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OPERATOR THEORY, ANALYSIS AND MATHEMATICAL PHYSICS

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ABSTRACTS

List of abstracts

Ahmed AL-RAWASHDEH
(*Jordan University of Science and Technology, Irbid, Jordan*)

Order isomorphisms and projections of $C(X)$

A mapping between projections of C^*-algebras preserving the orthogonality, is called an orthoisomorphism. We define the order-isomorphism mapping on C^*-algebras and using Dye's result, we prove in the case of commutative unital C^*-algebras that the concepts: order-isomorphism and the orthoisomorphism coincide. Also, we define the equipotence relation on the projections of $C(X)$, indeed, new concepts of finiteness are introduced. The classes of projections are represented by constructing a special diagram, we study the relation between the diagram and the topological space X. We prove that an order-isomorphism which preserves the equipotence of projections induces a diagram-isomorphism, also if two diagrams are isomorphic, then the C^*-algebras are isomorphic.

Jussi BEHRNDT
(*Technische Universität Berlin, Germany*)

Spectral properties of selfadjoint elliptic differential operators on bounded domains

In this talk we consider uniformly elliptic second order differential operators on bounded domains and we present a sufficient criterion for selfadjoint and dissipative boundary conditions. The resolvents and spectral properties of these selfadjoint or maximal dissipative elliptic operators can be described with Kreins formula and the Neumann-to-Dirichlet map.

The talk is based on a joint work with Matthias Langer (University of Strathclyde, Glasgow) and Hagen Neidhardt (WIAS, Berlin).

Malcolm BROWN
(*University of Cardiff, United Kingdom*)

A HELP inequality for trees

We show that the HELP inequality may be developed for both discrete and metric trees.

Dmitry CHELKAK
(*St. Petersburg State University, Russian Federation*)

The complete characterization of spectral data for the vector-valued Sturm-Liouville problem

We consider the (self-adjoint) vector-valued Sturm-Liouville operator $Hy = -y'' + Vy$ on $[0,1]$ with Dirichlet boundary conditions, where $V(x)$ is a self-adjoint $N \times N$ matrix-valued potential. We suppose that the mean value V_0 of $V(x)$ is fixed (the unitary transform leads to the diagonal V_0) and the eigenvalues of V_0 are different from each other (the "generic" case).

The spectral data of H consist of

(i) eigenvalues l_m (and their multiplicities k_m); and residues of the (matrix-valued) Weyl–Titchmarsh function. Each residue is a nonnegative self-adjoint $N \times N$ matrix of rank k_m and we consider it as pair of

(ii) orthogonal projector P_m in C^N, $\text{rank}(P_m) = k_m$;

(iii) positive quadratic form g_m in $\text{Ran}(P_m)$.

In "generic" case all sufficiently large eigenvalues are simple (and corresponding g_m can be viewed as positive "normalizing constants"), so we also use the double-indexing (n,j), $j = 1,\ldots,N$, for sufficiently large n.

We give the complete characterization of all spectral data that correspond to the square summable $V(x)$ with the given mean value V_0. The necessary and sufficient conditions are:

(a) asymptotics of eigenvalues in l^2 sense;

(b) asymptotics of normalizing constants in l^2_1 (i.e., l^2 weighted by n^2) sense;

(c) asymptotics of $P_(n,j)$. Their behavior is not trivial and involves two parts:

(c1) $P_(n,j)$ are asymptotically close to the coordinate projectors in l^2 sense;

(c2) sums of $P_(n,j)$ over $j = 1,\ldots,N$ are asymptotically close to the identity in l^2_1 sense (i.e., one order closer than it immediately follows from (c1));

(d) some algebraic restriction on l_m, P_m (but not on g_m) that can be formulated in the following "interpolation-type" way: if $f(z)$ is an entire vector-valued function and $P_m f(l_m) = 0$ for all m, then either $f = 0$ or f increases at least as $\sin z/z$ at infinity.

Asymptotics (a), (b) are similar to the scalar case and their leading terms are given by the Fourier coefficients of the diagonal entries of $V(x)$. Similarly, the leading terms in (c1), (c2) are given by the Fourier coefficients of nondiagonal entries of $V(x)$.

The proof is based on (some essential modification of) Trubowitz's approach to the inverse problems with purely discrete spectrum that doesn't require the explicit proof of the convergence of the reconstruction procedure.

Joint work with Evgeny Korotyaev (Humboldt Universität zu Berlin).

Vsevolod CHERNYSHEV
(*Bauman Moscow State Technical University, Russian Federation*)

Dynamics and statistics of gaussian packets on a geometrical graph

Let us consider a nonstationary Schrödinger equation in which the spatial variable varies on a metric graph. We describe the propagation of gaussian packets on a graph using semiclassical approximation (Maslov's complex germ method). Several examples are treated. Also we consider problems concerned with the statistics of the propagation of quantum packets. These problems turned out to be related to the well-known number-theoretic problem of evaluation of the number of integral points entering an extending polyhedron. Some properties can be easily established in the case of a hyperbolic equation on a graph as well.

Dariusz CICHOŃ
(*Jagiellonian University, Kraków, Poland*)

A class of indeterminate Hamburger moment sequences

This is an account of a joint paper prepared with J. Stochel and F. H. Szafraniec. The aim is to present an example of an indeterminate Hamburger moment sequence which has a continuum of representing measures with closed supports forming a partition of a real line. It is known that such phenomenon takes place for all indeterminate Hamburger moment sequences (as proved e.g. by B. Simon), hitherto there was no (more or less) constructive examples of this type. We also associate the representing measures to extensions of an apropriate symmetric operator discussing prospects for the representing measures to be N-extremal.

Stephen L. CLARK
(*Missouri University of Science and Technology, Rolla, MO, USA*)

Borg–Marchenko-type uniqueness results for CMV operators with Verblunsky coefficients

We discuss basic Weyl-Titchmarsh theory for CMV (Cantero, Moral, and Velazquez) operators with matrix-valued Verblunsky coefficients and local as well as global versions of Borg–Marchenko-type uniqueness theorems for half-lattice and full-lattice CMV operators with matrix-valued Verblunsky coefficients. Half-lattice results are formulated in terms of matrix-valued Weyl–Titchmarsh functions, while full-lattice results involve the diagonal and main off-diagonal Green's matrices.

Monique COMBESCURE
(*Université Claude Bernard, Villeurbanne, France*)

The theory of mutually unbiased bases, and circulant Hadamard matrices

This work is devoted to the theory of mutually unbiased bases, namely of finite dimensional orthonormal bases (dimension d) such that any vector in one base has a scalar product with any vector of an other base of modulus equal to the inverse of the square root of d. We show that the finite Fourier transform plays a crucial role in the solution of this problem, together with a set of circulant Hadamard matrices. The complete solution in the case of prime dimension d is exhibited. In the prime power dimension, the relavant matrices are block circulant with circulant blocks Hadamard matrices.

Maksym DEREVYAGIN
(*Donetsk Institute of Applied Mathematics and Mechanics, Donetsk, Ukraine*)

An operator approach to multipoint Padé approximations

A modification of the famous step-by-step process of solving the Nevanlinna–Pick problems for Nevanlinna functions is presented. The process in question gives rise to three-term recurrence relations with coefficients linearly depending on the spectral parameter. These relations can be rewritten in the matrix form by means of two Jacobi matrices. This form allows to consider multipoint Padé approximants to Nevanlinna functions as m-functions of linear pencils with two finite (truncated) Jacobi matrices. As a result, a convergence theorem for multipoint Padé approximants is proved.

Joint work with Alexei Zhedanov

Jan DEREZIŃSKI
(*University of Warsaw, Poland*)

Excitation spectrum of the Bose gas and superfluidity

I will discuss some facts and conjectures about the joint energy momentum spectrum of many body bosonic Schrödinger operators and their physical significance.

Joanne DOMBROWSKI
(*Wright State University, OH, United States*)

Spectral Properties of Unbounded Jacobi Matrices

This talk will present results on the existence of spectral gaps, and possible eigenvalues within spectral gaps, for Jacobi Matrices obtained by various types of oscillating perturbations of Jacobi Matrices with smooth weights. Related results on absolute continuity will also be presented.

Rupert FRANK
(*Royal Institute of Technology, Stockholm, Sweden*)

Non-linear ground state representations and sharp Hardy inequalities

We derive a ground state representation for arbitrary differential operators of fractional order. In the non-linear case, this representation holds in the form of an inequality. This allows us to find sharp constants in Hardy inequalities, to obtain remainder terms and to prove optimal Sobolev embedding theorems. We also discuss applications of the ground state representation to the study of Lieb–Thirring inequalities.

Parts of the talk are based on joint works with E. Lieb and R. Seiringer and with B. Simon and T. Weidl.

Vladimir GEORGESCU
(*University of Cergy-Pontoise 2, France*)

Spectral analysis of many-body systems

We introduce a general class of many channel systems and discuss their spectral properties. Our results concern the essential spectrum (HVZ type theorems), the Mourre estimate, and the absence of singular continuous spectrum. This covers many-body systems, i.e. systems of particles interacting between themselves through k-body forces (arbitrary k) but also subject to interactions consisting of creation-annihilation processes which allow the system to make transitions between states with different numbers of particles.

Abdelouahab KADEM
(*University of Sétif, Algeria*)

Spectral method for the fractional radiative transport equation in one-dimensional case

In our recent work we have presented a new approximation for the solving the one dimensional transport equation analytically, where we are using the Chebyshev polynomials combined with the Sumudu transform. The approach is based on expansion of the angular flux in a truncated series of Chebyshev polynomials in the angular variable. By replacing this development in the transport equation, this which will result a first-order linear differential system is solved for the spatial function coefficients by application of the Sumudu transform technique. The inversion of the transformed coefficients is obtained using Trzaska's method and the Heaviside expansion technique.

In order to obtain a new rule for the calculate the matrix exponential which arises in the formal solution of algebraic systems of differential equation without using the Sumudu transform and Trzaska's method for solving the first-order fractional linear

differential system, we fractionalize the one dimensional integro-differential equation and we try to convert it into a system of fractional differential equation.

Yulia KARPESHINA
(*University of Alabama at Birmingham, United States*)

Zero-range model of *p*-scattering by a potential well

The method of point-range potentials is widely used in quantum mechanics. This method consists of replacing a deep potential well of a small radius by a boundary condition at the point of the centrum of the well . However, in passing to the limit from a deep and narrow potential well to the zero-range model, information, concerning p-scattering and scatterings of higher orders, gets lost. The zero-range model describes approximately only bound states and s-scatterings.

The principal mathematical difficulty, which arises in the mathematical construction of a zero-range potential, considering p-scattering, is that p-scattering waves have a square nonintegrable singularity at the point where the well is located. That is why it is not possible to construct directly an energy operator in $L_2(R^3)$.

We construct the energy operator being selfadjoint in some Hilbert space, which naturally arises from the problem and includes $L_2(R^3)$. We construct the complete system of eigenfunctions in this space.

Witold KARWOWSKI
(*Opole University, Poland*)

Laplace operator with perturbation localized on the Cantor set

The problem we discuss is analytical but it has a natural connection with theory of stochastic processes. We consider a quadratic form in the real Hilbert space of square integrable functions of real variable defined as the sum of two forms. The first one is defined by the Laplace operator. The second form is defined by a Borel measure on the tensor product of the real lines off the diagonal and supported by the tensor product of the Cantor sets off the diagonal. There is a natural one-to-one map between the Cantor set and 2-adic unit ball. By this map the mentioned measure is related to a measure supported by tensor product of 2-adic unit balls. The last measure defines a Dirichlet form in the Hilbert space of square integrable functions on 2-adic numbers and hence a random process on the 2-adic unit ball.

We show that the form sum can be extended to a Dirichlet form in the Hilbert space of square integrable functions of real variable. We find a condition (in terms of the 2-adic processes) for the resulting form to be regular. The corresponding stochastic process consist of diffusion on real line and jumps on the Cantor set. The jumps preserve the non-archimedean structure of the 2-adic processes.

Alexander KHEIFETS
(*University of Massachusetts, Lowell, United States*)

Uniqueness of Inverse Scattering Problem for CMV Matrices: Parametric Description of Scattering Functions

A parametric description is given to scattering functions of CMV matrices that have uniqueness property for the inverse scattering problem. The result is formulated in terms terms of Arov-regularity. Bounded invertibility of the Gelfand–Levitan–Marchenko transformation operators is characterized in terms of the Hunt–Muckenhoupt–Wheeden condition. The key tool is recently discovered Perherstofer–Volberg–Yuditskii duality for new scattering models that extend the Faddev–Marchenko model.

Alexander KISELEV
(*St. Petersburg State University, Russian Federation*)

Spectral analysis of Matrix Model operators with almost Hermitian spectrum

We discuss the recent progress in the spectral analysis of non-self-adjoint operators with almost Hermitian spectrum. The simplest setting (although already revealing all the major difficulties to be found in the general case) in which this analysis can be carried out is the Matrix Model case, i.e., a rank two non-self-adjoint perturbation of a self-adjoint operator. In this situation, the analysis is reduced to the study of four scalar analytic functions (matrix elements of the characteristic function, which is in this case a two by two matrix-function). Some recently obtained results will be covered, including the so-called almost-spectral theorem for the operators of this class.

Joint work with Sergey Naboko (St. Petersburg State University).

Evgeny KOROTYAEV
(*Humboldt Universität zu Berlin, Germany*)

Nanoribbons in external electric fields

We consider the Schrödinger operator on nanoribbons (tight-binding models) in an external electric potentials V. The corresponding electric field is perpendicular to the axis of the nanoribbon. If $V = 0$, then the spectrum of the Schrödinger operator consists of two spectral bands and the eigenvalue (with infinite multiplicity, i.e., the flat band) between them. If we switch on an weak electric potential V_0, then we determine the asymptotics of the spectral bands for small fields. In particular, we describe all potentials when the unperturbed eigenvalue remains the flat band and when one becomes the small band of the continuous spectrum.

It is a joint work with Anton Kutsenko, St. Petersburg.

Roger KOUNKOU
(*Dublin, Ireland*)

Stokes' phenomenon and spectral properties of the Sturm-Liouville problem

We consider the general Sturm-Liouville problem, and we are concerned with investigating the existence of connections between certain aspects related to the Stokes' phenomenon (namely Stokes' lines and anti-Stokes' lines) and the spectral properties of the Sturm-Liouville operator.

Helge KRUEGER
(*Rice University, Houston, United States*)

Relative oscillation theory

We present an extension of classical oscillation theory, which works inside the gaps of the essential spectrum.

This is joint work with Gerald Teschl.

Matthias LANGER
(*University of Strathclyde, United Kingdom*)

The HELP inequality on trees

In this talk I will present a generalisation of the HELP (Hardy–Everitt–Littlewood–Polya) inequality to an abstract operator setting and its application to quantum and discrete trees.

Joint work with B. M. Brown and K. M. Schmidt (Cardiff University).

Annemarie LUGER
(*Lund University, Sweden*)

Approximation and associated models

We consider an approximation problem connected with the singular Bessel differential expression and its regularizations.

In particular we present and apply general results about the convergence of corresponding (Titchmarsh–Weyl) functions and associated operator models.

This talk is based on joint work with A. Dijksma and Yu. Shondin.

Maria MALEJKI
(*AGH University of Science and Technology, Kraków, Poland*)

Asymptotics of large eigenvalues for some discrete unbounded Jacobi matrices

This presentation is devoted to asymptotic behaviour for eigenvalues of self-adjoint discrete operators in $l^2(N)$ given by some infinite symmetric Jacobi matrices. We treat a given Jacobi matrix J as an operator in the complex Hilbert space $l^2(N)$. We use finite submatrices of the Jacobi matrix J and eigenvalues of such submatrices to express asymptotic formulas of the point spectrum of J. The approach used to calculate an asymptotic formulas is based on method of diagonalization, Janas and Naboko's lemma and the Rozenbljum theorem.

Victor MIKHAYLOV
(*University of Alaska, Fairbanks, United States*)

The boundary control approach to inverse spectral theory

We establish connections between four approaches to inverse spectral problems: the classical Gelfand–Levitan theory, the Simon theory, the approach proposed by Remling, and the Boundary Control method. We show that the Boundary Control approach provides simple and physically motivated proofs of the central results of other theories. We demonstrate also the connections between the dynamical and spectral data and derive the local version of the classical Gelfand–Levitan equations.

Joint work with S. A. Avdonin (University of Alaska Fairbanks, Department of Mathematics and Statistics).

Marcin MOSZYŃSKI
(*University of Warsaw, Poland*)

On some methods of spectral studies of Jacobi operators

The talk is devoted to some methods of studying spectral properties of self-adjoint Jacobi operators (absolute continuity, pure pointedness etc. in a region of real line), which have started to be popular recently (e.g., results of J. Janas, S. Naboko, L. Silva, and me). The presented methods include the so-called H-class method, and the Levinson method (i.e., a method based on Levinson type asymptotic studies of solutions of the difference system related to the operator), both of them strongly using the subordination theory for Jacobi operators.

The description of the methods will be illustrated by some concrete examples.

Wojciech MOTYKA
(*Polish Academy of Sciences, Kraków, Poland*)

The asymptotic analysis of a class of selfadjoint Jacobi matrices, double root case

We present two methods of finding asymptotic formulas for a basis of solutions of the generalized eigenequation of Jacobi operators in the double root case. An application to spectral analysis of Jacobi operators is also sketched.

Yaroslav MYKYTYUK
(*Lviv Ivan Franko National University, Ukraine*)

What spectra can non-self-adjoint Sturm–Liouville operators have?

We address the question, what spectra non-self-adjoint Sturm–Liouville operators on a finite interval can have. Although in the self-adjoint case the question is completely understood, the non-self-adjoint case is more difficult due to possibility of nonsimple and/or nonreal eigenvalues. We solve the inverse spectral problem of reconstructing the complex-valued potential of a Sturm-Liouville operator from two spectra or from a spectrum and the sequence of suitably defined norming constants. We also establish a criterion on solubility of the inverse spectral problem and thus answer the question posed in the title.

The talk is based on a joint work with S. Albeverio (Bonn) and R. Hryniv (Lviv).

Marlena NOWACZYK
(*Lund University, Sweden*)

Quantum graphs and vertex scattering matrices

Differential operators on metric graphs are investigated. It is proven that vertex boundary conditions can be successfully parameterized by the vertex scattering matrix at the energy equal to 1. Two new families of boundary conditions are investigated: hyperplanar Neumann and Dirichlet conditions. The spectral and algebraic multiplicities for energy parameter zero are calculated.

Joint work with Pavel Kurasov.

Leonid PARNOVSKI
(*University College London, United Kingdom*)

Recent results in periodic problems

I plan to discuss recent results in periodic elliptic pseudo-differential operators. I will concentrate on the Bethe–Sommerfeld conjecture and the asymptotic behaviour of the integrated density of states.

Iuliia PASHKOVA
(*Taurida National University, Simferopol, Ukraine*)

Ornstein's inequality for Orlicz spaces for a measurable function on positive semiaxis

Let μ be Lebesgue measure on $[0,+\infty)$, $S(0,\infty)$ be a space of all measurable functions on $[0,+\infty)$, such that a distribution function $n_f\tau = \mu\{t \in (0,\infty),\ |f(t)| > \tau\}$ is not equal to ∞ identically. By $f^*(t)$ we denote the decreasing rearrangement function for measurable function f on $[0,\infty)$. For any function $f \in L_1(0,\infty) + L_\infty(0,\infty)$ we define the Hardy-Littlewood maximal function f^{**}.

Let $T:\ L_1 + L_\infty \to L_1 + L_\infty$ be a positive contraction ($T \in PC$). Denote by

$$B_T(f) = \sup_n \frac{1}{n} \sum_{k=0}^{n-1} T^k |f|.$$

The Ornstein's inequality has the form

$$(OI) \qquad \frac{1}{2t} \int\limits_{\{|f|>t\}} |f|\, d\mu \le \mu\{B_T f > t\}.$$

Let $\Phi(u)$ be a Orlicz function. The function $\Phi_1(x) = x \cdot \int\limits_0^x \frac{\Phi'(u)}{u}\, du$ is also a Orlicz function. Spaces L_Φ and L_{Φ_1} are corresponding Orlicz spaces.

Theorem. *Let $T \in PC$ such that the Ornstein's inequality is realized, and $f \in L_1 + L_\infty,\ \ f^*(+\infty) = 0$. If $B_T f \in L_\Phi$, then $f \in L_{\Phi_1}$.*

Boris S. PAVLOV
(*St. Petersburg University, Russia, and University of Auckland, New Zealand*)

Compensation of singularities in Krein formula

The non-homogeneous equation with self-adjoint operators H_0, V in Hilbert space E

$$[H_0 - V - \lambda I]\, u = f \qquad (1)$$

is conveniently reduced to the corresponding Lippmann–Schwinger equation

$$u = [H_0 - \lambda I]^{-1} f + [H_0 - \lambda I]^{-1} V u := R^0_\lambda f + R^0_\lambda V u \tag{2}$$

Formal solution of this equation leads to the dual problem on localization of zeros of a relevant operator-function- the denominator $\mathcal{D} = \Theta_V - |V|^{1/2} R^0_\lambda |V|^{1/2}$ of the corresponding Krein–Naimark–Birman–Schwinger (KNBS) formula, see [2], [3], [4], [5], [6] for the solution of (1)

$$u = \left[R^0_\lambda + R^0_\lambda |V|^{1/2} \frac{I_V}{\Theta_V - |V|^{1/2} R^0_\lambda |V|^{1/2}} |V|^{1/2} R^0_\lambda \right] f, \tag{3}$$

or the corresponding formula for the perturbed resolvent:

$$R^V_\lambda := [H_0 - V - \lambda I]^{-1} = R^0_\lambda + R^0_\lambda |V|^{1/2} \frac{I_V}{\Theta_V - |V|^{1/2} R^0_\lambda |V|^{1/2}} |V|^{1/2} R^0_\lambda \tag{4}$$

which we refer to as *general Krein formula*. Both summands in the right side of (4) are singular at the eigenvalues of the non-perturbed operator H_0. Though it was commonly expected that the singularities of both summands in the original Krein formula compensate each other in case of isolated singularities, but the proof of the fact for a group of isolated eigenvalues requires a deeper insight into the problem and revealing connections with the Schmidt perturbation procedure, see [1]. We modify the Schmidt perturbation procedure extending the tight bounds of convergence

$$2\|V\| \ < \min_{\lambda_s \neq \lambda_0} |\lambda_s - \lambda_0|, \tag{5}$$

of the standard analytic perturbation procedure for an individual eigenvalue by considering the simultaneous perturbation of a group of eigenvalues, situated on a certain "essential" spectral interval Δ as zeros of the corresponding denominator $\mathbf{D}$. We apply the modified analytic perturbation procedure to the perturbation of terms [7], to the perturbation of the Dirichlet-to-Neumann map and to the Scattering matrix of a Quantum Network [8] and to fitting of the zero-range solvable model of a tectonic plate under a pointwise boundary stress, see [9].

[1] R. Newton, *Scattering theory of waves and particles*, Dover Publications, Inc., Mineola, NY, 2002, 745 pp.

[2] M. Krein, *Concerning the resolvents of an Hermitian operator with deficiency index* (m, m), Doklady AN USSR 52 (1946), 651.

[3] M. Naimark, *Self-adjoint extensions of the second kind of a symmetric operators*, Bull. AN USSR, Ser. Math. **4**, 53 (1946).

[4] J. Schwinger, *On bound states of a given potential*, Proc. Nat. Acad. Sci. USA 47 (1961), 122–129.

[5] M. Sh. Birman, *On the spectrum of singular boundary-value problems*, Mat. Sbornik (N.S.) 55 (1961), 125–174. English transl. in Amer. Math. Soc. Transl. 53 (1966), 23–80.

[6] B. Simon, *Trace Ideals and Their Applications*, London Math. Soc. Lecture Notes Series 35. Cambridge Univ. Press, Cambridge, NY, 1979, 134 p.

[7] C. Zener, *Non-adiabatic crossing of energy-levels*, Proc. R. Soc. A 137 (1932), 696.

[8] A. Mikhailova, B. Pavlov and L. Prokhorov, *Intermediate hamiltonian via Glazman's splitting and analytic perturbation for meromorphic matrix functions*, Mathematische Nachrichten 280 (2007), 1376–1416.

[9] L. Petrova, B. Pavlov, *Tectonic plate under a localized boundary stress: fitting of a zero-range solvable model*, Journal of Physics A 41 (2008), 085206 (15 pp.).

Irina PCHELINTSEVA
(*University College London, United Kingdom*)

Spectral properties of a class of Jacobi matrices

We study a Jacobi matrices whose entries are powers with a shift, and discuss their spectral properties, we investigate the behavior of the discrete spectrum as a function of the shift.

Olaf POST
(*Humboldt Universität zu Berlin, Germany*)

Convergence of the spectrum of the Hodge-Laplacian on graph-like manifolds

We consider Laplacians on a family of manifolds converging to a metric graph. In particular, we show that under suitable conditions, the spectrum of 1-forms (vector fields) converges to a natural operator on the metric graph (dual to the Kirchhoff one), the case of 0-forms was already treated by former works.

Roman ROMANOV
(*St. Petersburg State University, Russian Federation*)

Absolutely continuous spectrum of non-selfadjoint operators beyond the trace scattering theory

The talk will review the current situation in the theory of non-selfadjoint operators which cannot be treated by means of the nuclear scattering theory. A particular attention is paid to differential operators and, on the abstract level, to subtle distinctions between different notions of absolute continuity in the non-selfadjoint case.

Grigori ROZENBLIOUM
(*Chalmers University of Technology, Gothenburg, Sweden*)

Spectral properties of the Landau Hamiltonian and related Bargmann–Toeplitz operators

The Landau Hamiltonian describes the charged quantum particle (an electron) confined to a plane and moving under the action of a uniform magnetic field orthogonal to the plane. This is one of earliest explicitly solvable quantum models. The spectrum consists of infinitely degenerate eigenvalues placed at the points of an arithmetic progression, called Landau levels. A natural question arises, what happens with the spectrum when some kind of external perturbation appears. Generally, one should expect that Landau levels will split into clusters of eigenvalues, with Landau levels being their limit points. The general problem consists in finding quantitative characteristics of such clusters. Different kinds of such perturbations have been already considered by Raykov, Pushnitsky, Hempel, and others, including the speaker. Some new effects, not present in other types of spectral problems, were discovered. The new results to be presented in the talk concern two types of perturbations not studied previously.

First, it is introducing an obstacle, a compact region inaccessible for electrons. Here we find the rate of convergence the eigenvalues in clusters to their limits. Another type concerns the perturbation by an electric potential. Here we study the asymptotic distribution of eigenvalues in the cluster as the perturbation expands.

All existing results on perturbations of the spectrum of the Landau Hamiltonian are essentially based upon the spectral analysis of Toeplitz type operators in the Fock-Bargmann space. Some new spectral properties of such operators are established in relation with the problems considered in the talk. In particular, a complete answer to the long-standing question about finite rank Toeplitz operators is given.

The results were obtained jointly with A. Pushnitski and with A. Sobolev.

[1] Alexander Pushnitski, Grigori Rozenblum, *Eigenvalue clusters of the Landau Hamiltonian in the exterior of a compact domain*, arXiv:0707.4297.

[2] Grigori Rozenblum, Alexander V. Sobolev, *Discrete spectrum distribution of the Landau Operator Perturbed by an Expanding Electric Potential*, arXiv:0711.2158.

Inna SADOVNICHAYA
(*Moscow Lomonosov State University, Russian Federation*)

Uniconvergence theorems for Sturm–Liouville operators with distribution potentials

In the talk we deal with Sturm–Liouville operators with Dirichlet boundary conditions on the finite interval, where potential belongs to the scale of Sobolev spaces. We obtain uniconvergence theorems for such type operators for L-two functions. Moreover, we obtain a uniform estimate of the uniconvergence rate.

Artem SAVCHUK
(*Moscow Lomonosov State University, Russian Federation*)

Uniform stability theorems for inverse Sturm–Liouville problems

In the talk we deal with Sturm–Liouville operators with Dirichlet and Neumann–Dirichlet boundary conditions on the finite interval, where potential belongs to the scale of Sobolev spaces. We give complete characterizations of the sequences of eigenvalues of such operators, i.e. asymptotic formulae and interlacing condition (for real-valued potential). We also solve an inverse spectral problem of recovering of a real potential by two real sequences, which obey the same asymptotic and interlace. Moreover, we obtain a Lipschitz type uniform stability theorems.

Luis Octavio SILVA PEREYRA
(*Universidad Nacional Autónoma de México, Mexico*)

The two spectra inverse problem for Jacobi matrices: Necessary and sufficient conditions

In this talk we present necessary and sufficient conditions for two real sequences to be the spectra of:

a) two different rank-one perturbations of a self-adjoint Jacobi operator with discrete spectrum.
b) two different self-adjoint extensions of a Jacobi operator in the limit circle case.

Joint work with Ricardo Weder.

Sergey SIMONOV
(*St. Petersburg State University, Russian Federation*)

Singularities of the spectral density for Schrödinger operator with Wigner–von Neumann potential

Summable perturbations of the Schrödinger operator with Wigner–von Neumann potential are considered. We are interested in the behavior of the spectral density near the critical point where an eigenvalue embedded into the absolutely continuous spectrum can appear.

Alexey TIKHONOV
(*Taurida National University, Simferopol, Ukraine*)

Inverse problem for transfer function of conservative curved system

We study possibility to recover characteristic function $\Theta(z)$ for a given transfer function $\Upsilon(z) = M(T-z)^{-1}N$ of conservative curved system $\Sigma = (T, M, N)$ over multiply connected domain. The following results are established.

1) Uniqueness theorem for $\Theta(z)$ assuming $\Upsilon(z)$ has a scalar multiple.
2) Procedure of recovery employing generalization of Lax–Phillips–Adamyan–Arov scattering.
3) Constructing functional model for a given system.
4) Test a triple (T, M, N) to be a system.

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Joachim TOFT
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Wave-front sets of Fourier Lebesgue types

Roughly speaking, a wave-front set $WF_*(f)$ of the distribution f with respect to "something", give information *where* the distribution f has singularities with respect to this "something", as well as *what directions* in these points of singularities, the singularities propagates.

In this talk we introduce wave-front sets $WF_*(f) = WF_{\mathscr{F}L^q_{(\omega)}}(f)$ of the distribution f with respect to (weighted) Fourier Lebesgue spaces $\mathscr{F}L^q_{(\omega)}$, where ω is an appropriate weight function, and give links on how such wave-front sets can be used to get information on micro-local properties. An advantage with such wave-front sets comparing to the wave-front set of smoothness (i.e. wave-front sets with respect to C^∞), is that we may examine micro-local properties which are more close to differentiability up to a certain degree, instead of infinitely differentiability.

An important property that a useful type of wave-front set should fulfil is

$$WF_*(a(x,D)f) \subseteq WF_*(f) \subseteq WF_*(a(x,D)f) \bigcup \mathrm{Char}(a),$$

when $a(x,D)$ is a partial differential operator with smooth coefficients and $\mathrm{Char}(a)$ is the set of characteristics for a with respect to ω. It is well-known that these embeddings hold for the usual wave-front sets. We give motivations that they also hold for wave-front sets with respect to Fourier Lebesgue spaces. Moreover, the definition of $\mathrm{Char}(a)$ depends on the weight ω and is smaller than the usual definition of characteristic sets. For example, by an appropriate approach, one obtains that $\mathrm{Char}(a) = \emptyset$ when $a(x,D)$ is hypoelliptic. In particular, for such operators we have

$$WF_*(a(x,D)f) = WF_*(f).$$

Finally we remark that one may get the "usual" wave-front set (with respect to smoothness) due to Hörmander by considering sequences of wave-front sets of Fourier Lebesgue types. In particular it follows that the usual wave-front sets can be obtained by our wave-front sets.

This talk is based on results in collaborations with Stevan Pilipovic and Nenad Teofanov.

Françoise TRUC
(*University of Grenoble, St. Martin d'Heres, France*)

Spectral asymptotics for magnetic bottles on the hyperbolic half-plane

We consider a magnetic Laplacian on the hyperbolic half-plane, when the magnetic field $B = dA$ is infinite at the infinity such that the operator has pure discrete spectrum. We give the asymptotic behaviour of the counting function of the eigenvalues.

Joint work with A. Morame.

Carsten TRUNK
(*Technische Universität Ilmenau, Germany*)

On accumulation of non-real eigenvalues of indefinite Sturm–Liouville operators

We consider singular Sturm–Liouville operators with an indefinite weight. We discuss the location of the spectrum and the sign types of the spectrum.

Special attention is paid to the differential expression

$$\operatorname{sgn}(\cdot)(-\frac{d^2}{dx^2} + q)$$

on the real axis with the indefinite weight $x \mapsto \operatorname{sgn} x$. It is assumed that q is a real-valued, locally integrable potential and that $\pm\infty$ are in limit point case.

For a class of potentials with $\lim_{|x|\to\infty} q(x) = 0$, we discuss the accumulation of non-real eigenvalues of to zero

The talk is based on joint works with Q. Katatbeh (Irbid, Jordan) and J. Behrndt (Berlin, Germany).

Ivan VESELIC
(*Technische Universität Chemnitz, Germany*)

Spectral and geometric properties of percolation on general graphs

We discuss percolation on general quasi-transitive graphs. The first result establishes the sharpness of the phase transition, extendig earlier results by Menshikov and

Aizenman & Barsky. Then we consider the random Laplacian generated by percolation on a amenable Cayley graph. We analyse the low energy behaviour of the integrated density of states (IDS), relating it among others to so called Lifschitz tails. These two mentioned results are joint with Tonci Antunovic.

If time permits we discuss the structure of the set of jumps of the IDS. We give a characterisation of this set. Finally, we prove uniform convergence of finite volume approximands to the IDS. This result is a special case of a general theorem for ergodic Hamiltonians on discrete structures, which is joint work with Daniel Lenz.

Kazuo WATANABE
(*Gakushuin University, Tokyo, Japan*)

Wave equation on the strip domain

We consider the wave equation on the strip domain and study the limiting absorption principle for the differential operator.

Joint work with M. Kadowaki and H. Nakazawa.

Ian Geoffrey WOOD
(*Aberystwyth University, United Kingdom*)

Spectral theory of non-selfadjoint operators via boundary triplets

We consider generalisations of Dirichlet-to-Neumann operators (M-functions) for non-selfadjoint operators in an abstract setting. The aim is to use the theory of boundary triplets for non-selfadjoint operators to generalise as much as possible of the classical theory of the Weyl-Titchmarsh m-function for the Sturm Liouville problem on a half-line to a more general setting which includes elliptic PDEs. We prove that in our setting isolated eigenvalues of an operator correspond to poles of the associated M-function and discuss problems and partial results on identifying the essential spectrum via the M-function.

Lech ZIELIŃSKI
(*Université du Littoral, Calais, France*)

Eigenvalue asymptotics of some classes of Jacobi matrices

The estimates of the asymptotic behaviour of large eigenvalues are considered for some classes of Jacobi matrices with unbounded entries. The particular interest is given to operators of the type considered in Jaynes–Cummings models.

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