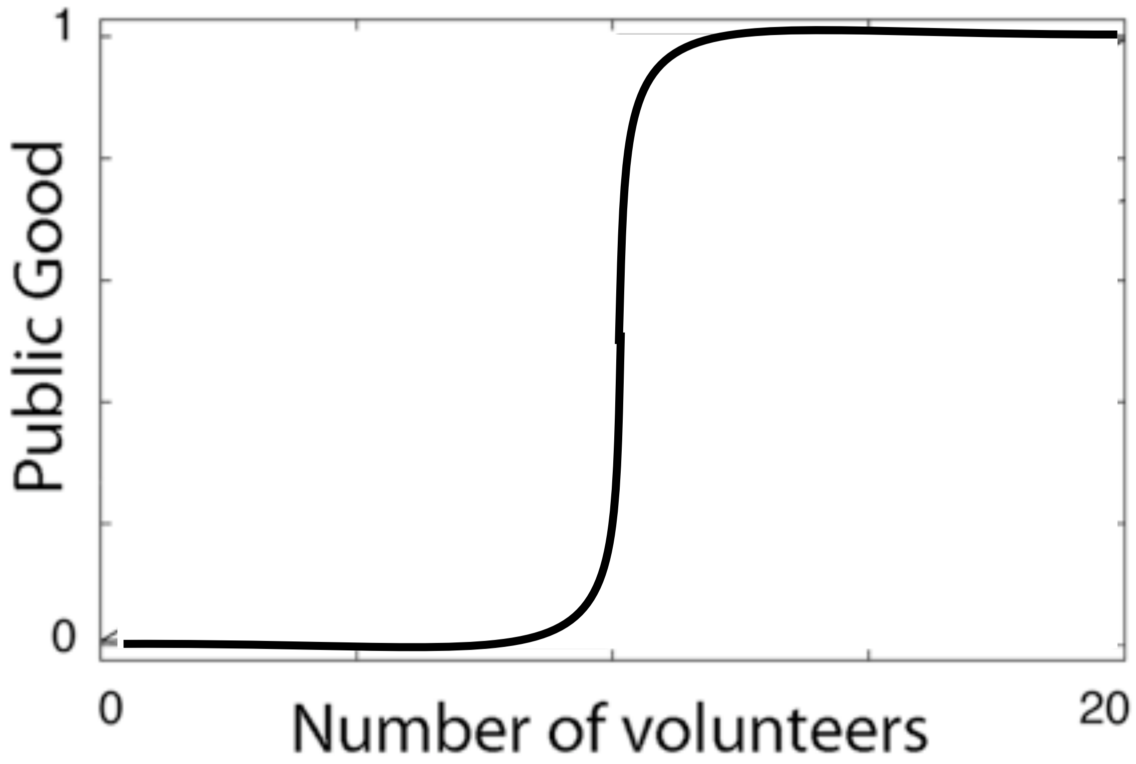
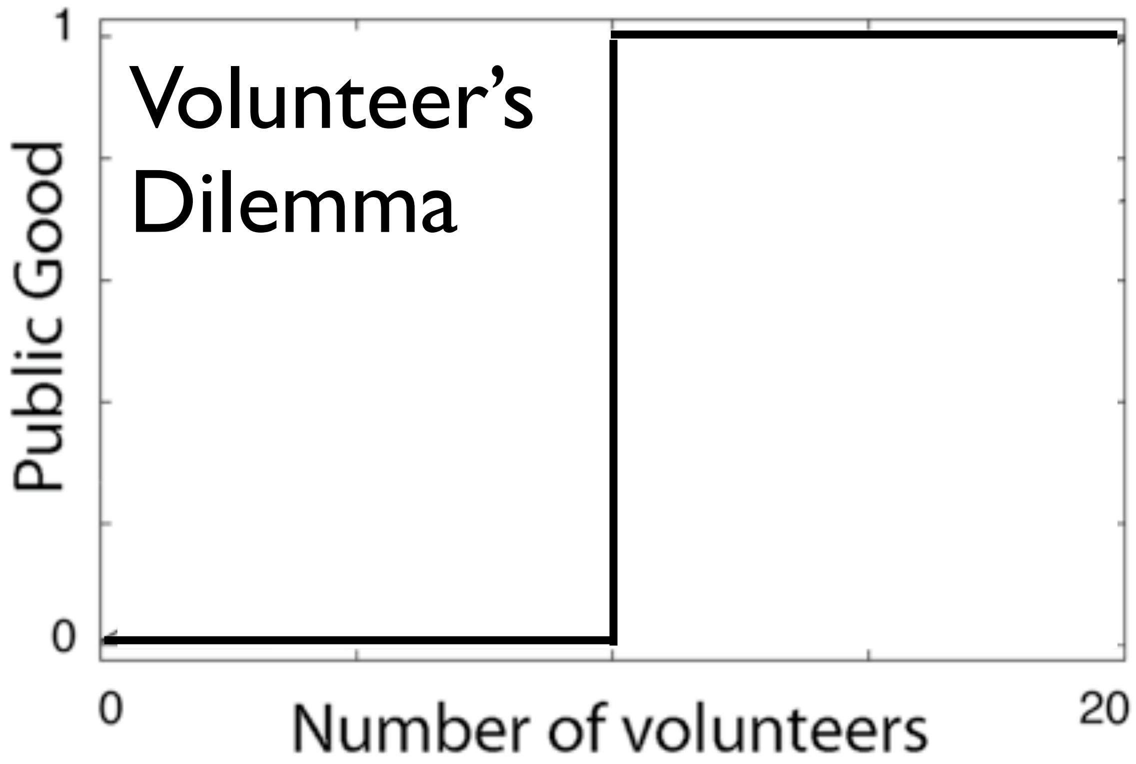
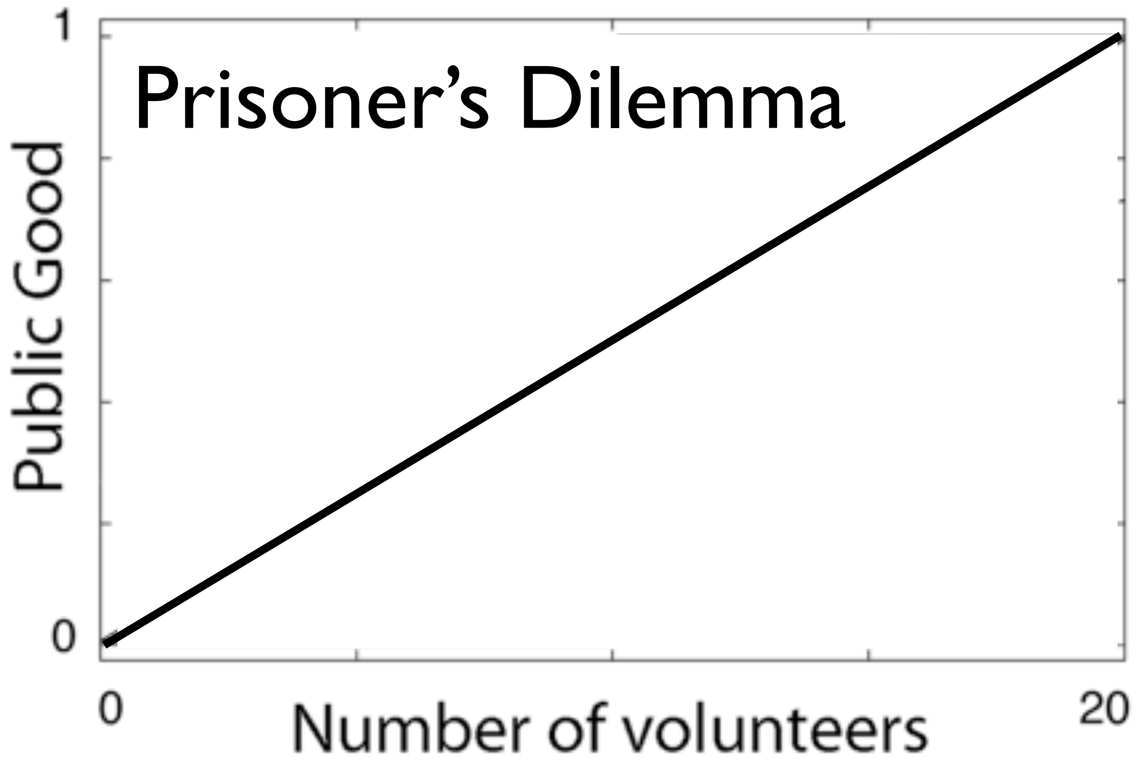
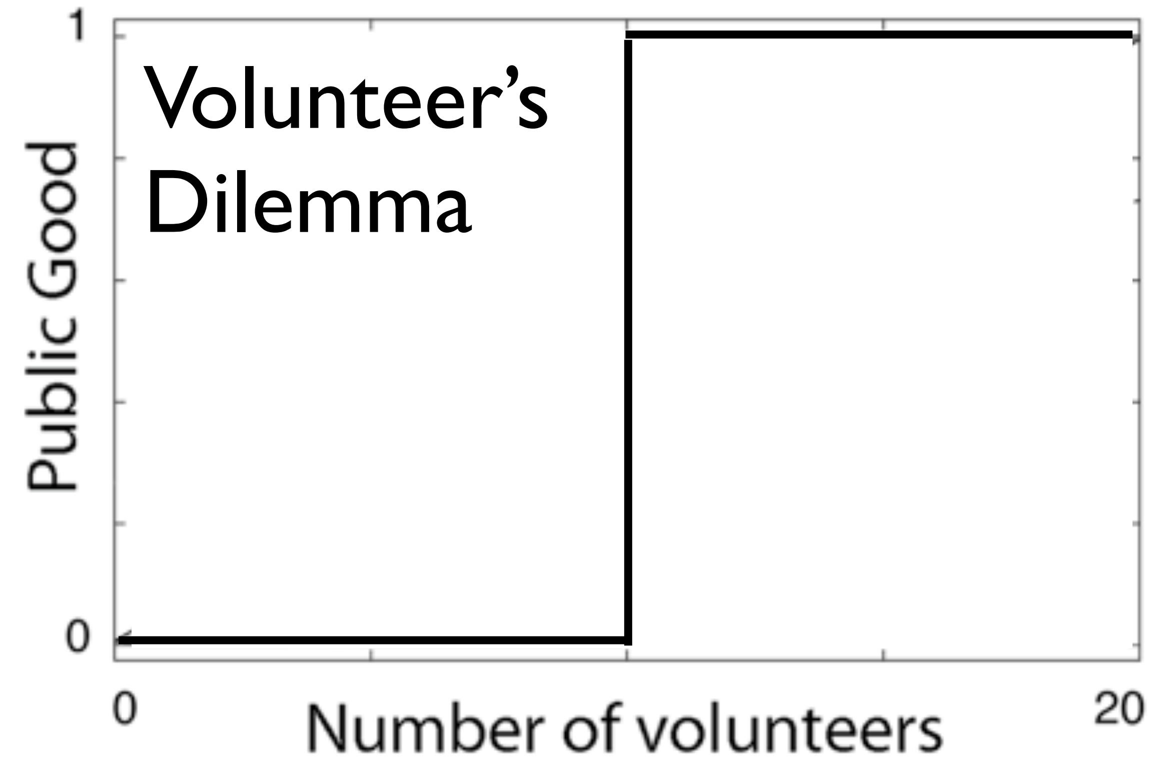

1
Public Good
0
0
Number of volunteers
20



Volunteer's Dilemma
Public Good
1
0
0
20
Number of volunteers





<u>NO</u> MIXED EQUILIBRIUM

Only Defectors

unless:

relatedness | **iterations**

MIXED EQUILIBRIUM

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probability that j of the other N-1 individuals play *Volunteer*

$$f_j = \binom{N-1}{j} \phi^{j} (1-\phi)^{N-1-j}$$

FITNESS:

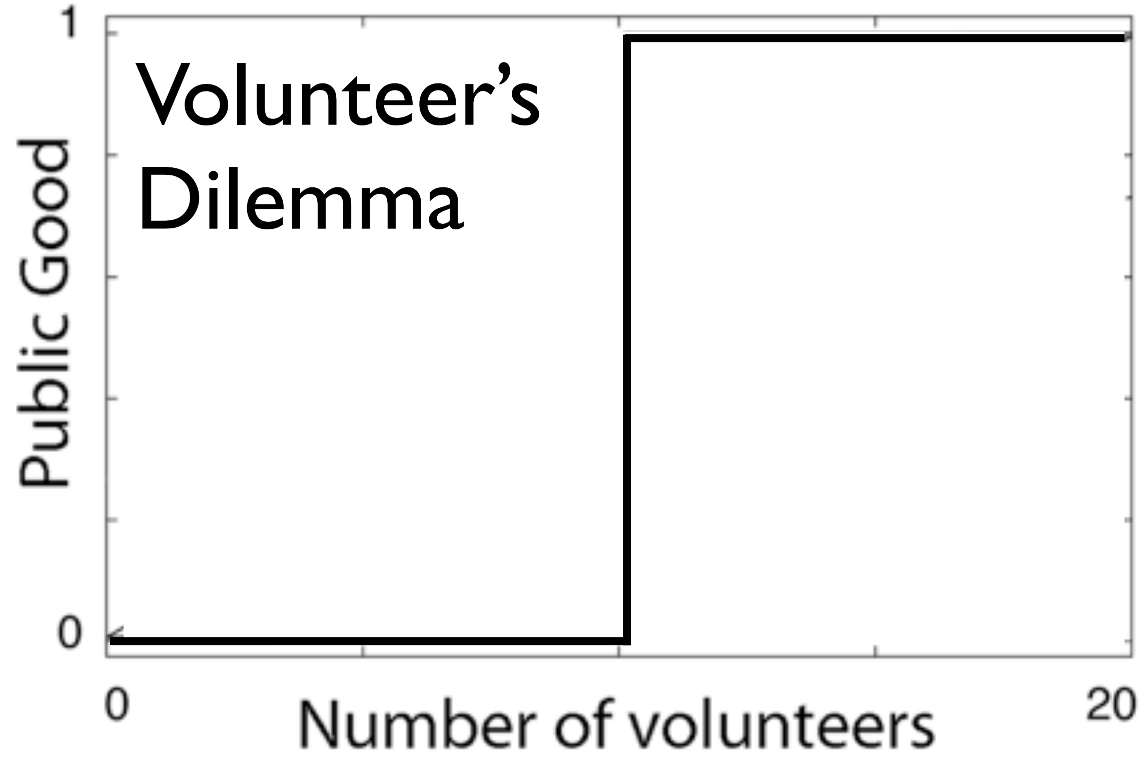
volunteer: $\displaystyle\sum_{j=0}^{N-1} f_j \, B_V(j) - c$

ignore: $\displaystyle\sum_{j=0}^{N-1} f_j \, B_I(j)$

MIXED EQUILIBRIUM

$$c/b = \binom{N-1}{N-k} \gamma^{N-k} (1-\gamma)^{N-1-(N-k)}$$

Public Goods Games as a Volunteer's Dilemma



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MIXED EQUILIBRIUM

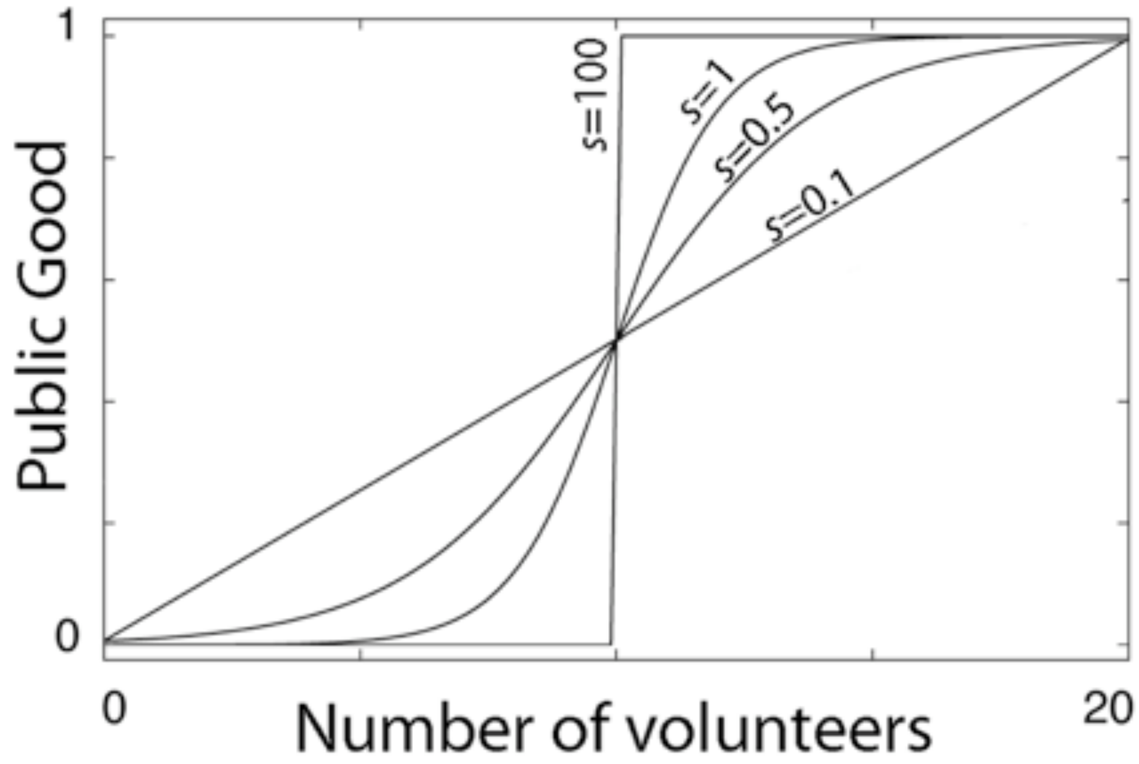
$$c/b = \binom{N-1}{N-k} \gamma^{N-k} (1-\gamma)^{N-1-(N-k)}$$

A Generalised Public Goods Game

Benefit due to the public good:

$$\beta_{\mathrm{I}}(j) = \frac{1}{1+e^{-s(j-k)}}$$

$$\beta_{\mathrm{V}}(j) = \frac{1}{1+e^{-s(j-k+1)}}$$



MIXED EQUILIBRIUM

something complicated....

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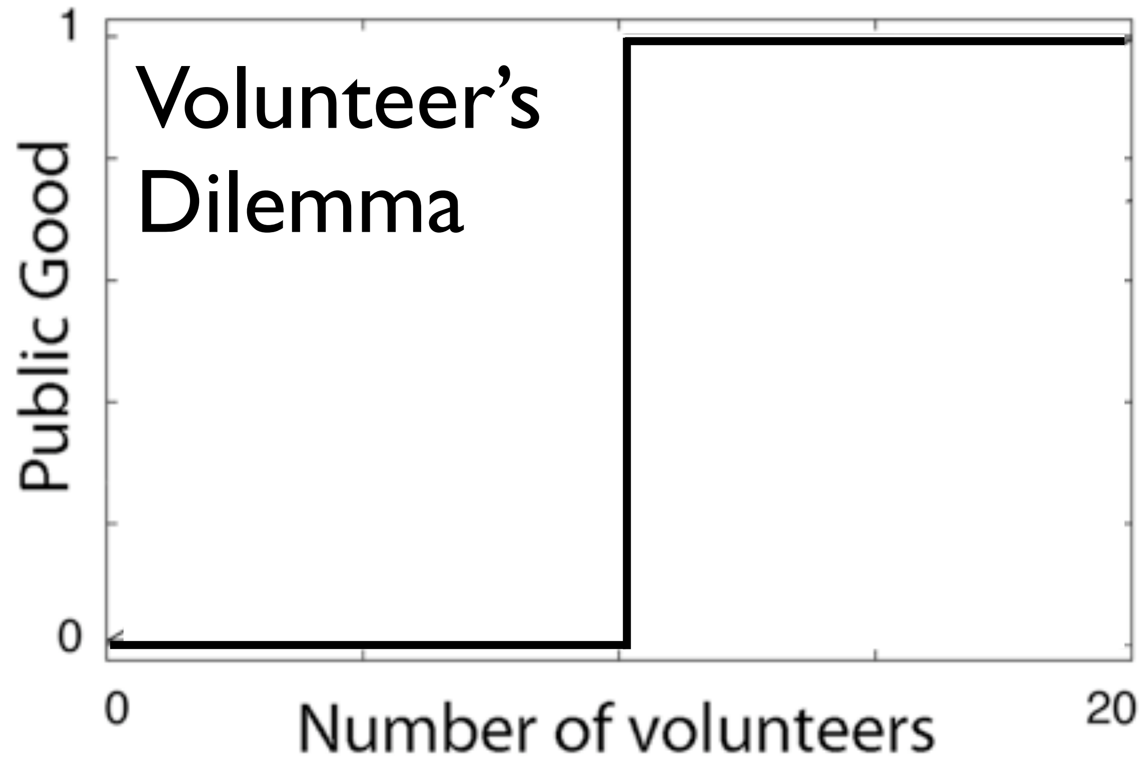

$$s \to \infty$$

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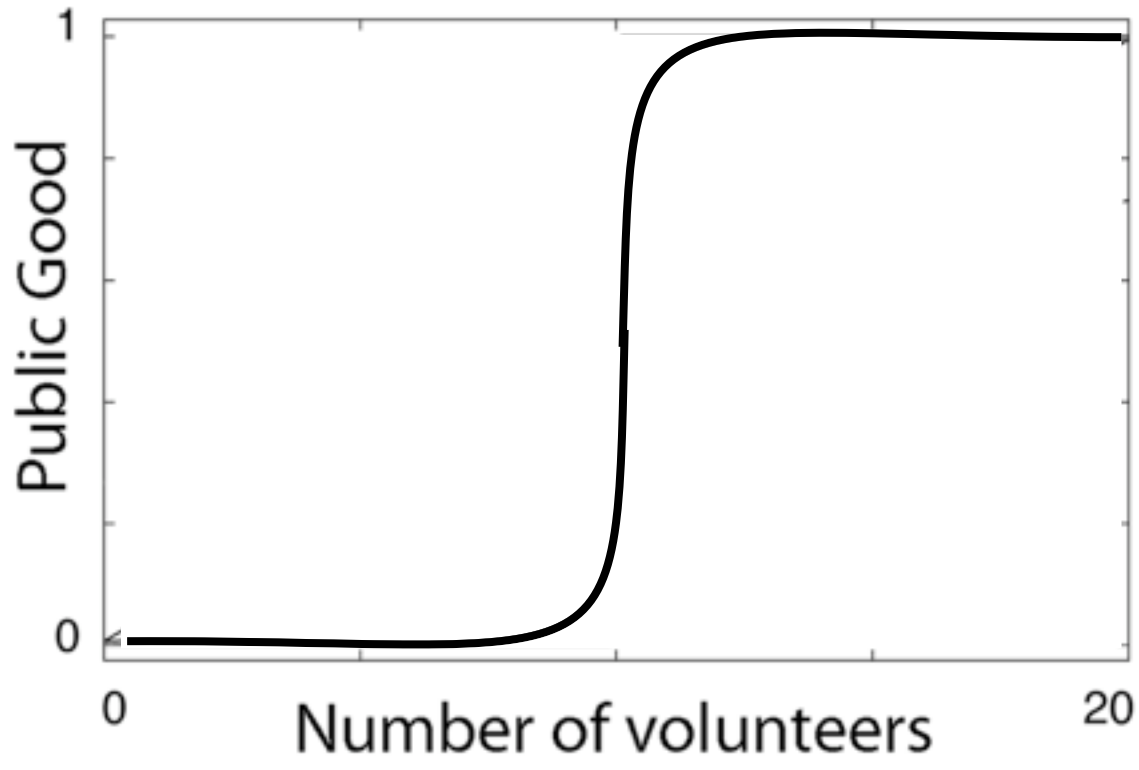


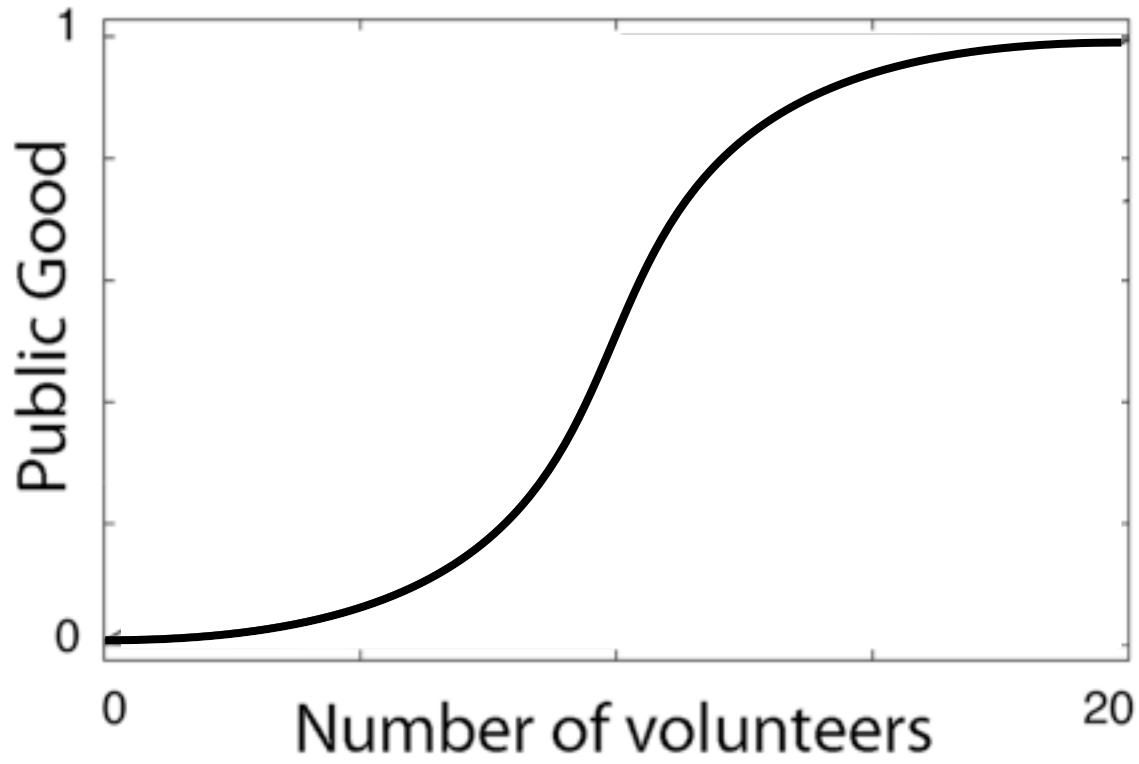
S=100

A Generalised Public Goods Game

Benefit due to the public good:

$$\beta_{\mathrm{I}}(j) = \frac{1}{1+e^{-s(j-k)}}$$

$$\beta_{\mathrm{V}}(j) = \frac{1}{1+e^{-s(j-k+1)}}$$



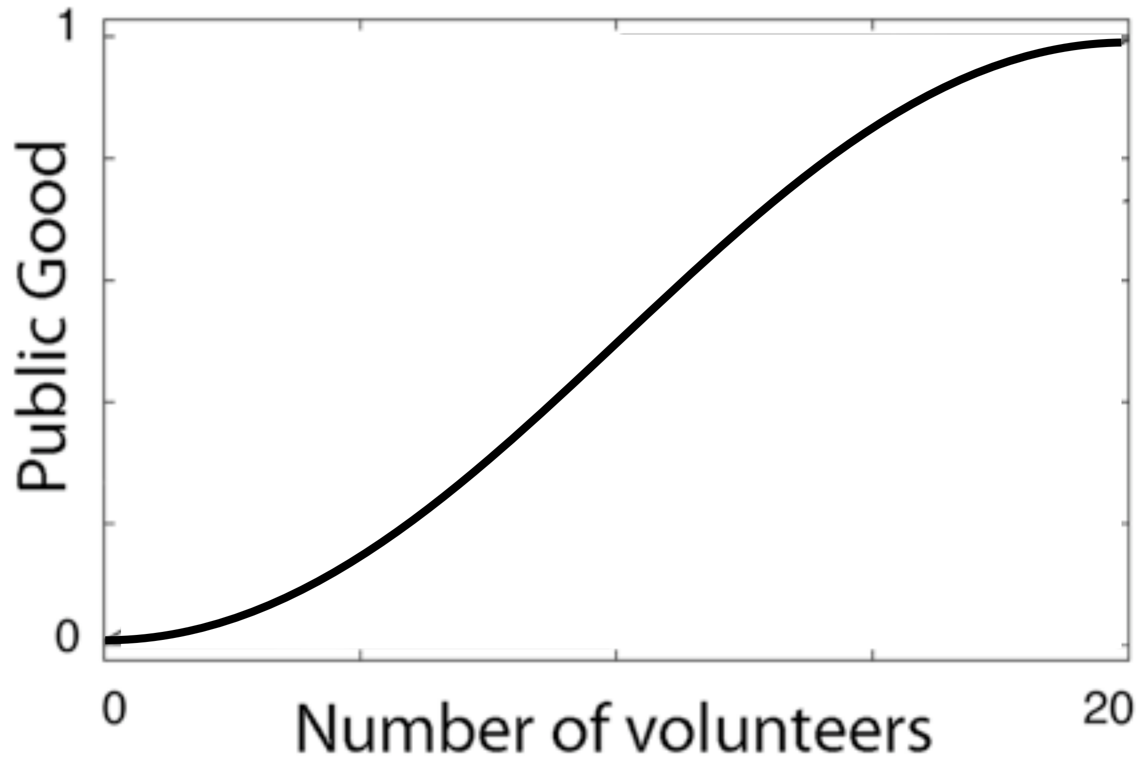
S=10

A Generalised Public Goods Game

Benefit due to the public good:

$$\beta_{\mathrm{I}}(j) = \frac{1}{1+e^{-s(j-k)}}$$

$$\beta_{\mathrm{V}}(j) = \frac{1}{1+e^{-s(j-k+1)}}$$



S=1

A Generalised Public Goods Game

Benefit due to the public good:

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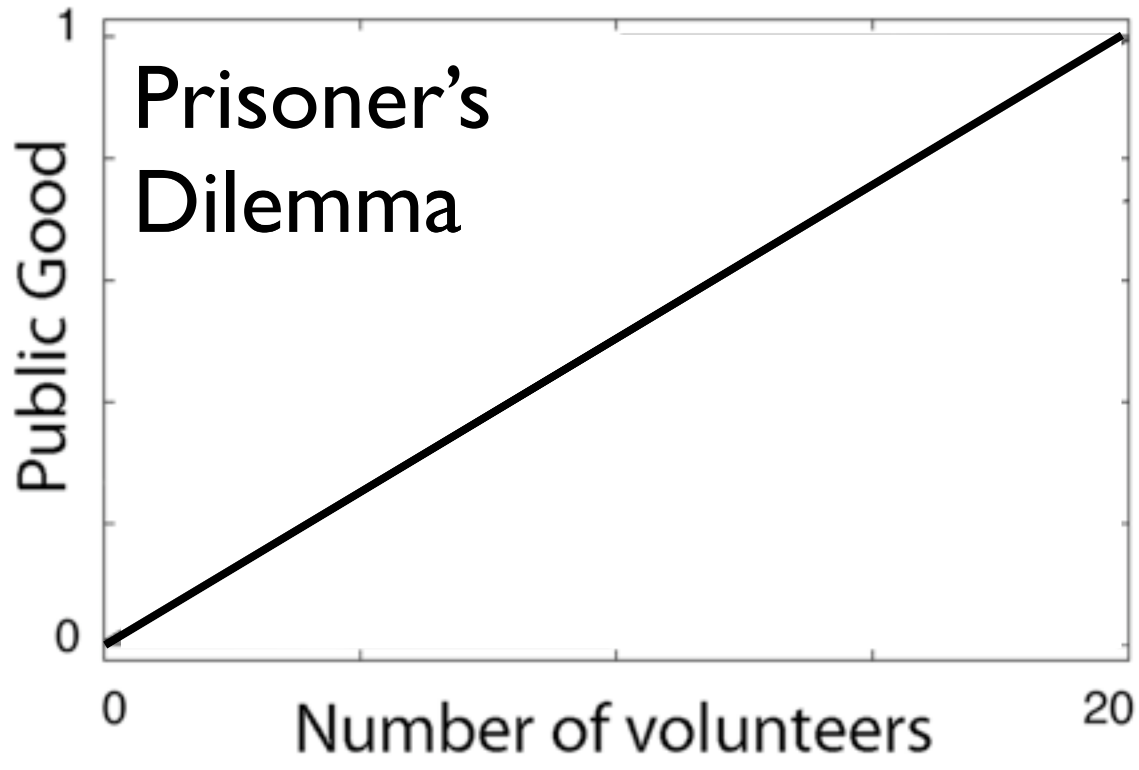
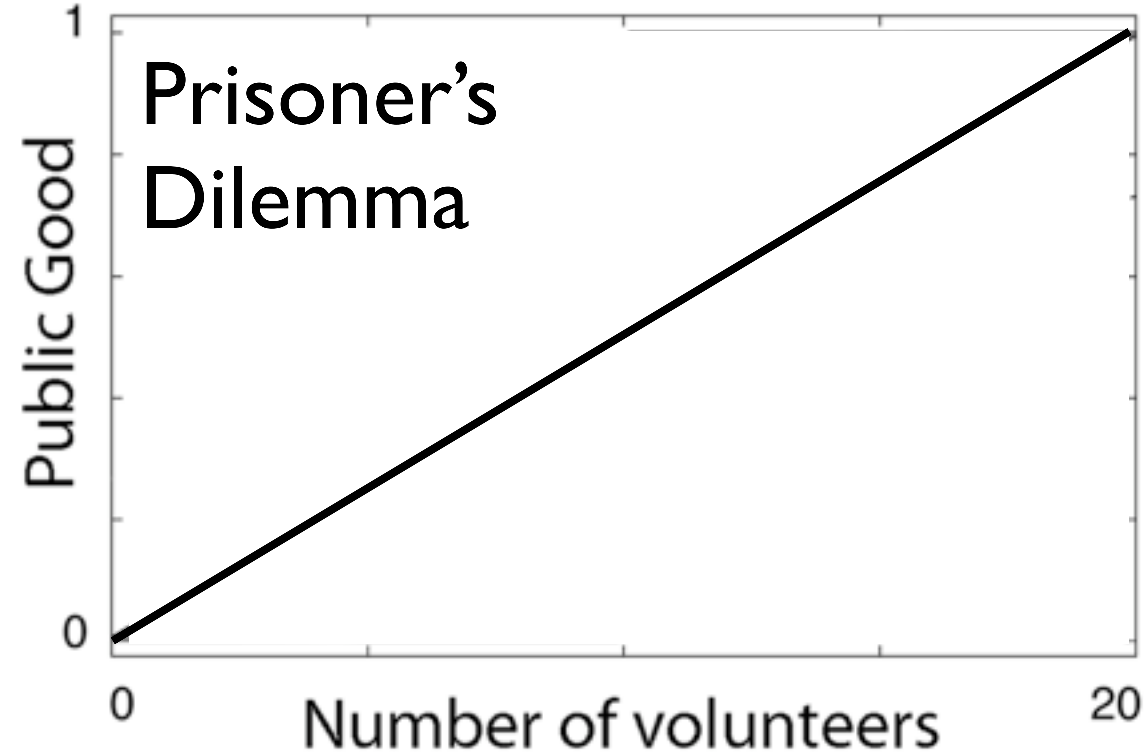
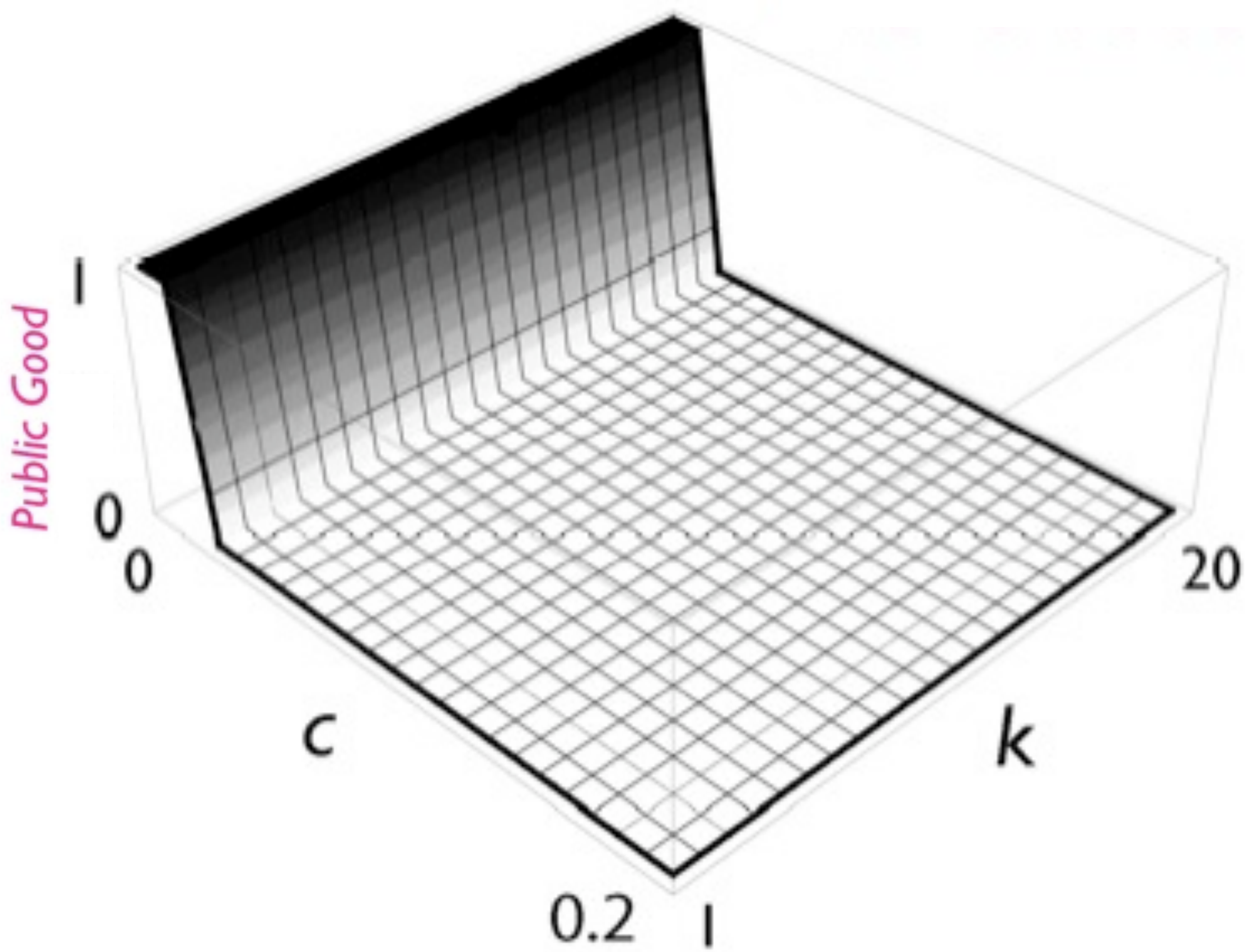
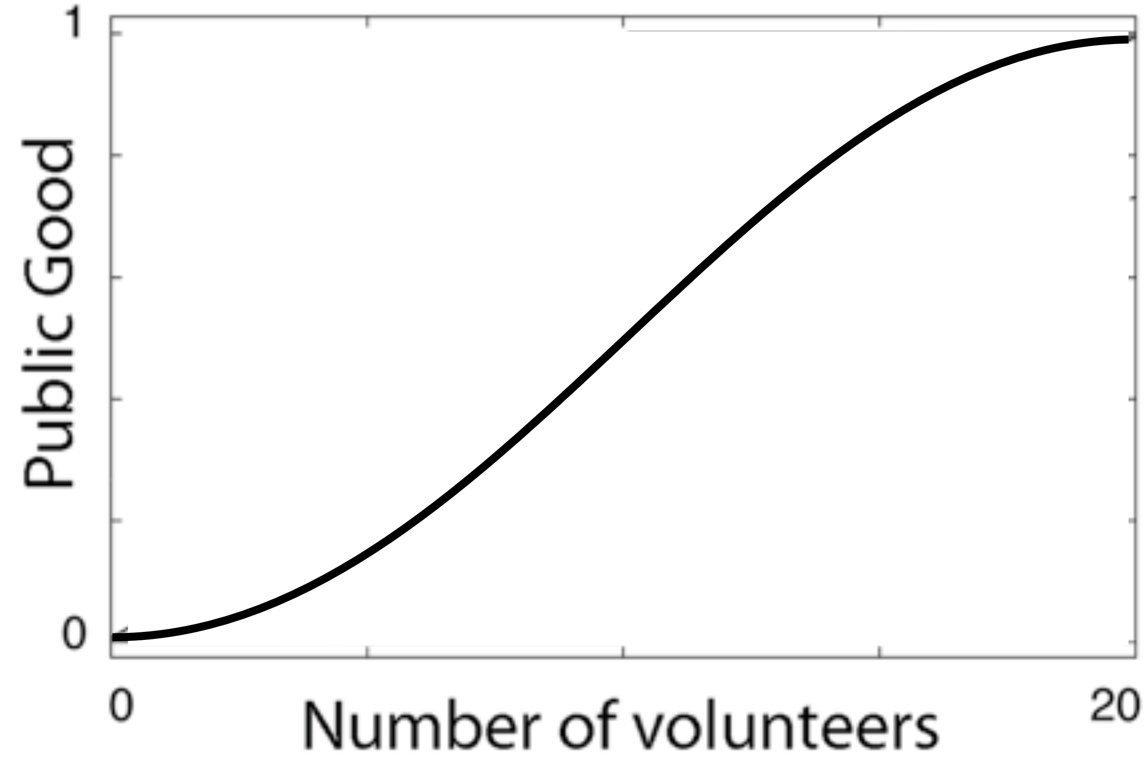


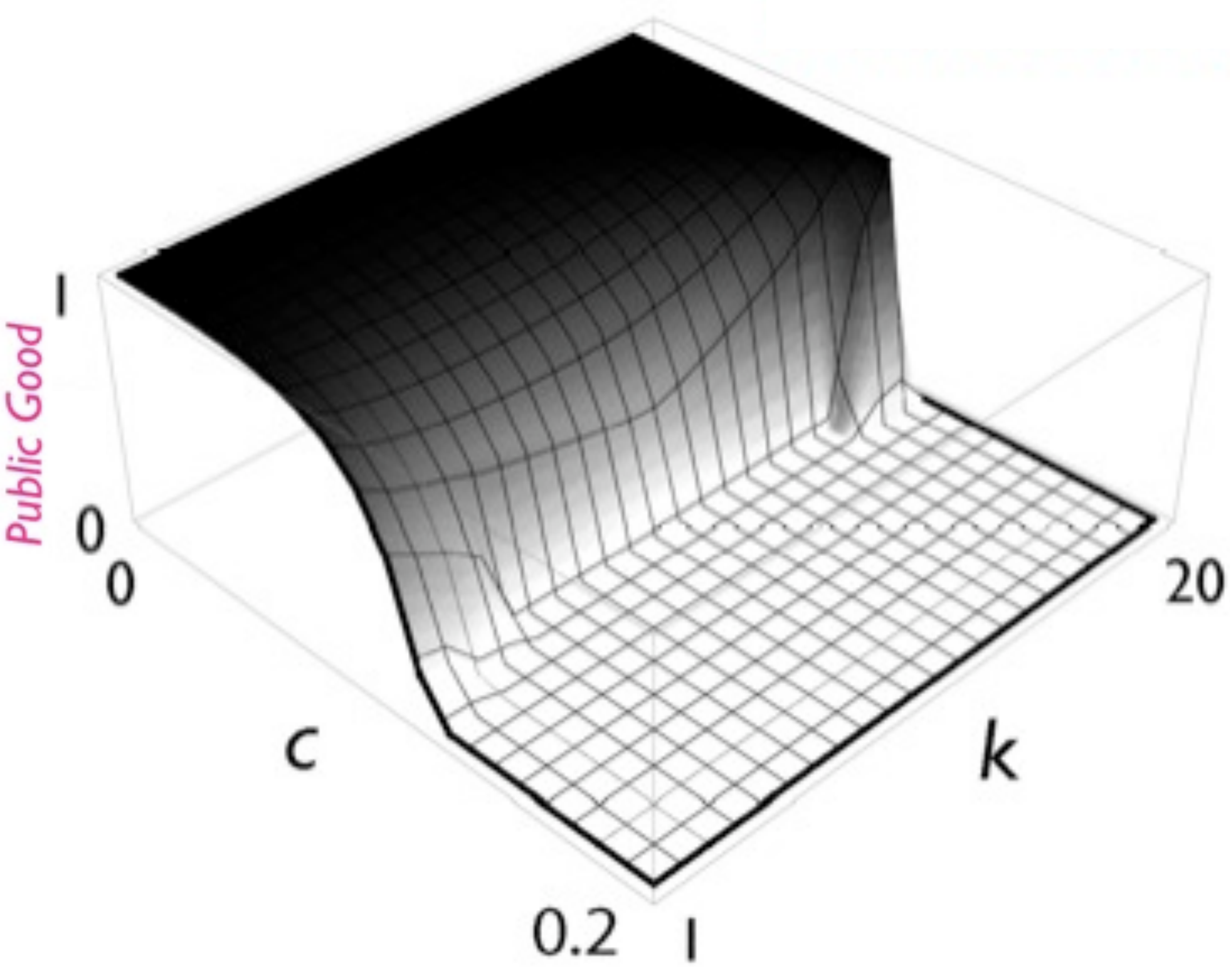
$$s \rightarrow 0$$


1
Prisoner's
Dilemma
Public Good
0
0
Number of volunteers
20

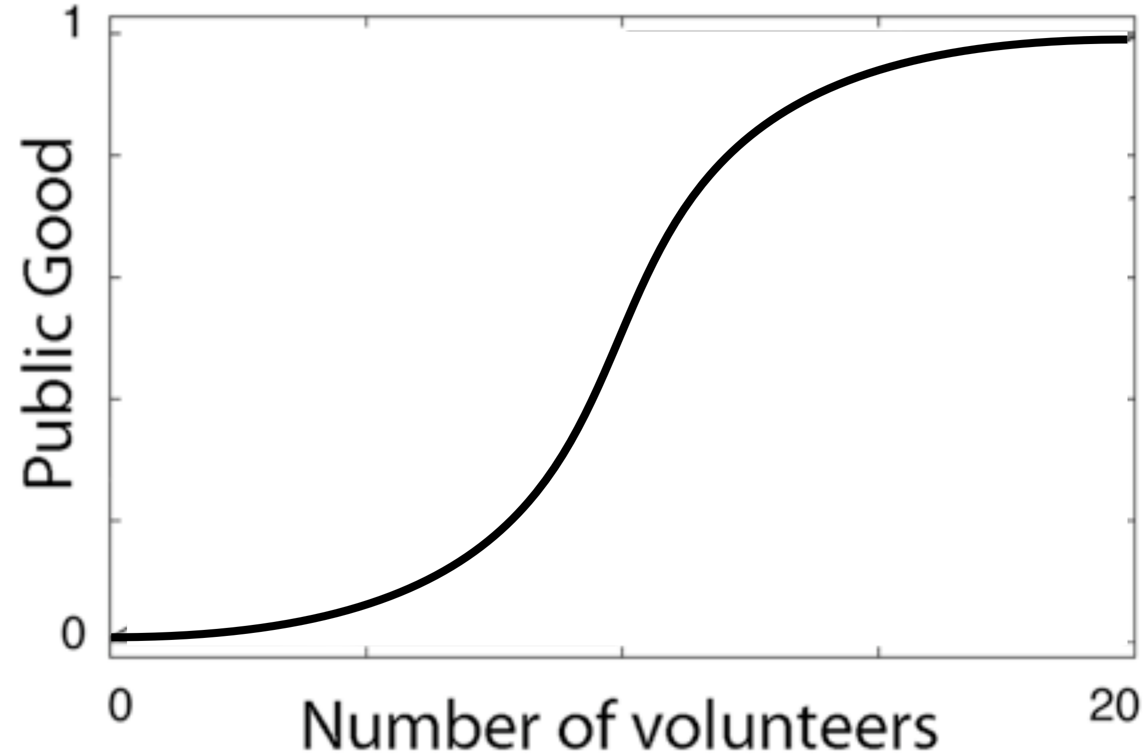

1
Public Good
0
0
c
k
20
0.2
1

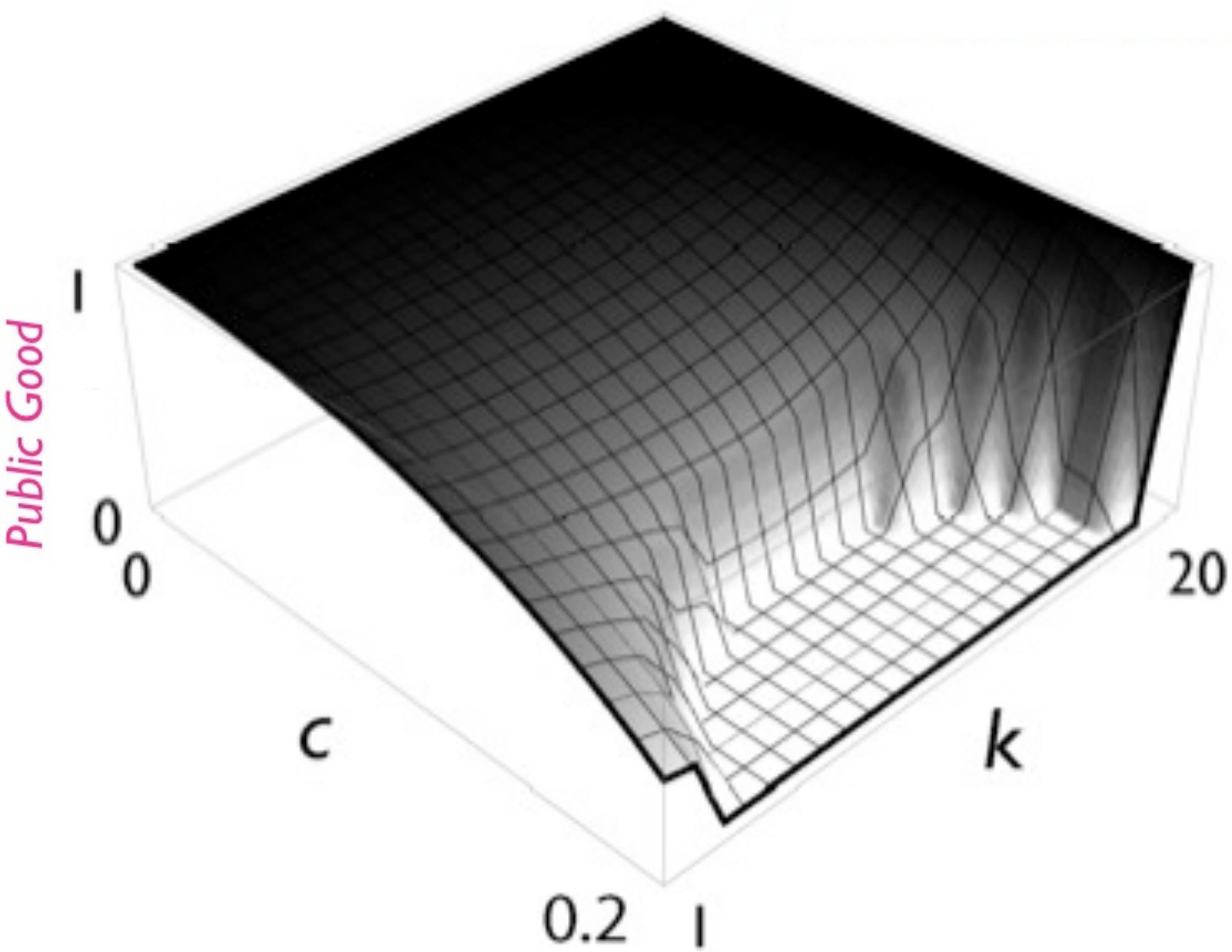


1
0
Public Good
0
20
Number of volunteers



Public Good
1
0
0
20
c
k
0.2
1


1
0
Public Good
0
Number of volunteers
20


Public Good
1
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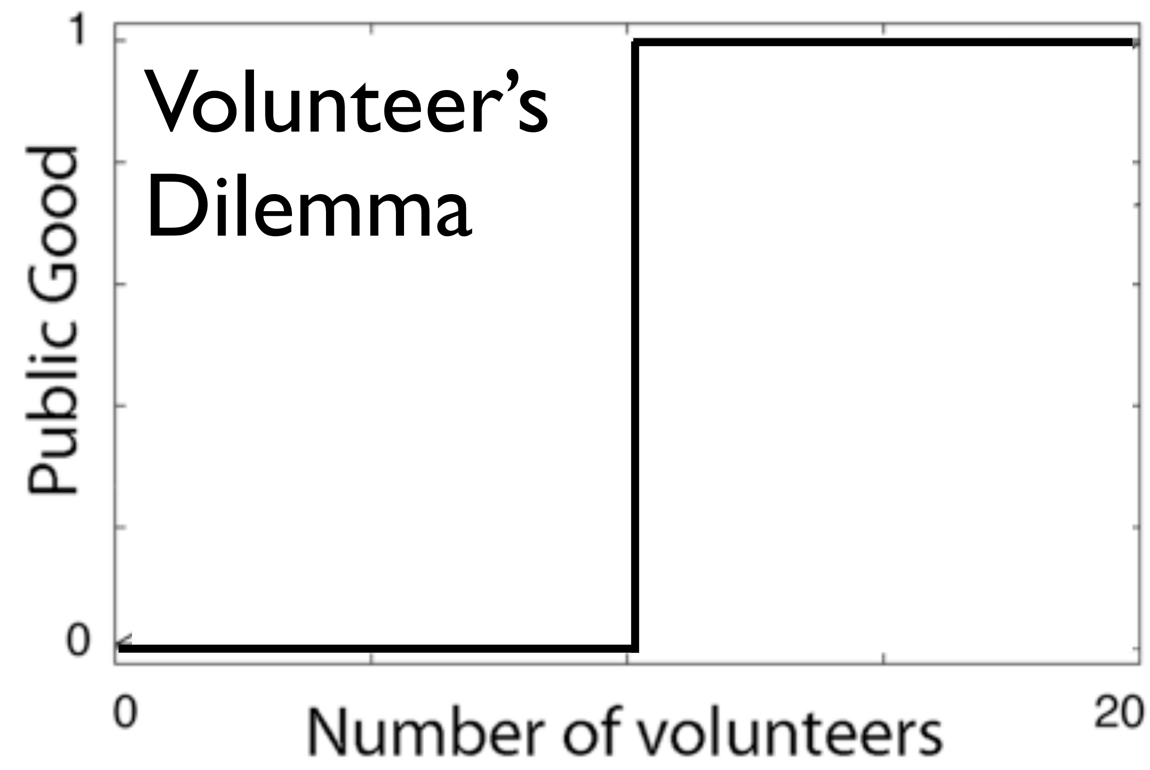
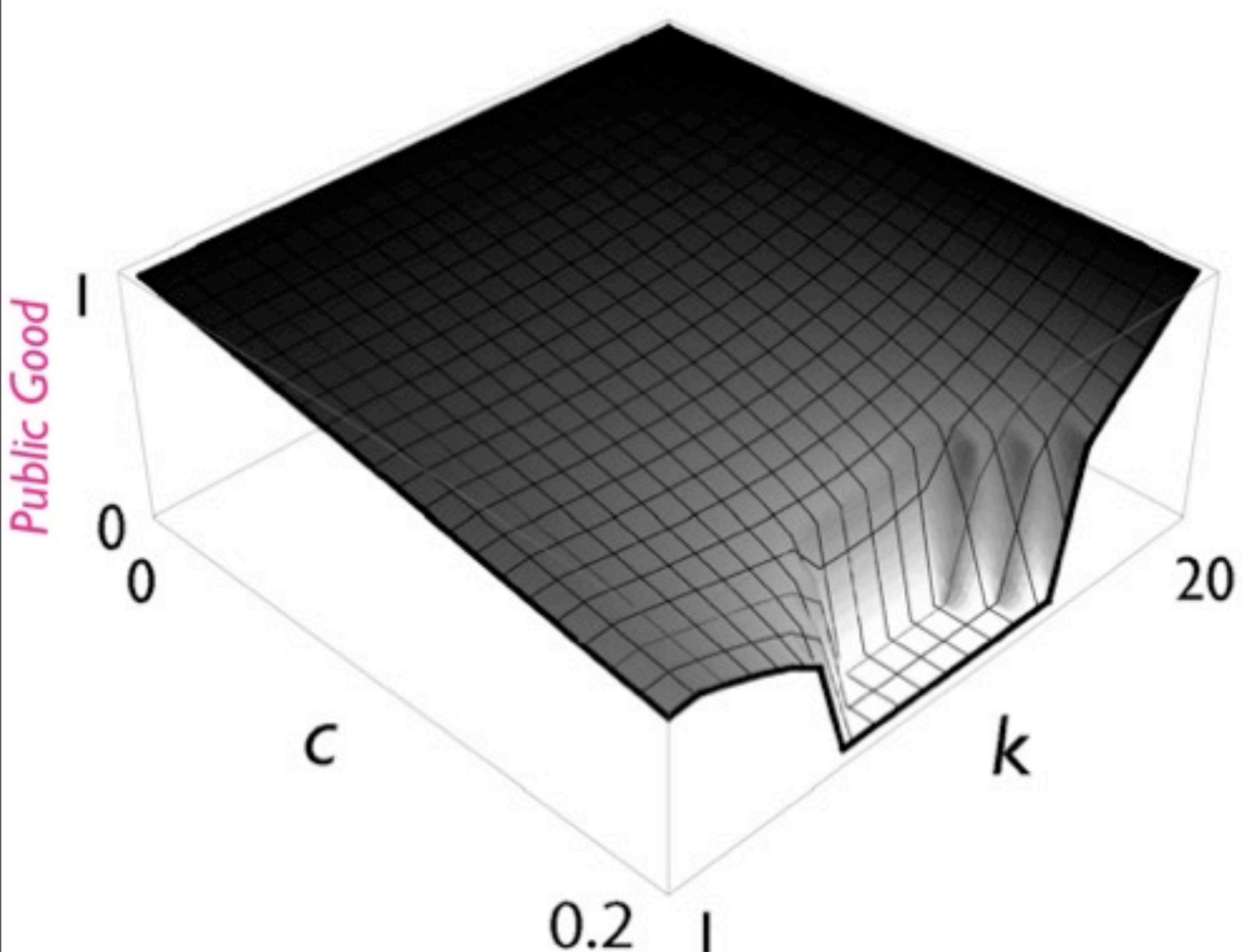
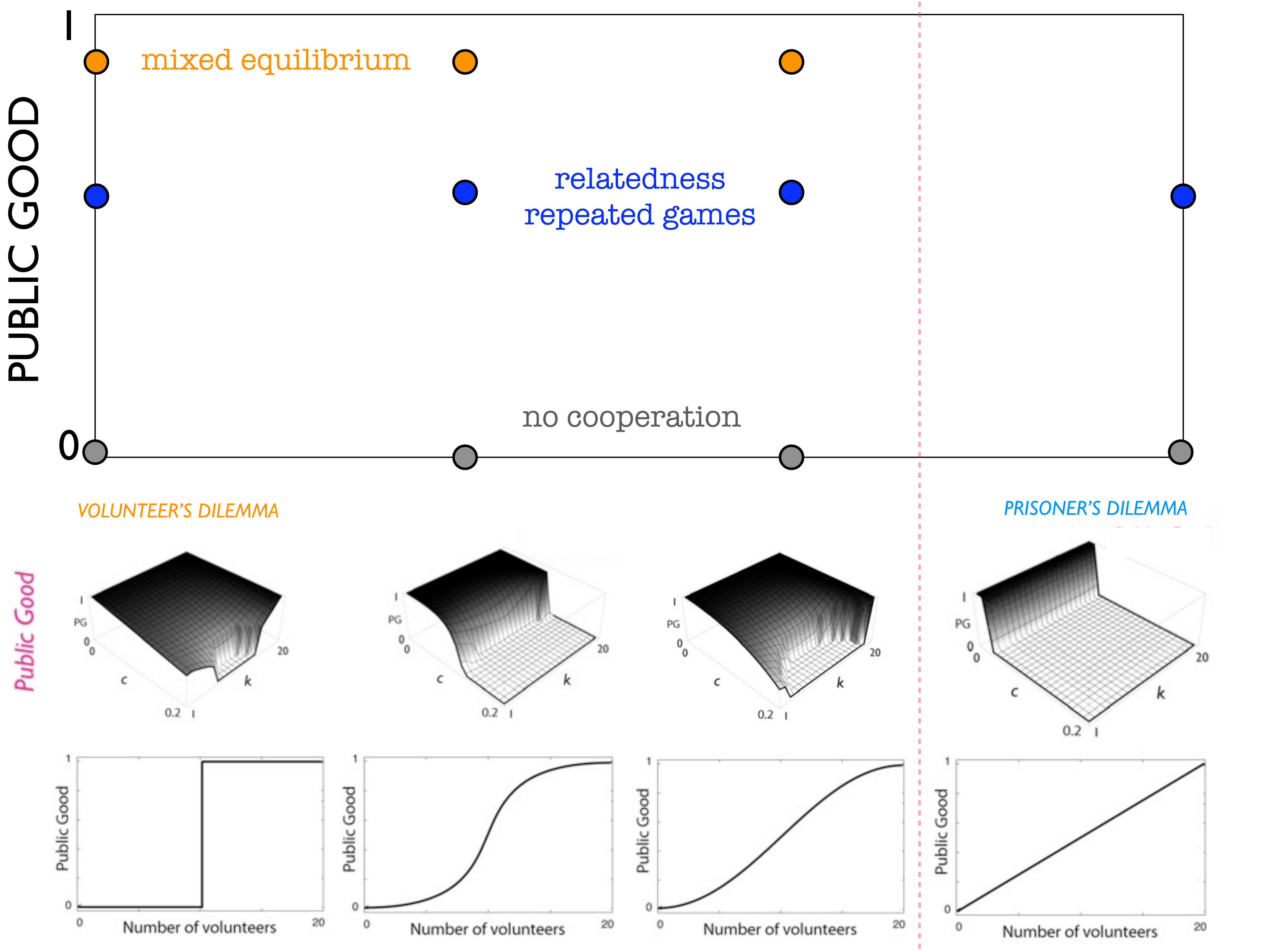
c
k
0.2
1


1
Volunteer's
Dilemma
Public Good
0
0
Number of volunteers
20


Public Good
1
0
0
20
c
k
0.2
1



PUBLIC GOOD
1
0
mixed equilibrium
relatedness
repeated games
no cooperation
VOLUNTEER'S DILEMMA
PRISONER'S DILEMMA
Public Good
PG
c
k
0.2
20
Public Good
Number of volunteers

MIXED EQUILIBRIUM

Coexistence of Cooperators + Defectors

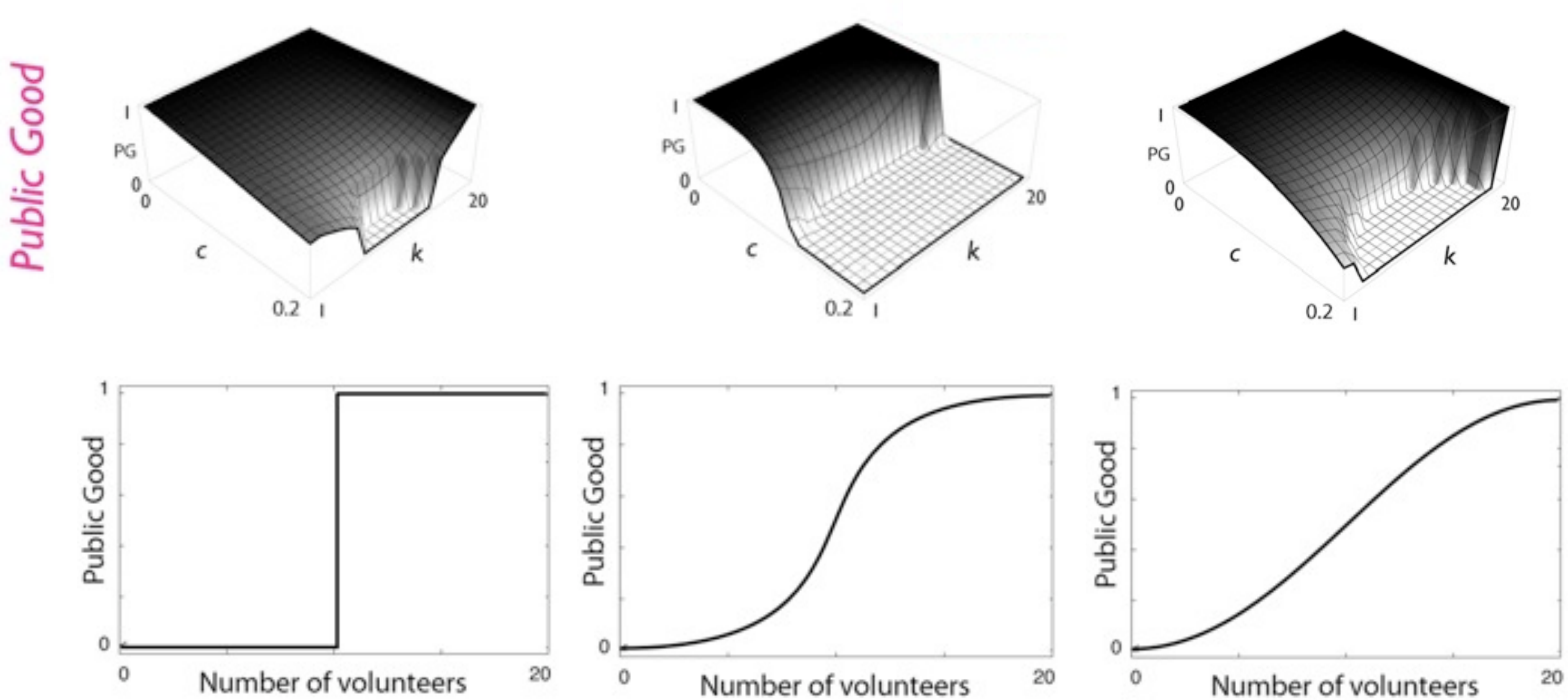
Cooperation **without relatedness | iterations**
Practical solutions to increase cooperation

NO
MIXED EQUILIBRIUM

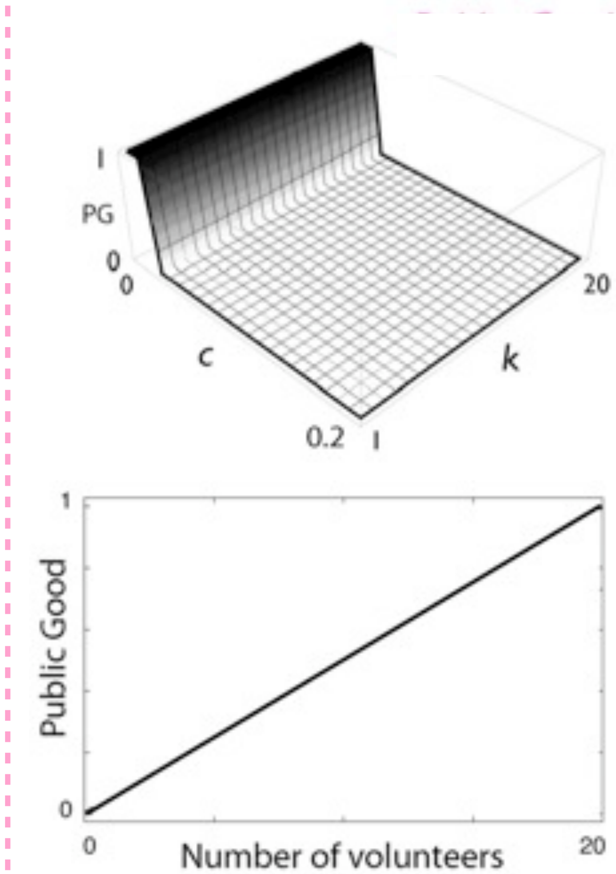
Only Defectors

unless:
relatedness | iterations

VOLUNTEER'S DILEMMA



PRISONER'S DILEMMA



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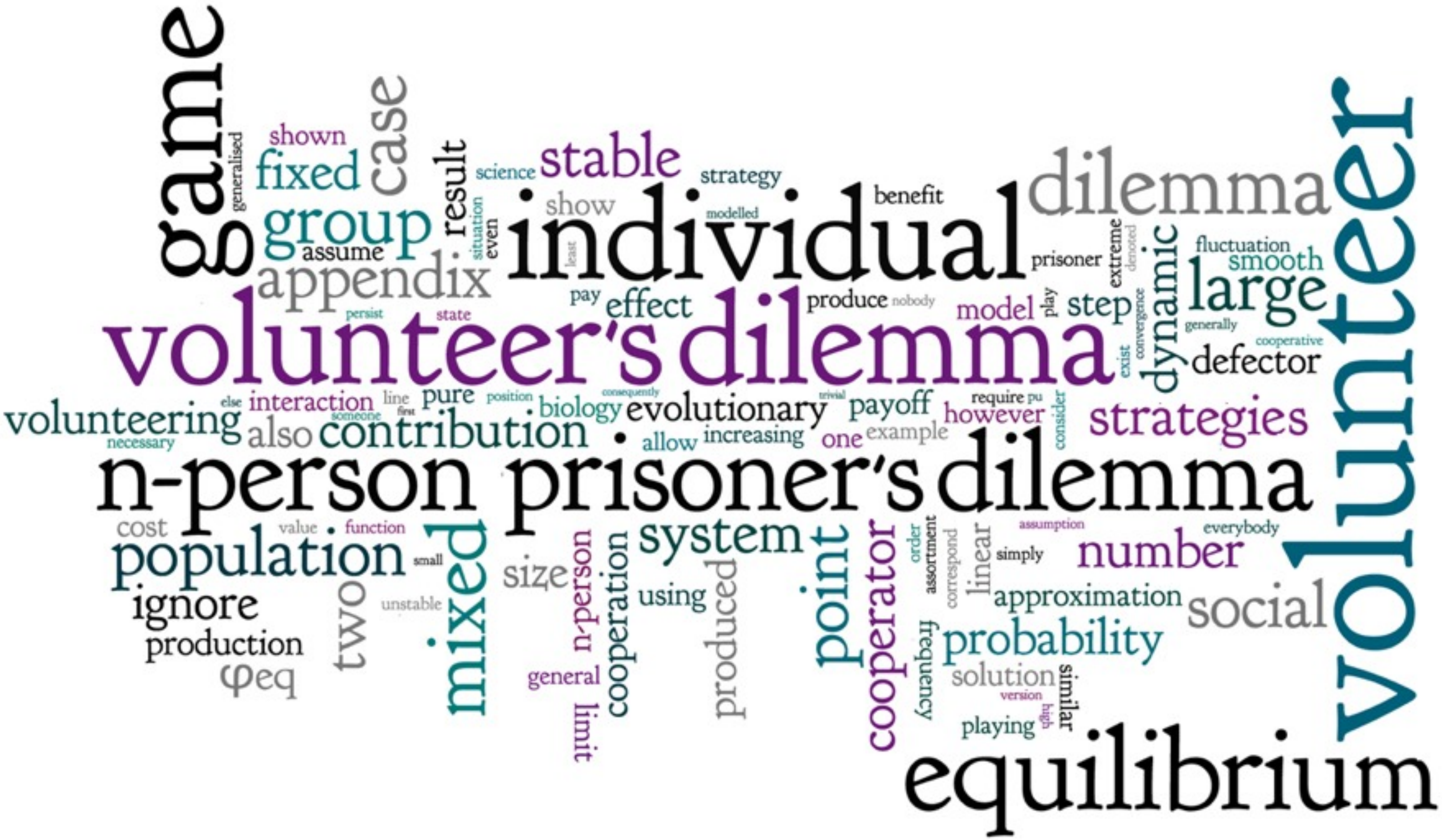
Thank You


game
generalised
shown
fixed
group
assume
case
result
science
stable
strategy
show
modelled
benefit
individual
dilemma
appendix
situation
even
least
prisoner
extreme
denoted
dynamic
fluctuation
smooth
large
pay
effect
produce
nobody
model
play
step
convergence
generally
cooperative
persist
state
volunteer's dilemma
exist
defector
volunteer
volunteering
necessary
else
interaction
someone
line
first
pure
position
biology
consequently
evolutionary
trivial
payoff
require
pu
however
consider
strategies
also
contribution
allow
increasing
one
example
n-person prisoner's dilemma
cost
value
function
population
small
ignore
production
φeq
two
unstable
mixed
size
general
n-person
limit
cooperation
using
system
produced
point
cooperator
order
assortment
frequency
correspond
linear
assumption
simply
everybody
number
approximation
social
probability
solution
version
similar
high
playing
equilibrium